21st Century Orchestral Instruments

Acoustic instruments for alternative tuning systems

Alternative Tuning Projects
in association with
London Guildhall University

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This discussion paper is part of a pilot feasibility study undertaken by Alternative Tuning Projects to establish a Centre in the UK for new acoustic musical instruments - particularly orchestral instruments for alternative tuning systems. The concept of establishing a collaborative 'network' based around one or more institutional nuclei each contributing to this evolved from the study. This was funded by the Arts Council of England (ACE) under the Arts for Everyone (A4E) scheme, and Alternative Tuning Projects is grateful to ACE for making this work possible. A further component of the study has been the making of two Renaissance tenor recorders in 19 division equal-temperament, commissioned (with funds from ACE) from Lewis Jones and David Armitage.

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During the 20th Century the primary advances of musical instrument design have been in electronics and computing. These new technologies have had considerable influence in many styles of music, and will increasingly do so in future. Considered as a global industry, digital technology will continue to sustain new developments of electronic instruments, providing new sounds and techniques for musicians working in many areas of music.

However, many composers, performers and listeners today remain passionate about purely acoustic instruments. This is in part due to basic qualitative differences between the physical and electronic production and emission of sound. It is commonly thought that little can be done to improve on existing models of mainstream orchestral instruments, in which instrument makers over the centuries have achieved great refinement. Since the mid 19th Century, no fundamentally new instrumental designs (excepting a few percussion instruments), and few radical instrumental adaptations have been adopted by the classical orchestra.

Drawing upon alternative approaches in instrument technology, acoustics and tuning theory, this paper shows that it would be both possible and advantageous to create '21st Century versions' of existing orchestral instruments. The purpose of this is not to replace existing instruments, but to extend the mainstream acoustic instrumentarium to reflect and stimulate new musical directions. It is argued that there would be great value in achieving this in parallel with the evolution of digital and electroacoustics. The central thesis of the paper is that a new collaborative framework and resources are necessary if radical developments of mainstream acoustic instruments are to be attained. A Centre and network dedicated to creating such instruments are therefore proposed.

The paper focuses on new instruments for 'alternative tuning systems' (ATS). A considerable proportion of new music explores ATS, and new instruments are needed for its dependable realisation - both now and in future. ATS are one amongst many aspects of recent classical (and popular) music, but have special implications for acoustic instruments. In particular, ATS bring compelling reasons for creating new versions of conventional instruments in a collaborative, long-term project that does not treat individual instruments in isolation. Moreover, the performance of music employing ATS is generally limited to contemporary music specialists - this need not be the case with new instruments. The latter would bring exciting new opportunities to soloists and specialist contemporary ensembles, as well as to orchestral and chamber music; new forms of mixed (acoustic/electroacoustic) work are also suggested; moreover, new acoustic instruments would certainly be taken up in popular and alternative musics.

Expressions of interest, in partnership, collaboration or support, are invited from research institutes, university departments, instrument manufacturers and builders, composers, performers, technologists, researchers in acoustics and psychoacoustics, music theorists and others. Responses are particularly invited regarding the following issues -

- **the foundation of a collaborative project, network and Centre;**
- **preferred alternative tuning system(s) for new acoustic orchestral instruments - why certain system(s) might be preferred and are also practical;**
- **the design and manufacture of new instruments;**
- **the performance of alternative tuning systems in general, and interest in performing works with new instruments in particular;**
- **funding and other support for this venture.**

CONTENTS

Prologue

This discussion paper proposes a long-term project to build conventional orchestral instruments, and perhaps completely new instruments, specifically designed or adapted for one or more *alternative tuning systems*.[1] The

[1] The term ‘microtonality’, which might seem to imply the use of very small adjustments of intonation, has in general been avoided. With the exception of instruments of fixed intonation, very fine tuning occurs as a matter of course in almost all music, especially in

first objective is to establish a permanent Centre from which to co-ordinate this activity. The paper brings together the reasons for initiating the project, which suggest professional and international collaboration. The reasons are ultimately musical, but are also argued in terms of new strategies of research, collaboration and funding. The aim is the professional realisation of music composed in alternative tuning systems, radical and otherwise, using specially built new instruments. It is envisaged that this would contribute to the general development of acoustic instruments in the 21st Century, and would be of value in all musical genres - classical and avant-garde, popular and alternative.

Such a proposal raises many practical questions:

- How many composers are (or will be) interested in composing in an alternative tuning system (or systems), and for appropriate new instruments?
- Are new instruments really needed - or can music composed in various systems be realised adequately on *existing* acoustic instruments? Or be realised with electronics?
- Is it feasible to build effective new acoustic instruments for new tuning systems?
- How much will new instruments cost, and who will build them?
- Will performers be interested in playing these new instruments? What special training will they need for learning to play them? How long will it take?
- ‘Quarter-tones’, in various guises, have been adopted amongst much of the European avant-garde almost as a new ‘standard’ system of tuning. Is this the best possible choice, or simply a practical compromise? What other emerging ‘standards’ are there?
- Is it in fact possible or desirable to agree a ‘standard’ alternative tuning system to sustain many musical aesthetics, in a similar way that (in various forms) twelve division equal-temperament has done for many years? Or would investing in a single system result in an unwelcome bias towards that particular system?
- How can we choose (and do we need to choose) from the many possible alternatives? Are there perhaps just a few outstanding systems in which a majority of composers would prefer to compose?
- Which tuning systems would best justify substantial investment in new instruments? What criteria are important for evaluating their advantages and disadvantages?
- Which new instrumental technologies would prove most successful for new instruments?
- For composers and performers who use electronics to aid the realisation of acoustic music in ATS, MIDI can be useful but is also awkward. What can be done about this?
- What agreement can be found for appropriate new notational standards?

These are just a few of the many controversial issues, broadly expressed, that the subject of new instruments for alternative tuning systems raises. The discussion paper therefore has two main purposes. The first is to stimulate an international debate, amongst acousticians, instrument makers, performers, composers, mathematicians and technologists, through which some firm answers to these questions might emerge. The second is to gain consensus from the widest range of informed musicians that new acoustic instruments are indeed necessary and will continue be so in the 21st Century. This would support the establishment a Centre, and the funding needed to research and commission new instruments, either ‘in house’, or from specialist manufacturers.

Project Strategy

Stage 1

As a result of the association of ***Alternative Tuning Projects*** with the ***Sir John Cass Department of Design and Technology*** at ***London Guildhall University***, a ***Centre for New Musical Instruments*** has been proposed, broadly along the lines discussed in this paper. This proposal is at an early stage of development. The Centre will be based at LGU; it is hoped that a number of other institutions, based in the UK and abroad, will together create a network, each forming a collaborative nucleus for the project.

many kinds of tonal music. Since this is not ordinarily what we are referring to when we use the phrase ‘microtonal music’, I prefer to talk about ‘alternative tuning systems’ or ‘ATS’, as explained below.

The key idea of the project is, at least initially, to focus attention on the building of new (and new versions of) mainstream acoustic instruments. As listed on the facing page, this raises many controversial issues. The discussion paper is intended to stimulate an international discussion of these, and especially to open out this discussion to instrument makers and manufacturers. The central issues are:

- *the collective need for new acoustic instruments in one or more alternative tuning systems*;
- *the value of a project dedicated to new developments of acoustic instrumental technology*;
- *methodologies for deciding which ATS would justify large-scale investment*;
- *the establishment of a single (or multiple) provisional ATS standard(s)*;
- *the relevance and interdependence of a wide range of research topics*;
- *the viability and appropriateness of new instrumental design technologies*;
- *the potential for international collaboration in the field*.

It is hoped this will attract not only composers but also performers, instrument designers, builders and manufacturers, acousticians, mathematicians and technologists - all of whom might ultimately collaborate on this venture.

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Stage 2

It is hoped that the *Pilot Study Report* will provide strong support for applications to fund a Centre. In the first instance it is envisaged that the Centre will co-ordinate a group of specialists - composers, performers, instrument builders, etc. - based in the UK and abroad. In the latter case collaboration would be via the World Wide Web and through programmes of exchange. An initial programme of research would include:

- *a methodology for choosing a provisional alternative tuning system standard (or standards);*
- *research into instrument design for alternative tuning systems, and prototyping;*
- *the formation of a creative link between the Centre and commercial instrument manufacturers;*
- *the building of new acoustic instruments, for one or more tuning systems.*

Long-term goals might include:

- *a programme of research in all aspects of music relating to alternative tuning systems;*
- *a permanent training Centre with appropriate facilities for performers and composers;*
- *equipping ensembles with new instruments and techniques;*
- *new standards of notation and performance in alternative tuning systems;*
- *an electronic studio for composition and research;*
- *a permanent residence, and full-time staff.*

1. *Overview*

Why 21st Century Orchestral Instruments?

Broadly speaking, there are five predominant approaches to musical instrument design, development and performance today. These are:

- refinements of conventional acoustic instruments and performance techniques;
- 'extended' or unconventional playing techniques for conventional instruments;
- the design and manufacture of original or unorthodox instruments (acoustic and semi-acoustic);
- the development of electronic and electroacoustic instruments and devices;
- the development of 'acoustic-electronic' adaptations of conventional instruments.

A further movement is conspicuously less widespread:

- the radical development of mainstream 'orchestral' instruments in purely acoustic form.

This discussion paper brings together developments and conjectures on the latter topic, especially in regard to designing or adapting orchestral instruments for alternative tuning systems. It is argued that there are particularly compelling reasons for creating such instruments. While there are many other areas of instrumental research with differing musical aims which are of equal value, it is likely that they complement the issues raised here rather than contradict them.[2]

'Alternative tuning systems' (ATS) are here defined as tuning systems which are intentionally specified and heard as differing from twelve division equal-temperament (12-ET). Examples of ATS are: various forms of Meantone and Well-Temperament; the Javanese slendro and Chinese pentatonic scales; Equal-Temperaments such as 9-ET, 19-ET, 24-ET ('quarter-tones'), 31-ET, 36-ET ('sixth-tones') etc.; various systems of Just Intonation - such as Harry Partch's 43-division system, or Erv Wilson's 'non-centric' JI systems; scales which do not repeat at the octave ('non-octave' scales); and many others. The immense variety of alternative systems is not discussed here - for the most part reference is made to the 'mainstream' alternatives most commonly used or referred to in recent Western music and theory. It is important to stress that no alternative system is here considered as in some way derivative of or subsidiary to 12-ET - the almost ubiquitous tuning system of Western classical and popular music.

It is assumed that the mainstream classical instrumentarium includes the voice, flutes, oboe and Cor Anglais, clarinets, bassoons, saxophones, horns, trumpets, trombones, tuba, harp, guitar, piano, organ, pitched percussion and strings. These instruments do not belong exclusively to the orchestra, and (excepting the voice) new versions of them might be useful in a very wide range of musical genres and instrumental combinations. Concomitantly, the classical instrumentarium exerts an influence over the nature of classical and other musics today, and yet is also the locus of considerable resistance to radical design innovation. Depending on one's aesthetic point of view, these reasons alone may suggest the value of 'modernising' orchestral instruments.[3]

Alternative tuning systems are now an important area of compositional interest, and one which will certainly increase in future. It is argued below that specifically designed new instruments would greatly aid (and are in

[2] There would be great value in establishing an independent Centre, in the UK or elsewhere, not only for new orchestral instruments but for new musical instruments in general. For example, a considerable proportion of work on new instruments today concentrates on experimental instruments which are unrelated to the orchestral mainstream. Makers and inventors of unorthodox instruments could be considerably encouraged by the existence of such a Centre. For a fascinating and accessible survey of experimental instruments see: Bart Hopkin, *Gravichords, Whirlies and Pyrophones: Experimental Musical Instruments* (CD and book), Ellipsis Arts, 1996. Some well known European experimental instrument makers not included there are Hugh Davies, Max Eastley, Hans-Karsten Raecke and David Sawyer. But see also the journal/newsletter *Experimental Musical Instruments,* which is produced by Bart Hopkin and available from: EMI, Box 784, Nicasio, CA 94946, USA, (or: http://www.thecombine.com/emi/).

[3] This paper discusses developments of the Western classical instrumentarium. It is taken for granted that there is no *a priori* reason why instruments from other traditions could not be developed in this context, but there may be acoustic reasons why the latter is not as simple as it first appears. The relationship between the design of an instrument and the sounds that can be made with it is fundamental in this paper, and the focus on Western instruments is not intended to be narrowly eurocentric.

many cases essential to) the dependable realisation of music composed in ATS. Existing orchestral instruments (which are generally optimised for 12-ET) are successful and their design is (within limits) largely understood. It is therefore logical to begin with new versions of these rather than completely new instruments,[4] although, as described later, the modifications required may be quite radical.

There are, however, three overriding factors which are decisive in choosing to focus on the orchestral mainstream. (Each of these initial topics is discussed in more detail later on).

(1) Standardisation: Currently there is no 'standard alternative tuning system' (excepting the emergence of quarter-tones as an alternative standard, at least in Europe). There is therefore little immediate incentive for instrument builders to fashion an instrument for an ATS. There are few existing instruments built specially for ATS with which a new instrument might combine in ensemble; [5] nor is there a fully established performance practice.[6] Similarly, although a one-off instrument might be much used (if it worked well for a specialist player), it would be extremely unlikely to be commercially successful in the absence of an agreed standard. The problem is acute in the case of orchestral instruments because of relatively high development costs, and because the tone quality, responsiveness and mechanism of new instruments will inevitably be compared to existing instruments. The latter are the product of many years of refinement, and without an alternative standard or standards, comparable refinements are unlikely to occur for new instruments.

(2)'*Instrumental harmony*': The design of an acoustic instrument, its characteristic timbre, and the tuning system for which it is optimised, should be considered inextricably interrelated. The elaboration of this statement is one of the major themes of this paper. The crucial point is that, given our current knowledge of the interrelationship between instrument design, tuning and timbre, it makes little sense to redesign in isolation a single (orchestral) instrument for an ATS. This is because a new scale (a chosen ATS) may be practical for one instrument but not for another; or the scale may be effective for the timbre of one instrument but not for another, or not for their combination; similarly, it is likely that effective new timbres and inharmonic characteristics are possible for some instruments but not for others,[7] and so on. Most importantly, the very notion of an alternative tuning system is tied to questions about *harmony*, understood in its broadest sense - that is, the way in which *pitched sounds of specific timbre or timbres interact*.

(3) Economics: The economics of building a *solitary* instance of a new version of an orchestral instrument (for an ATS) are not viable from either commercial and communal points of view. Very few individuals or companies could, at present, conceive of making a profit from, say, a 'quarter-tone oboe'; and it hardly needs saying that it is difficult to find public funding to build an instrument which would 'normally' be out of tune with all others. What is needed to kick-start the process is funding which will sponsor research into a *combined system of tuning and new instruments*. Clearly, such a project must be realised in stages, and will demand extensive theoretical work, prototyping, and finishing. From rough estimates, however, its cost would seem to be comparable with that of other large-scale arts ventures of similar importance. Moreover, there is already much relevant work in progress - but little in the way of institutional or project-based focus for these developments.

The value, therefore, of agreeing a *provisional standard alternative tuning system* (or systems) would be very considerable, both from the point of view of instrument manufacture as well as of composition and performance. However, from composers' perspectives, the idea of a *single* standard is controversial, and

[4] It is in not intended that acoustic instruments should only be developed for ATS - however, there are special reasons for developing instruments for ATS, and the means to achieve other kinds of adaptations are not addressed in this paper.

[5] To my knowledge the Osten-Brannen Kingma quarter-tone concert flute is the only commercially available, professional orchestral instrument which is specifically manufactured to achieve an ATS. This uses a mechanical adaptation of the Böhm system to provide a 24-ET gamut, thus including and extending twelve division equal-temperament. We may assume this flute would not be available commercially were it not for a considerable commitment to quarter-tones.

[6] The use of extended techniques to perform quarter-tones and other ATS is of course becoming well established, but as yet remains the province of specialists.

[7] I am greatly indebted to William Sethares' writings and advice in this paper. His revelatory analysis of the relation between tuning and timbre, in particular developing Plomp and Levelt's sensory consonance theory, is lucidly explained in an indispensable recent book: William A. Sethares, *Tuning, Timbre, Spectrum, Scale*, Springer Verlag, 1997. Sethares has suggested the creation of acoustic instruments with *inharmonic* timbres which are deliberately 'related' to, and consonant in, non-standard tuning systems. This is discussed more fully below.

perhaps premature. For this reason, the idea of designing acoustic instruments specifically to realise *multiple* tuning systems is worthy of serious investigation. But in either case these radical developments cannot be led by compositional or instrumental progress alone, since composers and instrument makers alike are in a 'catch-22' situation: acoustic new music using radical ATS cannot be ideally realised without new instruments (notwithstanding the use of 'extended techniques' with certain instruments); and instrument makers will not make radically new instruments without very good justification for doing so.

Approaching this project in terms of a complete and comprehensive new system of orchestral instruments may seem overly ambitious, and it might appear to be more realistic to concentrate, for example, on building a single new instrument. Of course, any such project has to progress one step at a time. Yet it is commonly agreed that the expenditure required to research and build even a single new orchestral woodwind or brass instrument of professional quality is difficult to justify without serious examination of all the above issues; and it makes little sense while there is no predominant and agreed alternative tuning standard, and for other practical and musical reasons which will be outlined. In fact, it is argued here that there are compelling reasons why no such standard should emerge without a collective and co-ordinated programme of research which brings the complete equation of "tuning + timbre + instrumental design" into focus.

There is one further reason for addressing this problem at the level of the 'orchestra' which is worth stating immediately. Some soloists and contemporary chamber ensembles perform works using quarter-tones and a limited number of other ATS, each of which require special techniques and a special level of instrumental skill and dedication. However, suppose that a composer were to write a work for chamber orchestra, which in general terms was of moderate difficulty, but instead of using twelve division equal-temperament the composer uses *nine* division equal-temperament. This is a perfectly workable but almost wholly unknown tuning system which has rather austere harmonic implications, but is none the less fascinating and musical. No current-day orchestra could attempt this work; one or two contemporary ensembles might try it out (but there would be a lot of fudging). Yet with acoustic instruments built specially for nine division equal-temperament (9-ET), and an appropriate degree of aural training, within a reasonably short time the work could become accessible to any orchestra or ensemble - as would its new harmonic universe. This example is compelling because it would seem that there are fewer difficulties for building instruments for tuning systems having less than 12 divisions per octave than for those having more.[8]

I am not suggesting we rush into building instruments for 9-ET. The example is merely an illustration of why it seems to me that the typical standard of musicianship of a professional orchestral player is a legitimate standard of excellence beyond which, especially for orchestral music, composers should begin to call for new and special acoustic *instruments* in addition to new and special performance *techniques.*

To summarise the reasons for focusing on orchestral instruments:

- mainstream 'orchestral' instruments are used in many musical styles;
- they are successful, familiar, and their design is largely understood;
- the economics of mass manufacture have indirectly stifled radical innovations of acoustic instruments, and an emphasis on the perfection of orchestral instruments has resulted in resistance to innovations in the mainstream;
- a provisional ATS standard is needed to encourage new activity;
- long-term economic factors favour a rational, collaborative approach;
- building a new instrument in isolation does not make sense in terms of 'instrumental harmony';
- new instrumental *sounds* are as important as new instrumental *scales*: yet, where this is possible, progress would greatly benefit from the *co-development* of both aspects;
- the classical orchestra is unlikely to conquer virtuoso extended techniques in the foreseeable future;
- there would be considerable value in encouraging, as far as possible, acoustic instruments to develop in conjunction with electronics and electroacoustics;

[8] This is not quite as simple as it appears, since in a system such as 9-ET in which there is no good fifth, the intonation of the overblown modes of brass and clarinet-type woodwind would pose problems, although these may not be insuperable. This is discussed further later on.

- new instruments would help the orchestra to keep pace with developments of contemporary classical music, and are also likely to encourage new developments in other areas of music making.

Intonational Characteristics of Pitched Acoustic Instruments

In this discussion it is essential to distinguish between the *intonational* characteristics of different pitched instruments. Broadly speaking, there are three categories - instruments of *fixed*, *fixed-but-variable* and *variable* intonation. Considering Western orchestral instruments, these terms are defined by examples: [9]

- *fixed* piano, harp, vibraphone, xylophone, marimba;
- *fixed-but-variable* woodwinds, valved brasses, guitar;
- *variable* strings, trombone, timpani.

For instruments of fixed intonation, the pitch of a note is not normally variable in performance; for the other categories, pitches may be instinctively and subtly inflected. Fixed-but-variable instruments differ from the variable in the sense that conventional woodwinds and valved brasses are designed to guide the reliable production of a specific scale or system of tuning. Despite the dependence on the embouchure and the ear of the performer, the mechanisms of discrete pitch production in woodwinds and valved brasses, if perhaps less so than the frets of the guitar, strongly influence the system of pitches which comes most naturally to the instrument.

We should also bear in mind a distinction between instruments of variable and less-variable *timbre*. While the characteristic timbre or spectrum of an instrument is related to both the register and the dynamic at which it is played, some instruments allow more flexible and radical timbral variation than others. The relationship between pitch and timbre is extremely important to questions about new tuning systems. Sethares, for example, has suggested that that being '*in-spectrum*' or '*out-of-spectrum*' might be considered equivalent to being 'in-tune' or 'out-of-tune', in terms of what is called 'sensory consonance' (or 'sensory dissonance').[10] Performers instinctively vary timbre for expressive purposes, but to what extent this affects a subtle form of 'intonation' is unclear.

These distinctions are mentioned at the outset because they are of fundamental importance to '*a new system of instruments'*. In this paper, a '*system of (orchestral) instruments*' implies *all* the conventional orchestral instruments. Since it includes instruments of both fixed and non-fixed intonation, this determines some of the most important questions for the discussion. For example - is a there an alternative tuning system which would best encourage the musical integration of instruments of fixed and non-fixed intonation, given that these categories broadly correspond to the distinction between instruments having harmonic and inharmonic spectra? Which system will best encourage new repertoire (solo and mixed) for instruments of fixed intonation? What are the implications of the acoustic and timbral properties of each instrument, such that, considered together, they themselves might 'prefer' to sound in one tuning system rather than another? What are the design limitations of each of these instruments that might make one system possible but another impossible? Could specific developments of acoustic instrumental *timbres* be made to correlate and agree with particular tuning system(s)? Is it feasible, in terms of new orchestral instruments, to deliberately foster what Sethares has called the 'coevolution' of instruments and tuning systems?[11]

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[9] This tripartite distinction is maintained throughout this paper, but it will often be simpler to refer to instruments as being of either *fixed* or *non-fixed* intonation. This is because, for the purpose of discussing the performance of music using alternative tuning systems, instruments of *fixed-but-variable* and *variable* intonation may sometimes be considered equivalent.

[10] William A. Sethares, *Tuning, Timbre, Spectrum, Scale*, Springer Verlag, 1997, p. 276.

[11] Sethares' notion of 'coevolution' is derived from biology: 'suppose that in order to more effectively catch flies, some species of frogs evolve sticky tongues. In order to avoid sticky tongues some species of flies evolve slippery feet. The spectra of instruments and tunings may have similarly coevolved.' Sethares, op. cit., p. 273.

Twelve Division Equal-Temperament - in Theory and Practice

In this discussion, '*alternative tuning systems*' are to be understood as alternatives to 'twelve division equal-temperament'. From the outset, therefore, it is worth making absolutely clear the relationship between 12-ET in theory and in practical music making. Theoretically, 12-ET is arrived at by dividing the octave (normally considered as a frequency relationship of 1:2) into 12 perfectly equal steps. This means that the mathematically exact frequency ratio between each step is $1{:}\sqrt[12]{2}$. Instruments of fixed intonation are normally tuned as close to this theoretical ideal as is practical. However, the physical properties of the instruments themselves usually require adjustments to be made. For example, the scale in the mid-range of the piano keyboard corresponds fairly closely to the mathematical description of 12-ET, but to compensate for the material properties of the strings in its upper and lower registers, the octaves are stretched (i.e. the frequency relationship is 1:>2), and the twelve steps are therefore stretched correspondingly ($1{:}>\sqrt[12]{2}$).[12] The uppermost range of the piano is about half a semitone 'sharp' relative to the middle range, and the lowermost range is similarly 'flat'. This nevertheless gives the illusion of a perfectly equal-tempered scale across the breadth of the keyboard - a scale which is popularly understood as the archetypal manifestation of 12 division equal-temperament.

However, players of non-fixed intonation instruments do not necessarily attempt to achieve 'strict' 12-ET - they are first of all playing 'in tune', relative to the norms of the music in question. This implies frequent deviation from 'exact' 12-ET just as it implies frequent deviation from 'just intonations'. Highly trained string players fine tune instinctively and with great accuracy. Similarly, the intonation of woodwind and brass instruments depends on the instinctive co-ordination of the player's fingers, lips and breath control. Players harmonise in ensemble with the instruments around them and carefully sculpt the intonation of melodic figures when playing solo. Influenced by years of intensive training and experience, a degree of intonational subjectivity is thus part and parcel of making music on these instruments. What actually counts as being 'in tune' is difficult to define - be it in terms of mathematics and acoustics, or individual and consensual pleasure - and is subject to debate. The most significant factor is thought to be the relative absence of 'roughness' or beats between components of complex tones. The point is that, since this discussion is devoted to the integration of the theoretical and practical aspects of *alternatives* to 12-ET, it is important not to think of the current practice of 12-ET as if all players and singers were constantly trying to attain tunings corresponding to the fixed mathematical definition of that system, a fact made clear by the enharmonic notation of Western music.[13]

In practice, intonations vary according to the player's expressive intention, the combinations of voices and instruments involved, and the nature of the music itself (leaving aside other accidental factors). For example, string players playing in a piano quartet rather than a string quartet, will subtly temper their intonation to blend with the 'equal-tempered' piano. Similarly, in atonal music, it is often preferable that performers playing non-fixed intonation instruments attain closer to 12-ET than they would in tonal music, because it is necessary to make clear the conception of musical space which is intended.[14] And intonation in orchestras is liable to be less exacting, where performers are less exposed than in smaller chamber groups. Except in particularly fast music, where intonation is less affected by subjectivity or enharmonic content, performers achieve these subtleties

[12] Schuck and Young, 'Observations on the vibrations of piano strings', *Journal of the Acoustical Society of America*, 15, 1943, pp. 1-11; also, Neville Fletcher and Thomas Rossing, *The Physics of Musical Instruments*, Addison Wesley, 1993, p. 335. To avoid beats, the tuning of the central octave is also usually stretched (this is sometimes disputed), depending in degree on the instrument, but in any case less so than in the octaves above and below. See Arthur H. Benade, *Fundamentals of Musical Acoustics*, Dover, 1990, pp. 318-22. Additional reasons have been proposed for the effectiveness of stretched piano octaves, relating not only to string inharmonicity but also the non-linearity of the ear. See, for example, Ernst Terhardt, 'Pitch, Consonance and Harmony', *Journal of the Acoustical Society of America*, 1974, Vol. 55, No 5, pp. 1066-8.

[13] The same point has been made by many researchers - for a review of these see W. Dixon Ward, 'Musical Perception', in Jerry V. Tobias, ed., *Foundations of Auditory Theory*, New York, 1970, pp. 414-22. Interestingly, Nicholas Cook briefly explores the musicological implications of this, commenting that 'the semitone, as such, has no objective reality in what singers and violinists actually do… the semitone, like the other intervals or interval-classes on which formalised music theory is based, is a representation of this phenomenon, and not the phenomenon itself'. See Nicholas Cook, *Music, Imagination and Culture*, Clarendon Press, Oxford, 1992, pp. 235-9.

[14] On this topic compare M. Lindley and R. Turner-Smith, *Mathematical Models of Musical Scales - A New Approach*, Verlag für systematische Musikwissenschaft, Bonn, 1993, where they say, for example, 'Webern's atonal music does not use a chromatic scale, but only a scale of semitones. To have traversed that difference completely is an important technical aspect of the history of art music in the first quarter of the 20th Century.' (p. 48). Compare also G.W. Hopkins, 'Schönberg and the 'logic' of atonality', *Tempo*, No. 94, 1970, pp. 15-20.

almost unconsciously, and in doing so demonstrate the (sometimes underestimated) ability of musicians to act on very small distinctions of pitch.[15]

These distinctions are important, and will be assumed in all further references to performing in '12-ET', or any other tuning system. The prefix 'strict' (as in 'strict 12-ET') is used when referring to a more precise realisation of a tuning system - for example, when describing a tuning for instruments of *fixed* intonation - but even this is not necessarily exact.[16]

Practicalities of Intonation on New Acoustic Instruments

The reason for making the above remarks at the outset of this paper is that, in proposing the manufacture of new acoustic instruments for alternative tuning systems, the accuracy of intonation on new instruments of semi-variable intonation would be comparable to that on conventional semi-variable instruments in 12-ET.[17] This is neither a problem, nor does it mitigate the advantages of building new instruments. It is often said, for example, that if composers use alternative tunings when writing for conventional wind instruments they should specify the tuning but leave it to the ear and ingenuity of the performer to achieve the intended result; or, to consult a manual of extended fingerings. The argument is that intonation is in any case rather subjective, and making a new instrument with a subtly different set of intervallic properties will give no real advantage. The reason given is not only that the cost of building and learning new instruments is prohibitive, but also that (to some extent) ways of playing the required pitches can be found anyway.

However, for an expert performer who has acquired a good aural understanding of 19-ET, the intonational detail achievable on a '19-ET clarinet', for example, should be neither more nor less accurate relative to 19-ET than would performance on a conventional clarinet be to 12-ET. The same may be said for a '19-ET trumpet'. Performance would involve a similar attention to intonation, in response to the different set of intervallic properties which the instrument's altered physical construction implies. Uniformity of tone, and the accuracy and ease of realising a work composed in 19-ET, especially in fast passages, would almost certainly be greatly improved on a new instrument - that is, as compared to using a conventional '12-ET' instrument to perform in 19-ET.[18] Ideally, the new instrument will not only facilitate the discrete pitches of an alternative system, but also *confirm* them in the ear of the player, and in the ears of players of other instruments of non-fixed intonation when playing in ensemble. What would really be new is the interaction between the ear of the performer and the acoustic nature of the new instrument, allowing exploration of a new melodic and harmonic palette in ways which are perhaps not imaginable without its existence.[19]

A survey of existing and possible adaptations of conventional instrument designs, each conceived for alternative tuning systems, is given in Section 5 (pp. 62 - 105). Readers whose main interest is in new instruments, or who are already well-versed in the theory of tuning and related issues, may wish to turn directly to that discussion.

The Keyboard and the Success of Twelve Division Equal-Temperament

The adoption of 12-ET as a standard in Western music was the result of the collective desire, drawn out over the course of about 250 years, to find a *keyboard* temperament which privileged the most intuitively powerful intervals (or at least good approximations of them) and allowed unlimited modulations within the scale, combining these with a maximum (overall) degree of consonance. The irregular Baroque keyboard temperaments, such as those documented by Werckmeister, Neidhardt, Kirnberger and many others, aimed to

[15] Lindley and Turner-Smith have discussed the implications of intonational flexibility on the logical modelling of alternative scales and temperaments. See M. Lindley and R. Turner-Smith, ibid., Chapter 15, pp. 48-52.
[16] A discussion of how to achieve Just Intonation with acoustic instruments can be found in David Doty, *The Just Intonation Primer*, Just Intonation Network, 1994, pp. 60-72.
[17] All things being equal this should be so: but it may be held in mind that design changes affecting (for example) the mouth-hole size on the flute, the reed and lay on the clarinet, or the windway height on the recorder, may alter the range of possible intonational variation. Thanks to Lewis Jones for remarks on this topic.
[18] The fact that not all conventional orchestral instruments of semi-variable intonation are perfectly 'optimised' for 12-ET does not contradict this point. Similar conditions would undoubtedly apply, for example, to a '19-ET clarinet'.
[19] Electronic simulations of acoustic instruments using alternative tunings are useful as compositional, training and rehearsal tools, but pose practical problems which are discussed below. See Section 6.

maximise consonance in the most frequently used keys and to avoid the wolf fifths of meantone temperaments.[20] In these 'circulating' temperaments different tonalities can bestow distinctive harmonic colours and emotions to a given passage, and also help to articulate structural, modulatory relationships. It was out of a context of many competing alternatives that 12-ET slowly emerged and began its reign of dominance, both in the design and tuning of keyboards and other instruments.

In strict 12-ET no interval except the octave is 'perfectly' tuned - that is, none of its intervals match exactly the 'ideal' ratios of just tunings - but each interval is mistuned by a relatively small amount.[21] 12-ET emerged as a compromise: flexibility of modulation deepened the potential for the hierarchical ordering of key relationships and tensions - building blocks for extended harmonic forms - at the expense of harmonic purity and colour. We may also speculate that the development of the timbre of keyboard instruments was intimately bound up with this development. For example, the glistening brightness and slight instability of the modern piano sound may have 'coevolved' with the 'mistuned' consonances, or what one might call the particular 'xenharmonic' character of 12-ET.[22] It is also significant that perhaps no single factor has been more influential in the rise to dominance of 12-ET than the success of the piano, both in the concert hall and at home, and particularly as the instrument *par excellence* of composers from the late 18th through to the mid 20th Century.

The long process of refinement through Baroque and 'post-Baroque' tuning systems resulted in the adoption of 12-ET as a standard only in the latter half of the 19th Century. It seems that the specific properties of the tuning systems in this the period were highly influential, most obviously so in keyboard music, and it is not tautological to say that Mozart, Beethoven,[23] Schubert, Brahms, Debussy or Rachmaninoff would not have written the music they wrote if the history of tuning systems had taken a different course.[24] Nor is it tautological to say that dodecaphonic music would not have developed as it did if the octave had been divided into something other than twelve equal divisions.[25] Certainly, 12-ET lends itself to certain musical facts in ways that other *n*-ET and *n*-UT systems do not, and it is arguable that this has been a major factor in its extraordinary success. The most important of these are normally thought to be (i) the accuracy of the fifth, being only 2 cents narrower than pure, which is crucial in agreeing (albeit approximately) with the harmonic spectra of sustained acoustic instruments, (ii) 'equal-tempered-ness' itself - the complete consistency and flexibility of unlimited transposition within the scale, and (iii) the relative manageability of 12 divisions, particularly compared to other equal-tempered systems which possess an accurate fifth[26] - but these points are merely the tip of an iceberg.[27] The point here is that the overwhelming *musical* success of 12-ET cannot be overlooked in further researches, and three features of this should concern us particularly:

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[20] J. Murray Barbour, *Tuning and Temperament - A Historical Survey*, East Lansing, Michigan State College Press, 1951, Chapter VII.

[21] See APPENDIX II, Tables 12(a) and 12(b). Of course, the question whether there is any such thing as an 'ideal tuning' of various intervals, and whether just ratios do actually define such tunings, is controversial. We return to these questions later.

[22] Ivor Darreg adopted the term 'xenharmonic' (from the Greek for 'unfamiliar modes') to 'identify scales that sound noticeably *different* from the 12-tone equal-tempered scale', so I am slightly stretching the use of this term. However, Darreg's purpose was to call attention to the individual 'mood' (as he called it) of each 'alternative' equal-temperament - and 12-ET, like them, also has its own 'mood'. I would prefer the word 'colour' to 'mood', but the point is the same. I. Darreg & B. McLaren, 'Biases in Xenharmonic Scales', *Xenharmonikôn 13*, Spring 1991, p. 5. The relationship between tuning systems and the timbre of the piano is discussed below (pp. 85 - 92).

[23] For some remarks on performing early Beethoven Piano Sonatas in temperaments other than 12-ET, see M. Lindley and R. Turner-Smith, op. cit., pp. 206-10. A recent CD of '*Beethoven In The Temperaments*' has been recorded by Enid Katahn on Gasparo #332. Further details cam be found at http://www.uk-piano.org/edfoote/ and http://www.gasparo.com/ .

[24] Lindley and Turner-Smith argue for a similar conclusion for Western music as a whole, citing many examples in support of the claim. Op. cit., p. 155.

[25] Reports as to the date when 12-ET may be considered to have become 'standard' in Europe vary significantly, ranging from the 1840's to the late 1880's. Jorgensen, for example, has argued that 12-ET did not become common practice for tuning *pianos* until late in the 19th century. Owen H. Jorgensen, *Tuning*, Michigan State University Press, East Lansing, 1991. This would perhaps corroborate the idea that 12-ET was influential in the rise of atonality and dodecaphony. Although Schönberg, for example, was more accomplished a string player than pianist, he is reported to have had a piano tuned in 12-ET so as to have it available during rehearsals of his 3rd String Quartet (1927), demanding that the players adhere to the tuning of the piano as far as possible. It is difficult to know from this story (if it is true) whether Schönberg simply wanted to help the quartet play 'in tune', or whether he really wanted 'equal-temperament'. From Schönberg's correspondence with Joseph Yasser it is almost certain that he wanted equal-temperament, but since piano tuning is mildly stretched, the story provides slightly confusing evidence. Thanks to Joesph Monzo, Daniel Wolf, and other contributors on the Tuning list for help with this. See Tuning digest XXX.

[26] As discussed in more detail below, 29-ET is the ET with the smallest number of octave divisions which betters the accuracy of the fifth; 24-ET of course has the same fifth as 12-ET. See APPENDIX II, Tables 12, 24 and 29.

[27] An overview of tuning theory in general is given below.

- the success of the piano, particularly, as an instrument of fixed intonation in a 'compromise' tuning (but which is stretched for increased consonance);
- the success of 12-ET in the extent to which it supports the musical integration of instruments of fixed and non-fixed intonation;
- the success of 12-ET as a standard.

However, consider for a moment how things might have been otherwise. Suppose 31-ET had developed as the standard Western tuning system (which, despite appearances, is not inconceivable).[28] Western music would have been very different - often profoundly so. Composition would have taken place against the same background of 'acoustic facts' - but heard and manipulated as if through a different filter. No composer's music would have been as we know it now. Great riches would therefore have been lost - yet who knows what we would have gained? This is in fact our situation today - but with the tuning systems reversed. Alternatively, suppose the system had been 19-ET, and leave aside for a moment whether the history which gave rise to atonality could have occurred in this hypothesis. In this case, a telling example is that Webern would not have been Webern for the simple reason that the major seventh in 19-ET is a rather different sounding interval (as is its cousin the minor ninth) from those intervals as we know them in 12-ET. In that case Webern's extraordinary music could never have happened - at least not as we know it. But the roles are reversed - so what extraordinary music have we been missing?

Towards a Provisional Alternative Tuning System Standard (or Standards)

The music of the most famous 20th Century pioneers of alternative tunings - Ivan Wyschnegradsky (1893-1979), Charles Ives (1874-1954), Julián Carrillo (1875-1965), Alois Hába (1893-1973) and Harry Partch (1901-74) - is reasonably familiar to those with a specialist interest in the subject, and many recordings are available. Of these, only Ives and Partch are generally considered to be of major importance as composers, and only Ives' works are heard in concert halls with any widespread regularity. Ives employed quarter-tones in his experiments; Wyschnegradsky, Carrillo and Hába explored equal-tempered systems, mainly quarter-tones but also smaller divisions of the octave; and Partch devised his own unique 43-division system of Just Intonation.

A considerable proportion of more recent composers have explored ATS.[29] Apart from 'quarter-tones as inflections' and 'strict quarter-tones', there are composers working today with other *n*-division equal-temperaments, systems of just intonation, tuning systems based on harmonic spectra, *ad hoc* unsystematised

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[28] For example, a 19 division harpsichord was built for Zarlino as early as 1548, and Trasuntino built a keyboard in 1606 having 31 divisions. See below for further mention of these keyboards. Another astonishing fact of 'microtonal' history is that 'in the 1550s a talented French composer, Costeley, wrote a song for the 19 division'. Lindley and Turner-Smith, op. cit., p. 147. In my opinion, I think it is unlikely that 19-ET would have been sustained as a standard for very long - but not so 31-ET.

[29] For an encyclopaedic chronicle of microtonal music, see B. Mclaren, 'A brief history of microtonality in the twentieth century', *Xenharmonikôn* 17, Spring 1998, pp. 57-110. It may be worth registering a (necessarily incomplete) list of composers who have been or are particularly recognised for composing using ATS in one form or another. They are listed here, for simplicity, by country of origin: from Austria - Franz Richter Herf; from Belgium - Henri Pousseur; from Finland - Kaija Saariaho; from France - Claude Ballif, Alain Banquart, Pierre Boulez, Pascale Criton, Pascal Dusapin, Gerard Grisey, Jean-Etienne Marie, Tristan Murail; from Germany - Clarence Barlow, Ernst Krenek, Helmut Lachenmann, Manfred Stahnke, Karlheinz Stockhausen, Wolfgang Rihm, Martin Vogel; from Greece - Iannis Xenakis; from Holland - Henk Badings, Anton de Beer, Jan van Dijk, Adriaan Daniël Fokker, Hans Kox, Ton de Leuw; from Hungary - György Kurtág, György Ligeti; from Italy - Luigi Nono, Giacinto Scelsi, Salvatore Sciarrino; from Poland - Witold Lutoslawski, Krystof Penderecki; from Romania - Horatiu Radulescu; from South Africa - Kevin Volans; from Switzerland - Heinz Holliger; from the UK - Richard Barrett, George Benjamin, James Clarke, Justin Connolly, Chris Dench, Frank Denyer, James Dillon, Brian Ferneyhough, Michael Finnissy, Jonathan Harvey, Bill Hopkins, Roger Redgate and James Wood. These are only some of the better known names in Europe: amongst the younger generation of the modernist avant-garde, inflective quarter-tone writing is almost the norm. In the US the variety of musical styles being pursued using ATS is greater, and the following catalogue of composers is consequently very broad - ranging musically from classical to jazz/blues. Some of the better known names include: William Alves, Easley Blackwood, Glenn Branca, David Canright, Ivor Darreg, David Doty, Dean Drummond, John Eaton, Harold Fortuin, Clem Fortuna, Kyle Gann, Denny Genovese, Lou Harrison, Neil Haverstick, Ben Johnston, Henry Lowengard, Joel Mandelbaum, Gary Morrison, Frank Oteri, Larry Polansky, Johnny Reinhard, Terry Riley, Carter Scholz, Harold Seletsky, William Sethares, Elliott Sharp, Ezra Sims, James Tenney, Tui St. George Tucker, Ervin Wilson, and La Monte Young. There may be living Russian composers specifically exploring ATS, although they remain obscure to me. But it is worth mentioning the little known early 20th Century Russian experimental microtonal movement, which included not only Wyschnegradsky but also Leonid Sabaneev, A.M Avraamov (who worked with a 48-division scale), Lourié (who worked with quarter-tones), Sergei Protopopov, G. M. Rimsky-Korsakov and others. In the article mentioned above, Mclaren mentions Aleksei Oglovets and Evgeny Murzin working with microtonality as late as the 1940's - 50's, but little activity since then. See also note 303.

tuning relationships, non-octave scales and temperaments, historical and ethnic tunings, and tunings based on natural phenomena (for example, bird song). Microtonality is also intrinsic in electroacoustic music.

In Europe,[30] the 'dominant' alternative to (or extension of) 12-ET in avant-garde instrumental music is quarter-tones (24-ET).[31] The extent to which, as yet, composers and performers conceive the quarter-tone scale as consisting of inflections of the familiar pitches of 12-ET, or as an independent tuning system with unique intervallic properties of its own, probably varies widely. Nevertheless, the familiar aural reference points given by half of the quarter-tone scale, the design of existing instruments, and the ease of adapting conventional notation, have all contributed to the adoption of quarter-tones - almost as a new *de facto* 'standard'. Manuals of extended techniques for conventional instruments more often than not give fingerings for quarter-tones rather than other tunings; and (gratifyingly, of course), one or two specially constructed quarter-tone versions of conventional instruments may be purchased from specialist builders.[32]

Quarter-tones are often used to extend the conventional intonational flexibility used in 12-ET, as described above, allowing performers to discover expressive inflections around the 'nodal' intervallic relationships implicit in 12-ET. In a stricter use of 24-ET, as is implied by creating fixed intonation instruments in that system, or, for example, in compositions for two pianos tuned a quarter-tone apart, the tuning system unfolds its own 'xenharmonic' character more clearly. 24-ET is by no means unworkable, and some extraordinarily beautiful music has been written using quarter-tones (although a great deal has also been composed which is far from satisfactory). But as a new standard tuning system for the future of acoustic music, 24-ET has important limitations. The reason usually given is that the additional intervals in 24-ET which augment those of 12-ET do not closely approximate 'just' intervals.[33] The aural correlative is that *strict* 24-ET can be harmonically awkward and inflexible - although, as I maintain, it can also be extremely effective and rewarding.

However, the *dominance* of quarter-tones, and its quasi-adoption as a standard amongst the European avant-garde, might be seen as expedient, not at all a logical continuation of the principles of tuning which led through the Baroque temperaments and to equal-temperament as we know it. It may be argued that the expressive intentions of many 20th Century composers are far removed from those of the Baroque era, and it is unsurprising if the most basic principles of harmonic logic influencing instrumental and harmonic design are today completely different from those of our predecessors. But in the mainstream of contemporary music this question has hardly been accorded the importance or attention it deserves.

This paper is intended to stimulate discussion of which system(s) of instruments/tuning might facilitate, initially, the expansion of contemporary musical language (for acoustic instruments) in the most *compelling*, *productive* and *adaptable* ways. This discussion should *not* be about establishing a *permanent* alternative standard. If an alternative standard were adopted,[34] the particular properties of the chosen ATS would inevitably influence the nature and direction of new music, and this influence may have an unwanted or unfortunate bias. It is of great importance, however, that if a single, initial, alternative must be chosen, it should in some sense be 'quasi-universal' - that is, accepting that no discrete, fixed system of tuning could be truly 'universal'.

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[30] In the USA it appears there may be equal or more interest in various forms of Just Intonation. Also, in correspondence with the author, Ezra Sims has suggested that 31-ET (in Holland) and 72-ET (as a result of 20 years of teaching by Joe Maneri in New England) may have a greater following than 24-ET.

[31] The dominance of quarter-tones amongst the European avant-garde may be waning, and expressions to the effect that 24-ET is already 'passé' or a cul-de-sac are quite common. The term 'quarter-tone' is frequently used, naturally enough, to denote an intervallic 'zone' rather than the interval of 50 cents. J. Murray Barbour, for example, uses the term to describe any interval from 26/25 (68 cents) to 49/48 (36 cents). J. Murray Barbour, op. cit., p. 228. Unfortunately, in the contemporary vernacular, 'microtone' and 'quarter-tone' often remain irritatingly synonymous - but the term 'quarter-tones' most frequently refers to a system of '24-ET' - performed with intonations appropriate to the instruments employed.

[32] For example the *Osten-Brannen* flute mentioned above, and also quarter-tone percussion which may be built to order by the French firm *Bergerault*.

[33] The interval of eleven quarter-tones (550 cents) is only one cent flat of the eleventh harmonic (11/8 or 551 cents). The interval of five quarter-tones (250 cents) lies roughly midway between 7/6 and 8/7, and nine quarter-tones (450 cents) is 15 cents flat of 9/7. While some writers have therefore seen in 24-ET reasonable approximations of intervals in the 7-Limit, as APPENDIX II Table 24 shows, it is perhaps more plausible to view strict 24-ET as being divided between intervals which belong mainly in 5-, 7- and 11-Limits. However, this is not the whole story - and only one way of looking at it - as is shown below.

[34] The intention here is not to 'replace' existing instruments or traditions - see below.

In the first instance, *one* integrated and fully developed alternative (Western) system of acoustic instruments would be better than none (there are of course other systems in other cultures). It would also be preferable to develop a unique system, and the instrumental and compositional skills needed to explore it fully, than to leave developments to isolated experimentation. If one alternative system was successful, further instrumental developments would also be made less difficult. It will also be borne in mind here that some new instruments might individually be made dependable in a *variety* of tuning systems. Although this possibility might seem to obviate a part of this debate, it is extremely unlikely that this would be possible for all orchestral instruments; nor, in the foreseeable future, will large numbers of musicians be willing to learn a wide variety of new tuning systems and notations. The debate should therefore almost certainly pose the question of choosing a unique system (or no more than a few alternatives). The choice cannot be aesthetically neutral, but for *acoustic instruments* it can be relatively objective.[35]

There can be no doubt as to the *musical* importance of these questions, which foster intractable debates about *which* system is most appropriate, and about whether a *single* alternative is appropriate at all. But I doubt that they are unanswerable. Wherever our sympathies lie, however, our answer bears crucially on long-term investment - not only in terms of the substantial funding required for research and for building new instruments, but also in terms of the time, effort, expertise and commitment of instrument makers, acousticians, performers and composers.

A Centre for 21st Century Acoustic Instruments

Amongst musicians working in contemporary music, ATS are no longer a minority interest. Today, every young classical composer is aware of ‘microtonality’, as are most young performers; in popular music there is increasing interest in ATS, and contemporary familiarity with world musics also has increasing influence.[36] While classical composers discuss the relative merits of quarter- and sixth-tones, teenagers discuss Kirnberger and Werckmeister III - often found as ‘presets’ on MIDI keyboards. Yet only virtuoso performers specialising in contemporary music can attempt to overcome the difficulties of playing works in radical ATS with conventional acoustic instruments; moreover, the alternative systems that are normally attempted represent a tiny selection of the possibilities.

Various centres and groups around the world are devoted to projects related to ATS.[37] But the development of new instruments is costly, and, to my knowledge, there exists no centre currently running a well-funded project co-ordinating the building of mainstream instruments in ATS.[38] Yet new instruments are urgently needed to respond to their increasing importance in Western music.

This paper attempts to illustrate the necessary interrelationships of research which a Centre could foster and co-ordinate, and the enormous potential of a ‘project network’. The project would be of particular value in encouraging:

- collaborative work - local and international, musical and technological;
- a focus for existing but currently disparate activities;

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[35] As far as possible a project to make new instruments should nurture a potential beyond what it can foresee, and ensure that investment supports a diversity of aesthetic approaches. Which tuning system(s) are most worthy of investment depends on collective musical intentions and practical feasibility as much as mathematical and acoustic theory. Individual composers are free to compose in any tuning system they like, and need not be constrained by a collectively agreed standard. But the current absence of an agreed ATS standard has the result of a kind of dis-empowerment, wherein composers depend on very exceptional performers, with or without their own special instruments. However, the question of individuality can be exaggerated - have not the majority of composers been content with a single standard for some time?

[36] The creation of new instruments in ATS may indirectly help to counteract the colonisation of ethnic musics by Western equal-temperament and instrument technology; it may also suggest new ways of overcoming barriers that exist between musics of different cultures.

[37] Amongst others - the *American Festival of Microtonal Music* (New York), the *Centre for Microtonal Music* (London), the *Interval Foundation* (San Diego), the *Just Intonation Network* (USA), the *Richter Herf Institute* (Salzburg), the *South-East Just Intonation Centre* (Florida), ‘*Système Micro-Intervalles*’ at IRCAM (Paris), *Stichting Huygens Fokker* (Holland), the *Boston Microtonal Society* (Boston), *Dedalus* (France), *Xenharmonikôn* (USA). The Stichting Huygens-Fokker Yearbook for 1994 lists about 20 different societies or groups for microtonal music of differing kinds around the world.

[38] The possible exception to this is the project ‘Système Micro-Intervalles’ at IRCAM.

- research into general as well as particular solutions, which cannot be undertaken on a smaller scale.

The paper therefore proposes the founding of a wide-ranging collaborative project to create new acoustic orchestral instruments. It is envisaged that a *Centre for New Musical Instruments* would be able to provide a co-ordinating focus for this, and it is hoped that a number of institutions internationally - commercial and academic - might form core 'nuclei' around which research activities would flourish. A vision of this project is given at the end of the paper (pp. 113 - 114). An independent *Centre for New Musical Instruments* would perhaps also help to mediate between the commercial interests of instrument manufacturers and makers and the creative interests of composers and performers. In the first instance it would focus on creating a new, integrated system of acoustic orchestral instruments in a provisional ATS standard. The Centre would be invaluable for helping to focus a range of existing activities, and to develop new ideas. It would also create an extraordinary opportunity for individuals from a very wide sphere - composers, theorists, performers, instrument designers and builders, electronics and computer experts, mathematicians and acousticians, tuners and technicians - to collaborate on a single goal: the perfect realisation of new acoustic music.

Personal and Impartial

This discussion paper cannot be other than personal. As a composer my primary interest is acoustic instrumental music. The 20th Century composers who have most influenced my compositional thinking are Schönberg, Webern, Nono and Barraqué. The arguments I pursue here are, of course, motivated by a dream of a certain kind of *different* music - a music whose strength and identity is dependent upon the power and emotional colour of precisely realised alternative intervals and harmony. This idea of a music - the 'air from other planets' which motivates me - could not be the same for everyone.

Aside from my contemporary interests, for me the highest musical pantheon comprises Bach, Mozart, Beethoven, Schubert and Brahms. And, to me, the need for new acoustic instruments seems to follow particularly from the development of 20th Century orchestral and chamber music in the classical tradition. As well as the composers I mentioned in the previous paragraph, I am thinking of Janácek, Debussy, Sibelius, Ives, Bartók, Stravinsky, Varèse, Berg, Shostakovich, Messiaen, Carter, Lutoslawski, Xenakis, Ligeti, Boulez, Feldman, Stockhausen, Birtwistle, Reich, Ferneyhough and beyond. These are personal preferences, and naturally my hope is to interest especially other composers and musicians who share similar roots, and who would contribute to this venture. But I hope equally that composers from other backgrounds - those working in more radical areas (for example, for whom Partch's work is especially important), as well as those working in more conservative idioms - will also support the aims of this paper. *The question of new instruments transcends questions of style and personality*.[39]

This discussion therefore aims to have no affiliation with any particular group or aesthetic manifesto. The idea of creating a project and co-ordinating Centre is opposed, as I see it, to the institutionalisation of creativity. For example, developing new instruments for a single, provisional, ATS would not compromise this aim, and would benefit a wide range of composers and listeners. Achieving this depends on considerable collaboration, co-ordination and funding. First of all, however, we need to be clear about how new instruments will really fit into the broader scheme of contemporary music making.

[39] New instruments should, if funded from public resources, be appropriate for a variety of musical languages. I therefore omit, for example, questions of whether atonal and serial music 'depart from the Western musical tradition' - as argued by Enrique Moreno (*Expanded Tunings in Contemporary Music: Theoretical Innovations and Practical Applications*, Edwin Melon Press, 1992, p. 90), and Martin Vogel (*On the Relations of Tone*, translated from the German by Vincent J. Kisselbach, edited by Carl A. Poldy, Verlag für systematische Musikwissenschaft, Bonn, 1993, p. 278).

2. Towards the 'Acoustic Orchestra of the 21st Century'.

Continuity and Disturbance

In 1986 the composer and percussionist James Wood, now director of the *Centre for Microtonal Music*[40] and the London-based ensemble *Critical Band,* was posing questions particularly close to those addressed in this paper:

> Today, through the development and the growing availability of electronic sound sources, we have an infinite number of octave divisions at our disposal. So it is hardly surprising if it is to this medium that the composer today most readily turns to realise his microtonal music. Hence the evolution of acoustic microtonal instruments has faltered even before reaching its adolescence. But if there is to be a future for acoustic music, should we not encourage the continuation of this evolution? is there not a greater likelihood that acoustic music will become museum-bound if it continues to be limited to the 12-note octave we have used since the 18th century? if so, which system should be adopted as universal? how many notes should there be to the octave?[41]

To put these issues into a broad perspective, particularly with reference to today's orchestras and contemporary chamber ensembles, I want to invoke briefly the words of the composer Alexander Goehr, looking back in a recent publication to his *Reith Lectures* of 1987. In those lectures, entitled '*The Survival of the Symphony*', he argued for the support and renewal of our symphonic institutions. In doing so, he examined

> the nature and likelihood of survival of the public institution through which music in the Western world has been broadcast since at least the time of the Viennese Classics. Its survival, in the face of colossal changes brought about by the mechanical reproduction of music and the emergence of a powerful and influential music industry… would seem to be in doubt. Furthermore the institution of the public concert and the symphony orchestra that performed it were further weakened by the emergence of specialised interest groups, such as those devoted to new, authentic, and ethnic musics, which are seen to require their own appropriate performing conditions and to cultivate their own exclusive publics.[42]

In the lectures themselves Goehr argued that much of the value of the 'traditional symphony concert' lies in the fact that new and old works are heard side by side, and that the new is best appreciated (and judged) in this context. As the traditional 'flagship' of classical music, the orchestra and the symphony are considered the form and forum where large-scale musical argument is most seriously presented. At least within a sector of Western society, the orchestra and orchestral concert represent (or used to represent) a kind of 'continuity' - the collective, large-scale culmination of music making - from music in schools to high-art chamber music. The argument is that the orchestra will only survive as a *living* (rather than archaic) forum if audiences continue to benefit from the 'friction' between past and present, and between a dialogue of styles and movements; and if the orchestra continues to serve as a 'culminative' forum for large-scale expression.

This view is controversial. Today, the dissemination of serious music is probably more private than public. Disparate musics coexist and rub shoulders in our living rooms (on the radio or CD player) more than in individual concerts, and it is arguable whether the musical (or social) power of music is consequently diminished or increased. Perhaps there is no need for the orchestra to serve as a 'flagship' of classical music. Yet, as I try to show, I believe Goehr was right to view the orchestra, in both its human and technological aspects, as a renewable forum - for continuity and conflict, change and (maybe) 'progress'.

In contrast, Trevor Wishart has expressed a more pessimistic view. He writes:

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[40] The *Centre for Microtonal Music* website can be found at: http://www.brailsford.demon.co.uk/cmm.htm.

[41] James Wood, 'Microtonality: aesthetics and practicality', *Musical Times*, June 1986, p. 328.

[42] Alexander Goehr, *Finding the Key*, Faber and Faber, 1998, p. viii. Goehr seems to have renounced some elements of his *Reith Lectures*, but I feel it is unfortunate they have not been republished for their broad and argumentative view of things. Alexander Goehr, 'The Survival of the Symphony', *The Listener*, 19 November 1987, Vol. 118, No. 3038, and the following five issues (to 3043).

> Regrettably I feel that the orchestra, as a possible arena for contemporary music, will be abandoned. The orchestra is already a moribund institution, centralised and bureaucratic, dependent on golden oldies to maintain its income… it is not just the in-built conservatism of most orchestras, but the overwhelming weight of tradition, which makes the further development of this medium problematic… the extra musical factors which help to maintain [the orchestra]… [such as] film, television and advertising industries, are likely to opt for electronic substitutes… because those are cheaper… Only the heritage industry will require the survival of this large ensemble…[43]

Wishart is discussing the situation of the orchestra rather than of acoustic instruments in general - and there is much truth in what he says. However, this view seems to be coloured by a general acceptance that acoustic instruments will be *abandoned* in favour of electronics,[44] and an acceptance that the development of the Western acoustic instrumentarium is 'over'. Yet, on the contrary, it seems that a majority of composers are far from ready to abandon either acoustic instruments or the orchestra. What many composers really want is that the orchestra and its instruments are open to change. Some orchestras today have adopted a multi-form infrastructure and some members play in related contemporary ensembles. But excepting some modern percussion and many (more or less incidental) instrumental refinements, contemporary orchestral instruments are much the same as they were in about the 1860's.

The argument presented in this discussion paper is that there would be great value in renewing acoustic instrumental resources *whether or not* new instruments are destined for the 'orchestra' as such, but it is extremely important that new instruments are designed to be played *together*. In this sense, the relevance of the orchestra is twofold: firstly, it represents the totality of potential (Western) acoustic ensembles; secondly, the creation of new instruments makes most sense within the context of an examination of 'instrumental harmony'. But the real vision behind this paper is the creation of a parallel instrumentarium - *a new orchestra* - so different, strange and exciting, that composers will not turn their back on it for another hundred years.

Thus, the general situation of the orchestra remains relevant to the discussion. To continue to be both a catholic and truly contemporary platform, one amongst others, the orchestra must do more than survive, especially considering the widening break between what can be realised by today's orchestra and what can be realised by contemporary chamber ensembles. This break is, of course, even more pronounced considering other new forms of music making. And, as Goehr put it:

> The future Kafka, Strindberg or Webern has to be the one who takes the lid off convention and gets below its surface. If modern music no longer actively disturbs society, it has no further function and cannot contribute to any renewal of the symphony. At present, no such renewal seems likely.[45]

New Instruments in the Orchestra?

As has been said, one of the reasons why the interest of the avant-garde in the orchestra has (at various times) seemed to wane, has been that the assimilation of new instruments into the orchestra has virtually ceased. From the composer's point of view, the size of the orchestra makes it an inflexible, unwieldy apparatus in which to transcend that inertia. During the early part of the 20th century, there was an important movement of composers turning towards smaller ensembles, for example, following Schönberg's *First Chamber Symphony* and *Pierrot Lunaire*. In both, Schönberg achieved a revolutionary range of expression. The reduced scale and increased intimacy of smaller ensembles, and their improved opportunities for virtuosity and co-ordination, encouraged composers to work in greater detail and to be adventurous with instrumental techniques. Both the rise of the contemporary chamber ensemble and the development of 'extended techniques' are descendants of that

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[43] Trevor Wishart, 'Connections', in 'Leaving the Twentieth Century - Ideas and Visions of New Musics', *Contemporary Music Review*, ed. Leigh Landy and Frank Denyer, Vol. 15, Parts 3-4, pp. 93-6.
[44] Wishart pleads a special case for the voice, ibid. pp. 95-6.
[45] I take it that with the term 'symphony', in this passage, Goehr means the forum as well as the form. *The Listener*, 31 December 1987, Vol. 118, No. 3043, p. 14.

tendency, and in some contemporary music there remains a cult of virtuosity or complexity associated with them.[46]

In the main, composers 'make do' with available instruments rather than making or commissioning new ones for specific expressive purposes.[47] There are good reasons for this: composers seldom have the knowledge or skills of instrument designers, and advisedly leave the task to those who know better. Alternatively, as has been implied, composers demand new results from existing instruments. 'Extended techniques', for some conventional instruments, have widened the composer's timbral (and harmonic) palette significantly, and the interest of many composers has shifted from more traditional concerns (melody, harmony) towards an emphasis on sound itself. Pure electroacoustic music has distilled this interest, and has evolved into an art which can deal with its musical material very directly. Extended techniques, on the other hand, provide a way round the problems posed by alternative tuning systems, rather than dealing with them head on; consequently, the ability to realise works using radically alternative tuning systems (or alternative intonations) with extended techniques remains limited.[48]

It has already been said that a significant proportion of works composed for acoustic instruments, mostly but not exclusively in the avant-garde, use ATS, and that in the foreseeable future interest in ATS will increase. Inevitably, some composers restrict their musical language when composing for orchestra, but in some areas of music we are approaching (if we have not already arrived at) a situation in which orchestras will simply not be equipped to attempt the language of much new music. In terms of the relationship between instruments, virtuosity and ATS, this break in continuity will continue to widen unless there is some intervention.[49] We surely want orchestras to be able to present the broad picture of contemporary music in the future. To ensure a continuity between the avant-garde and the symphony orchestra, creating new versions of acoustic orchestral instruments is the most obvious (though by no means easy) way of doing this.

Of course, I am not arguing that the 'survival of the orchestra' depends on the existence of new instruments - that would be absurd. Both the orchestra and the contemporary chamber ensemble are financially precarious. The survival of either, at least as contemporary platforms, depends on the public and political will to subsidise them, on the quality of new music, and on audiences continuing to want to hear orchestras play contemporary music. Chamber ensembles are less expensive and more flexible, and in large part this is why they are now the central platform for contemporary instrumental music.[50] While new instruments could bring new blood to specialist contemporary ensembles, and subsequently to orchestras, there is no guarantee they would bring increased revenues to either (although they might). New acoustic instruments in ATS will 'save' neither the orchestra nor the contemporary chamber ensemble, but they would do them both a power of good. And in

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[46] These comments are something of a simplification, as the diversity of contemporary orchestral music shows - compare, for example, the works of Ferneyhough, Lachenmann, Nono or many others. Of course, I am simply describing, not criticising, this movement.

[47] In the 20th Century there have been a number of exceptions to this rule - Partch, Hába, Wyschnegradsky and Carrillo being the most well-known examples. Varèse also comes to mind as a composer who, tragically, was born too early for the electroacoustic era, and was bitterly frustrated by the limitations of existing instrumental resources. In an interview with the *New York Telegraph* in 1916, he is reported as saying: "Our musical alphabet must be enriched. We also need new instruments very badly. The futurists (Marinetti and his noise-artists) have made a serious mistake in this respect. Instruments, after all, must only be temporary means of expression. Musicians should take up this question in deep earnest with the help of machinery specialists. I have always felt the need of new mediums of expression in my own work. I refuse to submit myself only to sounds that have already been heard. What I am looking for are new technical mediums which can lend themselves to every expression of thought and can keep up with thought". Fernand Ouellette, *Edgar Varese: A musical biography*, trans. Derek Coltman, Caldar & Boyars,1973, pp. 46-7. This statement should stand as a motto to this paper, with the word 'harmonies' substituted for the word 'sounds'.

[48] This is discussed in more detail in Section 4.

[49] This is true not only in purely instrumental terms, but also in mixed ensembles where composers attempt to integrate conventional instruments with electroacoustics. Hybrid instruments which combine acoustic and electronic technologies might be seen as addressing this need - for instance, the 'hyperinstruments' (the 'hyperviolin', 'hypercello' and others) developed at MIT under the direction of Tod Machover. The latter is an electric cello which incorporates sensors of different kinds - sound pickups and touch sensitive sensors on the instrument itself, and movement sensors worn by the player. The electronic cello sound is 'accompanied' by a computer which combines and transforms the output of these devices - much as if the player were at once soloist, conductor and orchestra. The hyperinstrument concept extends an instrument's range and power, capitalising on the performer's traditional skills at the same time as extending the range of interactive modes available. See: http://brainop.media.mit.edu/~tod/Tod/hyper.html. For a popular account of this work, see Thomas Levenson, *Measure for Measure - how music and science together have explored the universe*, Oxford University Press, 1997, pp. 304-313.

[50] Some would say that, seen from a broad perspective, electronics are already beginning to usurp this function, but this seems exaggerated.

purely musical terms, writing for new instruments is the one glaringly obvious route for new acoustic instrumental music to take.

It is surely desirable (whether or not it is affordable)[51] that our large-scale platforms for musical expression maintain a broadly catholic and contemporary status. Moreover, orchestral instruments embody catholic possibilities (leaving aside electroacoustics) because they serve a wide variety of aesthetics and audiences, and function well in both large and small collective units; similarly, the contemporary chamber ensemble perhaps presents a variety of music equal in range to the orchestra.[52] New versions of orchestral instruments would therefore contribute considerably to this end. But perhaps it will therefore be asked: would not music composed using alternative tuning systems be heard only in special concerts, given by performers specialising on new instruments? Would not new orchestral instruments therefore invite a further splintering of our discourse? I believe this is not the case. Certainly, further fragmentation is not what I am arguing for.

In Section 5 a brief survey is presented of ideas for building new instruments for ATS. As stated above, because existing instruments are successful, it seems logical to begin with the idea of making new versions of them rather than wholly new instruments. In most of the possibilities presented, new or adapted versions will function much like existing instruments. Woodwind players, for example, are used to doubling on different instruments, and with practice are unlikely to be unduly phased by switching to a version of their instrument specifically built for a new tuning system - although this would of course depend on the kind of adaptation involved. However, since singers and most instrumentalists must imagine a note before sounding it, the difficulty of switching from one radically 'xenharmonic' sound world to another in a single concert poses problems - which are acute for singers, brass and string players. Obviously, helping musicians to 'think' in a new pitch system is at least as important as new instruments themselves. Interactive CD-ROMs and other kinds of computerised ear-training aids could be useful in this respect, as would be specially composed technical studies for new instruments, and providing the simplest possible notation which capitalises on existing conventions. Similarly, maximising the number of new instruments which will produce reference tones, thereby minimising the intonational subjectivity demanded of players, would help considerably in overcoming these challenges. Initially, the need for performers to immerse themselves in a new sound world would be absolute, but the problem of switching between new and old systems could be overcome. Some experienced professional performers are doing this already, using extended techniques on existing instruments.

It is not intended here that new instruments for ATS should *replace* existing instruments or existing performing practices. New instruments would primarily be used for music written specifically for them, and for existing music in the particular ATS for which they are capable. Depending on which ATS new instruments were built for, it might be interesting to use them to perform music which was originally conceived in 12-ET (which can sometimes be successfully 'transcribed' into an ATS). And it is by no means necessary that new instruments for ATS would impose such musical and stylistic constraints as to limit their usefulness to the avant-garde, especially if they afford a reliable palette of tones which is technically accessible to the average orchestral player.[53]

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[51] The idea that renewing the acoustic instrumentarium is not affordable is easily exaggerated, relative to the intrinsic cost of sustaining a classical music culture which is fundamentally based around acoustic instruments, and all that this entails. Seen from one extreme, a project to create new acoustic instruments might be viewed as a futile diversion from an inevitably electronic future; from another extreme, as an attempt to 'force' the natural evolution of 'harmonic' language. In this paper these extreme positions have largely been ignored - because they don't match the facts. The pursuit of classical music has never been an entirely popular activity - its audience has always, to some extent, been specialist. The overwhelming majority of classical music making today, at least in Europe, is based on acoustic instrument performance (this is almost a tautology), and is still supported by a thriving gamut of teaching, master-classes, concerts and festivals. Admittedly, of these only a small proportion are of new or experimental music. The costs of this are not met entirely by box-offices, but no-one would seriously suggest that this is a reason to disband these activities - nor to argue that they should cease to *develop*. It is argued here that the development of new acoustic instruments is the next logical step in this tradition, and should not be sabotaged by extreme pessimism about the viability of this tradition as a whole.

[52] For example recent CD catalogues increasingly show contemporary ensembles recording arrangements of popular music.

[53] This does not necessarily depend on the tuning system *per se*. For example, some of the musical examples on the CD which accompanies Sethares' book *Tuning, Timbre, Spectrum, Scale* clearly show that tuning systems which might be thought completely unusable outside the context of the avant-garde can be made perfectly accessible by matching scale and timbre - this is at least the case for electronic composition. For acoustic instruments there are greater limitations, but in fact almost all such systems (except the most limited) can be used successfully, within limitations, for a wide variety of purposes.

The advantage of new orchestral instruments is that they would have something new to say. If new instruments were successful, composers would initially have the opportunity to write invigoratingly new chamber music, in which the special beauties of alternative tunings and intonation could be realised accurately, along with new timbres, and with their full expressive potential. In the long run, if the characters (and foibles) of new instruments gained acceptance, I believe that orchestras would be persuaded to partake in a new evolutionary stage in the language of Western music. For Goehr was right, broadly speaking, when he wrote:

> However much composers go on searching for new sound worlds - and platforms - most have to acknowledge a secret hankering for the luxuriousness and infinite variety of the orchestra, if only they could get their hands on one.[54]

The 'Orchestra of the 21st Century', if it is to survive as a living forum, must surely incorporate new instruments. Perhaps it will be able to shift easily between a number of different tuning and instrument systems. New instruments, if successful, would probably, by such a time, already have become attractive to jazz and popular music.[55] I believe we will really need *living* orchestras in the 21st Century. We need them to continue to master a wide spectrum of contemporary music, and to astonish and move us with the new. An adapted or new system of conventional orchestral instruments must surely play a part in this, and would encourage a new mainstream to be created, perhaps incorporating completely original instruments, and adaptations of instruments from other cultures.

Acoustic & Electronic Instruments, Funding and Market Forces

There has been a phenomenal acceleration of interest in ATS since the advent of digital music technology in the 1980's. The technology makes it easy (relatively speaking) for composers to create and hear music in radically alternative tuning structures, in ways that were previously impossible. It also offers the means of exploring other newer, even more radical musical ontologies, and some argue that the 'next stage' of Western musical evolution (if we are not already in it) will be electronic, because tradition will reformulate itself from the intertwining of contemporary technology and sensibility.[56] Yet acoustic music inevitably differs from electronic and electroacoustic in terms of distinctive qualities of sound.[57] Leaving aside questions about 'museum culture',[58] - the future is not *inevitably* electronic, even if, to some, acoustic instruments seem *passé*. Some electronics already do too.

Musical pluralism is here to stay. Therefore, enabling acoustic music to try to keep pace with the global force of digital technology has a value in itself. With a few exceptions, advances in electronics and computers have virtually replaced radical advances in acoustic instrument design in the latter half of this century. Fifteen years ago a computer and electronic music set-up was an expensive and enviable gadget. A new computer, synthesiser/sampler, with sequencer and sound modelling software, can cost considerably less than a brand new, none-too-musical sounding, lower-end of the range piano.[59] This trend will accelerate as the power and speed of processors and software continues to accelerate, which, in itself, can only be welcomed. But while on the one hand digital technology sustains electroacoustic music, it also makes possible the (albeit rather stiff)

[54] *The Listener*, 19 November 1987, Vol. 118, No. 3038, p. 16.

[55] The are a number of pop and jazz musicians based in the USA making music in various form of ATS: for example, Pat Metheny has used guitars refretted for 31-ET and other ATS; similarly, Neil Haverstick explores 19 and 34-ET; Jon Catler uses 31-ET and 13-limit JI.

[56] See, for example, Joel Chadabe, *Electric Sound - the past and promise of electronic music*, Prentice Hall, 1997.

[57] I intend no value judgement by this remark. The fundamental distinction between acoustic and electronic instruments is considered here to be whether or not an instrument's sound is primarily emitted through a loudspeaker. There may come a point in future when loudspeakers become acoustic instruments: for example, one might listen to a string quartet through a hi-fi in which the loudspeakers are resonating string instrument bodies - but this is some way off…

[58] In referring to 'museum culture' I do not intend to denigrate it. Who does not listen to brilliantly performed good music of the past in preference to poor contemporary music?

[59] '[C]onventional piano sales fell by approximately one-third between 1980 and 1989. During this same time, electronic keyboards proliferated with ever decreasing costs, and ever increasing sophistication. In 1985, ten times more electronic keyboards were sold than conventional pianos. In 1986, seventeen times more electronic keyboards were sold, and in 1987, over 30 times more were sold. It should be noted that these figures compare all electronic keyboards to all pianos. Therefore sales of very low-cost instruments are compared with much more expensive ones'. *Encyclopaedia of the Piano*, ed. Robert Palmieri, Garland, 1996, p. 143.

simulation of acoustic instruments playing in ATS. As young composers increasingly turn to the new technology, the number of ATS being explored will multiply, as will the diversity of musical styles.[60]

Today, far greater research budgets are being directed to computer and electronic music than to acoustic instrument research. Perhaps there are more individuals who want to research in electronics, which are more exciting (and easier) than *refining* conventional acoustic instruments. But perhaps this is also because the idea of creating *new* versions of acoustic instruments is not widespread, and is a by-product of the opportunities available. The advance of the technologies which are outpacing the development of acoustic instruments is also largely the result of economic factors. In saying this, I am not in any way disparaging electronic or electroacoustic music. It is a marvellous but mundane fact that many electroacoustic composers are able to work in university studios and at home because the relatively low price of their tools is an indirect result of the popular music market, and of the computer industry in general.[61] Not altogether dissimilarly, the development of the modern grand arose out of the sustained piano boom of the late 19th Century, and which peaked around 1910.[62] Unfortunately, acoustic instrument research today is not served directly by an equivalently wealthy economic engine.[63]

It is well known that little of the technology of existing orchestral instruments is originally of the 20th Century. In proposing this project, the hope is that new developments of acoustic instruments will take advantage of and develop in parallel with the development of electronics and electronic instruments. With the exception of manufacturing refinements and some modern percussion,[64] the most recent features of the orchestra are the systems of levers, rods and valves of woodwind and brass, and some aspects of the piano. For example, a Buffet 'Böhm system' clarinet, designed in 1839 and patented in 1844 was made of 'boxwood and brass, but otherwise hardly differ[s] from the modern [clarinet] save in the more delicate keywork';[65] the saxophone was patented by Adolphe Sax in 1846; the Böhm flute in 1847; and '[i]n 1859, Henry Steinway Jr. obtained a patent for cross-stringing grands… [the] 1864 Steinway is in all essentials a modern piano and sounds like one'.[66] Yet, for example, the deliberate development of tuning and timbral relationships in acoustic instruments - for example by making a woodwind with a square bore and relating its scale to its resultant sound, or by creating computerised electro-mechanical or piezoelectric keywork and interfaces for various acoustic instruments - would take them fully into the 21st Century, while remaining wholly acoustic and 'personal' in terms of sound emission and human artistry.[67] Similarly, the 'logical feedback' mechanism suggested below (although in this case entirely speculative) may be a similar example.

Obviously the task is not defined solely by technological challenges - the choice of a provisional ATS standard, the relationship between tuning and timbre, musical and aesthetic considerations, and instrumental feasibility - are all interdependent. As stated earlier, the cost of developing, say, a single high-quality orchestral instrument,

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[60] There is no question of renouncing this diversity. But which future institutions will be able, in Goehr's terms, to provide forums for comparison, conversation, and friction? What kind of future institutions *could* provide a truly plural forum - one which includes the modern orchestra? Concert attendance is tiny in comparison to television and home audio. The internet, too, may become increasingly participative - perhaps it will become a central forum where composers swap 21st Century versions of 'General MIDI' or 'General Digital' files, realised on powerful home studios, in a virtual realisation of Schönberg's *'Society for Private Musical Performances*'. Some may see this as a logical end to public alienation from contemporary music, and individuals may find it exciting or nightmarish. It would be wrong to exaggerate this, but it does pose the serious question - what are the alternatives for the development of acoustic music, and especially the *modern orchestra*? Some already think that a 'modern orchestra' is a contradiction in terms.

[61] The same might be said for composers using MIDI as a compositional tool. Classical musicians are often horrified when they hear electronic reproductions of conventional instruments - how could such a thing threaten the future of acoustic music? But the new technology is in its infancy. In any case, the reproduction of conventional instrumental sounds is not what normally interests contemporary composers about digital music technology.

[62] See: Pianoforte, §I, 8, *The New Grove Dictionary of Musical Instruments,* MacMillan.

[63] An exception is computerised manufacturing techniques: for example, the computerisation of brass component manufacture means that cheaper instruments can today be made to the same tolerances or better than the most expensive brass instruments of the past.

[64] For example, the Celesta was invented in 1886 by Auguste Mustel in Paris; the first vibraphone was created in 1922 by Hermann Winterhof in the USA. *Encyclopedia of Percussion,* ed., John H Beck, Garland, 1995.

[65] Anthony Baines, 'Clarinet', *The Oxford Companion to Musical Instruments*, Oxford University Press, 1992.

[66] Edwin M. Good, *Giraffes, Black Dragons and Other Pianos, A technological history from Cristofori to the modern concert grand*, Stanford University Press,1982, p. 178. 'Numerous technical refinements since the invention of the Steinway overstrung grand have promoted stability and durability, but the fundamentals of the modern concert grand were present by 1860...', Pianoforte, §I, 7, *The New Grove Dictionary of Musical Instruments*, MacMillan;

[67] I am referring particularly to the principles behind Giles Brindley's 'Logical Bassoon', and the extension of that principle to a range of other instruments, and related adaptations, which are described in Section 5.

in a given ATS, is difficult to justify (i) without an ATS standard or standards; (ii) without developing a system of instruments designed to be played together; and (iii) unless new instruments can in some sense be made available as a public resource. Moreover, there are more systematic reasons why orchestral elements should not be created in isolation, or without an agreed set of standards:

- a given number of scale degrees may be practicable on some instruments but not others;[68]
- for a non-12 scale (equal-tempered or otherwise), instrumental design, pitch standard (e.g. A=440 Hz) and notational standard(s), are inextricable;
- the ATS standard itself and a collaborative effort to examine the interrelations of the many criteria which are relevant to that choice. (In Section 4, pp. 25 - 61, 25 broad criteria are presented, each of which suggests a research project in itself);
- the relation between timbre and tuning - in terms of both musicality and feasibility (this is discussed particularly on pp. 39 - 49 and pp. 101 - 104);
- the possibility of creating standard electronic keying systems which are standardised for all woodwind would have to be done collaboratively; other advantages of collaborative development of 'virtual interfaces' for acoustic instruments are also suggested below;
- the development of new MIDI protocols (or their equivalent), and special new electronic interfaces and controllers, is only practical if new tuning and notational standards are in place;
- performers need to be confident that the time they invest in learning a new fingering, a new technique or a new notation is not time wasted - and that they are not going to have to learn something all over again to achieve a similar result;[69]
- composers are in a similar position, needing established standards for pitch system, notation and instrumentation.

A task of this magnitude can only be achieved by a wide ranging collaborative project, organised and co-ordinated by a permanent Centre, or group of 'Centres'. Ideally, these may be able to mediate and focus the interests of instrument manufacturers or builders, and of composers, performers, technologists, and so on. The value of collaborative research should certainly outweigh reservations about institutionalisation, because there are well defined research tasks and goals. There may also be benefit for composers and performers to have an independent institutional channel as a link with commercial instrument manufacturers.

An element of this discussion which is somewhat taken for granted, but which is of primary importance, is the need for training in ATS - particularly aural. The inner hearing of melody and harmony in ATS is perhaps the most important aspect of the whole project, for which new instruments are, in a sense, merely the physical vehicles of its realisation. The creation of interactive CD-ROMs and other programs as ear-training tools is one avenue to pursue, as will be specially composed technical studies for new instruments. But again, performers and composers especially would benefit immeasurably from a more widespread availability of training in ATS, and the more established knowledge that would result from intensive collaborative research.

The facilities needed to co-ordinate this work, and ultimately the need to realise these objectives in the context of professional music making, must be available. As suggested, the project need not be housed under one roof - indeed, as an international collaboration it could not be - but would require a central residence, and considerable funding and organisation; some full- or part-time posts would certainly be necessary. New strategies for long-term funding must therefore be found. Realistically, the cost of establishing this project compares favourably with other large-scale arts ventures of comparable importance. In the UK, for example, recent investment in new concert halls has been considerable, as are their annual running costs. The manufacture of new acoustic instruments would constitute a tiny fraction of such investment, and the long-term potential for the possibilities and quality of new music it would bring is very considerable. What could be *said*

[68] The relevant criteria are highly miscellaneous: singers', brass and string players' abilities to pitch large intervals accurately; string players fingertip widths; the length of the piano (due to the relationship between harmonicity and the length of piano strings); the number of effective tone-holes that can be placed on a woodwind instrument; keyboard design; the sizes of vocal and instrumental vibrato, etc. These are discussed separately in the relevant categories below.

[69] This depends on the instrumental designs that are feasible, and are adopted. As discussed below, 'logical' woodwind, brass or keyboards may allow multiple ATS to be controlled, for example, using generic fingering-systems.

in music in those new concert halls could be profoundly enriched, and it is strongly predicted that a Centre and new acoustic instrumental resources would make an important contribution not only to contemporary classical music, but ultimately to every domain of musical life.

3. Virtuosity and Otherness

Extended Techniques[70]

In the same article quoted above, James Wood wrote:

> Many would argue that a more realistic future for acoustic microtonal music lies in writing for instruments which do not have to be specially built such as strings, wind, brass and voices, thereby employing the finer ear and technique that many of today's musicians can boast. It is, however, precisely these conventional instruments which in practical terms are incapable of consistently accurate realisation of micro-intervals because of the subjectivity involved... if we want to achieve any degree of precision, we have to build special instruments.[71]

Wood perhaps overstated here (1986) the impossibility of aural and psycho-motor learning. Since the publication of that article great strides have been made in achieving alternative tunings, as performers have become more familiar with alternative intervals and harmony, particularly in 24-ET but also in other tuning systems.[72] The ability of performers to adapt to these demands cannot be overemphasised. Nevertheless, in important respects Wood's argument still holds true.

In much contemporary music heroic efforts are made to overcome the difficulties of playing music requiring sounds and intonations that differ, both from that for which the instrument was originally designed, and from performers' original training. Brilliant woodwind players, in particular, exploit their instruments to the full with a virtuosic mastery of unconventional fingerings, and to an extent their efforts are successful. In 1990, for example, the magazine *Pitch - for the International Microtonalist* published an extensive list of special fingerings for orchestral woodwinds (also the recorder and horn) covering a number of alternative tunings - including just intonations, quarter-tones, and even 31-ET and 72-ET for some instruments.[73] It is important to stress that I am not arguing *against* the development of extended techniques. On the contrary, it is intriguing to imagine what might be possible with new 'extended' techniques on new instruments: the design of new instruments should surely acknowledge these techniques, and build on them.

However, if an instrumental technique requires heroics and still remains tonally variable and somewhat finicky, we should not necessarily expect it to be standard fare for an orchestral musician. Goehr argues, for example, that unless orchestral performers - and the argument applies equally in other music - are made to feel they are interpreters, a work is unlikely to become part of the repertoire.[74] This is because, at various levels, the interpretative involvement of performing a work is crucial to it coming alive. This argument has repercussions for the use of extended techniques because, arguably, performers will have more opportunity for interpretative involvement if they can, with confidence, achieve precise tunings on an instrument specifically designed for the purpose, rather than struggling for a pitch or tone with a make-do technique, and being less than certain of success. The 'danger' of the latter pleases some players, but equally they may enjoy new challenges on new instruments.

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[70] The phrase 'extended techniques' is used here to designate not only advanced or unusual techniques, but those which at the same time transcend the original instrumental design or intention.
[71] James Wood, op. cit., pp. 328-9.
[72] Points similar to these were made by James Wood himself in a personal communication to the author in 1996.
[73] *Pitch: for the International Microtonalist*, Vol. 1, No. 4, Spring 1990. Other manuals of microtonal instrumental techniques include the *Microtonality Manual*, co-ordinated by James Wood and co-produced by the *Centre for Microtonal Music* and the *SPNM*, July 1991; Robert Dick, *The Other Flute - A Performance Manual of Contemporary Techniques*, Multiple Breath Music Company, 1989; Thomas Howell, *The Avant-Garde Flute: A Handbook for Composers and Flutists*, Berkeley, University of California Press, 1974; Peter Veale, et al., *The Techniques of Oboe Playing: A Compendium with Additional Remarks on the Oboe d'Amore and the Cor Anglais*, Bärenreiter, 1995; Philip Rehfeldt, *New Directions for the Clarinet*, University of California Press, 1994; Gerald James Farmer, *Multiphonics and other Contemporary Clarinet Techniques*, Rochester, New York, Shall-u-mo Publications, 1982. In terms of microtonality there is an emphasis on quarter-tones in each of these manuals except *Pitch*.
[74] *The Listener*, 31 December 1987, Vol. 118, No. 3043, pp. 15-16.

The view I am advocating here is controversial. For example, some very experienced wind players would argue that achieving ATS with conventional woodwinds is not as difficult as it is sometimes made out to be.[75] This view is based on two main points: firstly, good intonation on any wind instrument depends on well-practised, pre-emptive anticipation of the precise way each pitch of a scale is produced, through a combination of physical and aural memory and intention. No matter how well an instrument is designed to produce a particular scale, these skills are necessary for playing in tune. So realising music in an ATS is not so different from playing in 12-ET - given the time, ear and aptitude to learn an ATS. On this view, heroics aren't necessary: if a musician can acquire an ear for an alternative scale, performing it accurately on a conventional instrument is not exceptionally or impossibly difficult.

There is a lot of truth in this. The bassoon, for example, is commonly thought to be particularly difficult for a beginner to play in tune, precisely because it is far from being of intonationally perfect design. As Edgar Brown writes:

> It is commonly reported by players - and especially beginners - on the bassoon that the existing instrument suffers from problems of poor intonation and variability of tone-quality between one note and another. In the hands of an expert player, nothing seems to be amiss and any attempt to change the bassoon might seem presumptuous. However, the same could be said of several instruments which, for reasons of state of development or desire for historical authenticity rely on an advanced degree of skill for an acceptable performance.[76]

In short, playing a woodwind 'in tune' has more to do with the performer than the instrument. However, the 'non-heroic' view also rests on a second point. This is that all (or almost all) possible tones may be produced on a conventional woodwind[77] using a combination of embouchure and fingering in which (a) there is no loss of tone quality (or at least none so great as to be considered a problem), and (b) which are no more difficult than those of 12-ET (or none so difficult as to cause a loss of facility). This can hardly be true.

If it were true, why would it matter where the tone-holes were placed on the instrument in the first place? Surely it matters because it makes playing in-tune (and 'in-timbre') easier. Woodwind and brass players sometimes complain that an instrument is 'unplayable'. This may be because a tone-hole is badly positioned, or because of ill-designed trumpet draw lengths, or for some other reason. For example, if a clarinet was built for a pitch standard other than A440 - eg., A435 or A455 - then with the barrel pulled out (or pushed in) to tune to A440, the scale may become unplayable.[78]

David Doty summarises these points accurately:

> You can't just 'push and blow' and expect to get accurate intonation. You have to use your ears, breath and embouchure in co-ordination (this is, of course, just as true of equal temperament and standard fingerings)… Alternate fingerings on wind instruments will produce variations in timbre and volume as well as pitch. The resulting tones may or may not be acceptable in a given musical context.[79]

To make an analogy, a professional is able to do much more with a defective instrument than an amateur, and, to an audience that doesn't know any better, may be able to disguise the defects entirely. This is, I think, not an

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[75] Some of the following text is taken from my part in a discussion, particularly with Johnny Reinhard, on the Tuning List (internet discussion list) at tuning@eartha.mills.edu. (Digests 1230-1247, November 1997). Reinhard is a composer, bassoonist and the director of the *American Festival of Microtonal Music.* It should be pointed out that Reinhard has been performing music composed in quarter-tones, 31-ET and the like with his colleagues for over a decade. Unfortunately I have not heard Reinhard play, but I have heard celebrated reports of his ability with microtonal music on the bassoon.

[76] A. Edgar Brown, 'An experimental bassoon', *Proceedings of the Institute of Acoustics*, Vol. 19, Part 5, 1997, p. 279.

[77] This discussion focuses on woodwind, but the principles are equally applicable to brass and strings. The necessary qualifications are perhaps not obvious, but familiarity with Section 5 below should make them clear.

[78] See O. Lee Gibson, *Clarinet Acoustics*, Indiana University Press, 1994, p. 26.

[79] Doty, op. cit., p. 67. Doty's book was unknown to me when I began writing this paper, so I was delighted to discover the useful and highly relevant discussion in his last chapter - 'Practical Just Intonation with real instruments'. Where appropriate I have tried (somewhat belatedly) to reference points of overlap between Doty's chapter and this text, although for the most part Doty discusses (and advocates) achieving JI on existing instruments, in contrast to the present discussion.

unreasonable parallel to the situation of performing ATS on woodwind today. The extraordinary fact that players such as Pierre Yves Artaud, Anne La Berge, Laura Chislett, Ian Greitzer, Roger Heaton, Heinz Holliger, Kate Lukas, Kathy Milliken, and Johnny Reinhard, to name only a few, can achieve approximations of some ATS with existing woodwind makes one wonder what *more* they might achieve on an instrument which is not (in effect) a 'defective' instrument relative to whatever ATS is being realised. More to the point, and this is crucial to my reasons for writing this paper, it suggests that very many *more* players would be able to play works using ATS if effective new instruments were available.

Roger Heaton, for example, writing about the clarinet, suggests that:

> Quarter-tones can be articulated and heard very clearly, eighth-tones are much more approximate and rely on lip bending… Quarter-tones, in the main, use real fingerings but are often of poor quality. Always play them in context, *in relation to the fixed chromatic notes.* You may have to widen micro-intervals, depending on the voice leading, so that the quarter-tone becomes a discernible pitch rather than sounding like bad intonation![80] (my italics)

This makes sense regarding quarter-tones, particularly in music in which the additional pitches are of ornamental or secondary structural importance to the normal 12 notes, and where strict intonation may be compromised to ameliorate timbral inconsistency. But it makes little sense for playing chromatic music in 19-ET, 31-ET or many other systems, because the 'relation to the fixed chromatic notes' (of 12-ET) of those systems is complex, and because of the tonal variability that will result in such systems. Imagining and projecting radically alternative scales on a conventional woodwind in the 'non-heroic' way described above seems to be the only way of achieving them successfully on existing instruments. But it is still rather like trying to play Bach on a shakuhachi, or Webern on a flageolet.[81] In the long run, specially made instruments are more likely to succeed for ATS than conventional instruments,[82] and are necessary for the real acceptance of ATS into the repertoire, especially in orchestras but in contemporary ensembles too.

Since contemporary music has made a virtue of extended techniques, and because any intonational subtlety seems possible, it is tempting to conclude that there is no need to build new instruments, only a requirement to study alternative techniques. It is true, for example, that many performers, especially from musical cultures other than our own, create music on Western instruments which sometimes sounds as if the instrument on which it is played is transformed, and in itself is not bound to an intonational system. For example, a more or less identical clarinet might be used for Mozart's Clarinet Quintet and the 'Bulgarian' jazz as played by the well known Bulgarian clarinettist Ivo Papasov. Studies of non-classical instrumental techniques are extremely worthwhile, and could benefit from the kind of co-ordination provided by the project envisioned in this paper. But this is only one aspect of how ATS might develop, and it does not make sense to limit the future possibilities of acoustic music to *already existing* experiences and conceptions of pitch. Nor should we let a fascination with and admiration of virtuosity rule out simple and perfect otherness.

To my knowledge, no definitive scientific study has been made of the results of using extended techniques on existing instruments with reference to the full range of ATS. Together with the issue of *Pitch* mentioned above, the nearest examples of comprehensive study/manuals are Robert Dick's '*The Other Flute*' and Philip Rehfeldt's '*New Directions for Clarinet*', but these are largely restricted to quarter-tones (Dick signals smaller distinctions by arrowheads, Rehfeldt includes eighth-tones). Studies are needed which make strict comparison between the result of conventional techniques in 12-ET, and extended techniques for a variety of ATS. These should show, relative to each tuning system, intonational (in)accuracy, timbral (ir)regularity and technical

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[80] Roger Heaton, 'The contemporary clarinet', in *The Cambridge Companion to the Clarinet*, ed. Colin Lawson, Cambridge University Press, p. 173.

[81] Suppose we imagine, for example, a virtuoso shakuhachi player, steeped in traditional Japanese music, who he has little knowledge of Western music, but listens repeatedly to recordings of Debussy's *Syrinx*, Varese's *Density 2.5*, and Boulez's *Sonatina*. He might be fascinated by what he imagines to be a kind of strange, altered shakuhachi music, with odd timbre and intonation, and will try to emulate it on the Shakuhachi. In trying to realise alternative tunings on conventional orchestral instruments, we are in a sense putting ourselves in an analogous position, but - unless we use electronic techniques similar to those described in Section 6 - without 'recordings' to help us.

[82] This discussion has focused on instruments of semi-variable intonation - clearly, for instruments of fixed intonation the point is even more obvious.

facility/difficulty. It is likely that this would confirm the need for new woodwind and brass instruments for ATS, and provide an objective measure against which the effectiveness of both conventional and unconventional techniques on new instruments might be judged.

Orchestral Electroacoustics

Alternative tunings are not a concern of all contemporary composers. Some may feel they are simply irrelevant or inappropriate.

There are others who use alternative tunings regularly, but who perhaps also feel that new instruments, built in a radical tuning system, are unnecessary to their own vision of music. I am thinking here of music which emphasises the timbral-harmonic sculpting of sound and texture, but in which pitch, although important, remains somewhat subsidiary to timbral and gestural elements. It might be argued, for example, that 'quasi-electroacoustic' chamber and orchestral works, 'spectral' music, or works of 'new complexity' fall into this category.[83] In these kinds of musics, performers are often required to bend notes more or less intuitively, while often being notated in quarter-tones. Here, 'strict 24-ET' is normally even less appropriate than 'strict 12-ET' would be in more conventional music. In these kinds of music, therefore, it is arguable whether there is benefit to explicitly composing in a more 'radical' or harmonic system than 24-ET, since the melodic and harmonic 'otherness' of the alternative can be greatly diminished if highly complex sound masses occur as frequent compositional structures. For this reason, composers, who on the face of it might be expected to support the development of new instruments, may in fact see current instruments, notation and performance practice as broadly satisfactory. Thus, for a range of composers, the effort proposed in this paper may seem unwarranted, or for nothing.

But equally, it may not. On the one hand, many kinds of tonal music (including minimal compositions, for example) would benefit considerably from an alternative system of instruments, perhaps enabling reliable and alternative structures of just intonation or other ATS. Gayle Young, for example, reporting her experience of building the 'Columbine' (a metallophone tuned in a just intonation system using 23 pitches per octave), writes:

> Some very simple pitch structures which sound banal when played on a piano take on a new vitality when played on the Columbine.[84]

On the other hand, composers working with highly complex sound masses, or exploring forms of 'spectral' harmony or post-serialism (especially if electronics are incorporated) may be crying out for new acoustic instruments. Later on, Sethares' notion of the *coevolution* of instruments, tuning and timbre is discussed in more detail (pp. 39 - 49), as is the idea of a radically 'inharmonic instrumentarium' (pp. 101 - 104). This is the idea that acoustic instruments might coevolve towards a complementary system of tuning and *deliberately* inharmonic timbre. This is a controversial suggestion, because it is as yet unknown how acoustic sustaining instruments could be made to achieve this. But it is likely that composers would quickly take up a new vocabulary of instrumental composition if alternative, controllable, inharmonic timbral structures were possible with acoustic instruments, and if composers were no longer restricted to acoustic instruments originally intended for earlier melodic and harmonic models. The possibility of integrating mutually conceived melodic and harmonic material within an appropriate inharmonic tuning/timbral system would surely suggest ideas for extraordinary music.[85] If acoustic instrument design is unable to keep pace with electronics - in terms of timbral manipulation, sonic dexterity, performance speeds, accuracy etc. - this may yet be an equivalently radical, yet purely acoustic, path to follow.

Similarly, when orchestral instruments are used in mixed acoustic and electroacoustic works, there is a tendency for them to sound anachronistic (no matter how many modern techniques are used). If this is to be

[83] My personal view is that this is not the case.
[84] Gayle Young, 'Pitch probe. Musical instruments for unusual tuning systems', *Contemporary Music Review*, Vol. 7, Part 1, p. 56, 1992. Of course, what is meant here is that the pitches are actually different.
[85] For example, if radically new timbral/harmonic universes become possible for acoustic instruments, they will engender new mutual processes of harmony and form. New acoustic instruments might thus offer radical solutions to the kind of questions that James Dillon has asked - 'what kinds of functionality can we assign to timbre within a musical form?' - James Dillon, 'Speculative instruments', *Eonta*, Vol. 2, No. 1, 1993/4, p. 33.

avoided, some degree of 'synchronisation' between electroacoustic and acoustic instrument design should be encouraged - that is, synchronisation between an electroacoustic conception of sound as an open continuum, evolving and in process, and an instrumental conception comprising relatively closed, discrete scales, and relatively stable, fused, harmonic timbres. If new acoustic instruments merely substitute one scale for another, with no adaptation of timbral production, then they may make little or no difference to this aesthetic divide. Yet even within the electroacoustic aesthetic context, there may be good reasons for developing new acoustic instruments in an alternative system of tuning, due to their role in the coevolutionary process. Certainly electronic and electroacoustic music, in both their 'academic' and popular forms, have an increasingly large influence on the broader 'coevolutionary' musical development we are witnessing.

4. Which Alternative Tuning Systems would be most effective for the 'Acoustic Orchestra of the 21st Century'?

Criteria

In the short term it is unlikely that financial commitment to developing more than one alternative system of orchestral instruments will be found. Moreover, the individual characters of different ATS correspond to a wide range of musical possibilities: a truly 'universal' system of fixed tuning is not possible because all discrete systems have a more or less unique 'xenharmonic' character, at least when considered from the point of view of instruments of fixed intonation.[86] Composers will want to embrace the system (or systems) which their own musical vision demands, and already some Western composers are committed to a chosen alternative system (or multiple systems). Therefore, if only one system of new instruments is affordable, for which ATS should they be built? Alternatively, is it at all conceivable to build woodwind, brass, keyboards, percussion and strings, such that each individual instrument is acoustically and ergonomically optimised for *multiple* tuning systems?[87]

As we have seen, one of the reasons in favour of retaining conventional woodwind is that extended techniques are already used to realise 'multiple' systems. If *new* woodwind were created to be equally or even more versatile, they must equal or improve on existing instruments in terms of intonational accuracy, quality of tone, ease of performance, and the number of alternative systems which they allow. Ideally, the set of ATS for which each new instrument would be capable would be the same (or similar) for every instrument. Various ideas are suggested in Section 5 with this in mind, and some may be worthy of further exploration. However, as stated previously, instruments themselves are merely the physical vehicles of music making, and it is unlikely that a majority of musicians will be prepared to master a wide variety of ATS and their corresponding notations. Therefore a 'unique' alternative system is the more likely option, at least in the first instance. A new system of instruments for a single ATS, each element of which maximises intonational accuracy, quality of tone etc., is already a very formidable task. However, the ideal, 'multiple system' option will be borne in mind in what follows.

For both approaches, we need to distinguish which tuning systems are the most suitable for research and prototyping, over and above individual preferences for particular alternatives.[88] What criteria, therefore, are relevant to choosing a unique system? Which ATS would be most effective for new instruments, and would justify long-term investment? Which system(s) will provide the widest intrinsic musical effectiveness?[89]

A set of 25 broad criteria is suggested below as a practical and theoretical means of comparing the character, feasibility, advantages and disadvantages of different ATS. It is not the intention to discuss each of these criteria in detail. But there are four main reasons for presenting them: firstly, to define the scale of the task before us; secondly, to suggest that, alone, no single theoretical perspective alone is adequate to this task;

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[86] Broadly speaking, the smaller the number of divisions in the octave the more specific the character of the system, the larger the number of divisions the more generalised. A system such as 72-ET, as has been adopted by the composer Ezra Sims, might be thought to transcend this difficulty. But, rightly or wrongly, I have deemed it not feasible to conceive an instrumentarium of acoustic instruments built specifically for 72-ET, or other very large division systems. In this paper I consider the upper practical limit of instrument design (considered en masse) to be 41-ET, but this is somewhat arbitrary.

[87] That is, so that each of a number of ATS can be realised ideally (or adequately) on every (or almost every) *single* instrument.

[88] If this would seem to place an inordinate power over the future of acoustic music in the hands of acousticians, instrument designers and tuning theorists, please refer to the conclusions of this paper.

[89] In addition to learning to identify and hear alternative systems in the inner ear, learning to sing them, following one's own ear in the compositional process, and experimenting on conventional instruments, the most practical approach to judging that a system 'sounds good' might be to build simple, relatively cheap, acoustic instruments in many different pitch systems, and experiment accordingly. My own efforts have focused similarly on creating high quality electronic simulations of orchestral instruments, and composing and creating simulations in specific ATS with these. My experience has been that, when imagining a work in my mind, it has often been clear that it could not be accurately expressed, for example, in 12 or 24-ET, but that it might be expressed and developed in *more than one other* ET system (I have limited myself , for the most part, to ET systems in the range 19 to 41 divisions). By creating two or more simulations in different ETs, of what would normally be considered to be one passage of music, I have sometimes gained strong confirmation that the music itself implies one system rather than another. I have used this as my primary criterion for deciding the tuning system for a work. This is invaluable, although inordinately time consuming. At the same time, these experiments have led to a more theoretical perspective.

thirdly, to present a common background for debate, and to distinguish various concepts;[90] and fourthly, to suggest the wide range of research areas which contributors might undertake, and which a Centre would endeavour to co-ordinate. The 'overriding' criterion would be if a composer created a body of work in an ATS which inspired other composers, performers and instrument makers to collaborate on new instruments for that system: but of course, without the instruments in the first place (other than electronics), this is unlikely to happen.

1. COMMITMENT, AVAILABILITY, COST

i) *Commitment*: the number of composers committed (or who would be committed) to composing in a given tuning system. Implicit, therefore, is that a system 'sounds good'.

ii) *Instrumental availability*: the availability of orchestral instruments already existing for non-12-ET systems (using extended techniques or otherwise).

iii) *Cost*: the relative costs of building and learning new instruments for different systems.

2. FEASIBILITY: INSTRUMENTS and PERFORMANCE

i) *Instrumental feasibility*: the number and variety of instruments which may be successfully built/adapted for a new pitch system, and their effectiveness.

ii) *Timbre*: the effectiveness of the timbres of new instruments relative to their use in different tuning systems.

iii) *Audibility*: the ability to distinguish the smallest intervals of a system, in a variety of contexts, and over a wide frequency range.

iv) *Intonational integration*: the effectiveness with which instruments of fixed and non-fixed intonation will work together in any given system.

v) *Reference tones*: the number of instruments which will produce reliable reference tones with relative ease in a given system.

vi) *Notation*: the feasibility of a simple and intuitive notational system for a given tuning; its intuitive correspondence to the conventional notation of 12-ET (and perhaps other successful notational systems - eg., 24-ET).

vii) *Intonational accuracy*: how accurately a new tuning system can be achieved in performance (solo and in ensemble) on new instruments.

3. INTERVALLIC PROPERTIES

i) *Purity*: the accuracy of approximations to (or coincidence with) *just* intervals.

ii) *Number*: the number of close approximations to (or coincidence with) *just* intervals expressed as a proportion of the available intervals of the scale.

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[90] The criteria include subjective and objective properties. Not all of them can be rigorously defined or quantified, and I doubt it would help us if they were. It goes without saying that understanding these criteria is not an abstract exercise: alternative systems must be *heard*, using electronics or whatever means are at the reader's disposal. Once heard, much of the difficulties surrounding these discussions fall away. However, there cannot exist a single tuning system which satisfies all these criteria equally: Some criteria contradict others; others are simply indexes. Consider, for example, 3.(ix) and 3.(x): Darreg and McLaren give an account of what they call 'xenharmonic bias' - that is, whether a tuning system is better suited to melodic or harmonic music. They argue, to some effect (that is, assuming harmonic spectra) that each of the equal temperaments between 7-ET and 31-ET are in varying degrees 'biased' towards either melody or harmony (Darreg and McLaren, op. cit., pp. 5-19). Nor is the *value* or relative importance of any criterion pre-judged: for example, criterion 3.(i) is not in itself an assertion that closeness to just intervals is preferred.

iii) *Distribution*: the distribution pattern across the scale of 'close' approximations to (or coincidence with) just intervals.

iv) *Primeness*: relative accuracy to ratios formed by *smallest* integers.[91]

v) *Tolerance*: the level of tuning deviation tolerance of a system.

vi) *Consistency*: the degree of intervallic coherence which a tuning system affords to melodic and chordal formations considering the available approximations to just intervals within a (tempered) scale. [92]

vii) *Transposition*: the properties of transposition and modulation for a given system.

viii) *Manageability*: the ease with which the number of scale degrees of a given system can be understood and acted upon.

ix) *Melody*: melodic effectiveness of the available intervals of a given system.

x) *Harmony*: harmonic effectiveness of the available chords of a given system.

xi) *Agility*: the extent to which both fast and slow musics can be effective in a given tuning system (conceived as an intervallic property).

xii) *Stretched-ness*: the extent to which the intervals of a system correspond to the well documented tendency of performers to pitch the higher note of (some) rising intervals slightly sharp relative to just intonations (low integer ratios).

xii) *Flexibility*: the extent to which a given system provides acceptable approximations of an extended set of scales and harmonic resources (all of which may be more or less accurately realised on instruments of non-fixed and fixed intonation).

4. FORMAL, LOGICAL and STYLISTIC PROPERTIES

i) *Form*: the extent to which a system allows or encourages musical forms - that is, forms which are acoustically significant in terms of the integration of tuning ontology and compositional ontology.

ii) *Logic*: the musical significance of the correspondence between the logical structures implicit in a given tuning system and its acoustic properties.

iii) *Style*: the variety of musical styles or modes of expression for which a given tuning would be effective.

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[91] *Primeness* is a subset of *purity*: it is listed independently because of the importance of the 5th (and the 4th) and other intervals approximating very low integer ratios. This is influential even in instrumental music in which these intervals do not necessarily figure prominently (or directly) - such as atonal or serial music. This is because these ratios are in any case implicit in implicit in harmonic tones. Although the ratios may not be critical in simultaneities, they are nevertheless influential in all harmonic implications.

[92] Many formal and mathematical analyses of what I have loosely called the 'intervallic properties' of ATS have been developed - by Balzano, Barlow (harmonicity), Darreg and Mclaren, Erlich (consistency), Euler (gradus suavitatis), Fokker (deficiency), Hahn (completeness, diameter), Krantz and Douthett, Polansky (morphological metrics), Rothenburg (propriety), and Tenney (distance functions) - as well as others, such as symmetry, diatonicity, etc. The 'criteria' listed here are intended to introduce the general issues; typically, the mathematical approaches formulate these and related criteria in some detail. As an example, however, I have singled out 'consistency' as a powerful concept which appeals to musical common sense - it is the sole 'mathematical' approach discussed here in any detail. The notion of 'consistency' was first developed by Paul Erlich - see pp. 54-56 and APPENDIX IV for further explanation. John Chalmers' book *Divisions of the Tetrachord* (Frog Peak Music, 1993) is good starting point for researching mathematical approaches in general, as is the magazine *Xenharmonikôn*. There are also software suites available, such as 'SCALA' (created by Manuel Op de Coul), which is a useful tool for the analysis, construction, and electronic realisation of ATS. Details of SCALA may be found at: http://www.tiac.net/users/xen/scala/.

Commitment, Availability, Cost

The criteria of *commitment* and *instrumental availability*, and to an extent *cost*, favour the further development of new instruments built specifically for 24-ET. This is due to the large number of musicians (at least in Europe) already experienced in working in 24-ET, the fact that some quarter-tone instruments exist and are in use, and the extent of research already carried out into quarter-tone designs (for example at IRCAM and elsewhere). The degree of aural familiarity with 24-ET, and the fact that instrumental practice is already relatively advanced compared to techniques for performing more radical systems, also endorse further commitment to 24-ET.[93]

Broadly speaking, the difficulty and cost of designing and building new instruments for ATS will increase in proportion to the number of divisions per octave, whether or not the system is equal-tempered.[94] However, research such as that carried out at IRCAM, for example, into using 'branched-tube' design to produce quarter-tone length corrections for some wind instruments, may have a more general application, making it possible to adapt wind instruments built fundamentally for (say) 11, 17 or 19-ET, to produce 22-, 34- or 38-ET - that is, without the cost becoming absurd.[95] Of course the practicality of this depends not only on this particular technology, but also on the feasibility of adapting and performing brass, strings, keyboard and percussion instruments in the same tuning system.

Other factors which may have an influence on cost include:

- acoustic instruments which simultaneously act as an electronic controller (MIDI or other);
- acoustic instruments whose *interface* for pitch control is electronic (see below for discussions of 'logical' woodwind (pp. 65 - 68 and 73 - 74), brass (pp. 82 - 83), strings (pp. 86 - 86), and keyboards (pp. 96 - 97);
- 'adaptive' acoustic instruments - that is, instruments normally of fixed intonation, but modified to incorporate the intonational flexibility of instruments of non-fixed intonation, enabling *automatic* (and configurable) intonational adjustment - normally to increase harmonic purity;[96]
- attempts to create instruments for 'multiple' ATS.

Electronic woodwind and brass MIDI controllers[97] may be used to perform ATS without insuperable difficulties. Additional MIDI functionality in acoustic or acoustic-electronic hybrid instruments, and the creation of a true acoustic-electronic hybrid capable of ATS, in which sound generation is physical (or physical-electronic) and amplification, pitch control, expression etc., are electronic (or physical-electronic) may have a bearing on this paper, but are not considered here.[98] As discussed below, 'logical woodwind' may cost considerably less than conventional woodwind, both in prototype and production, but as yet the concept of

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[93] Sims' comments on 31 and 72-ET (see note30) should also be borne in mind in this context.

[94] Similarly, for example, the cost of developing woodwinds or brass for non-octave systems would probably be astronomic - unless, for example, the 'pitch-sensor' approach discussed below was perhaps viable.

[95] The system developed at IRCAM, in collaboration with the acoustical laboratory at the Université du Maine, is protected by French patent. J. Kergomard and X. Meynial, 'Systèmes micro-intervalles pour les instruments de musique à vent avec trous latéraux', *Journale Acoustique* 1, 1988, pp. 255-70. See below, pp. 66 - 67.

[96] The idea of adaptive acoustic keyboard instruments is not new: 'In 1936 Eivind Groven, a Norwegian composer and musicologist, built a harmonium with 36 pitches per octave tuned to form an extension of Helmholtz's quasi-just-intonation scheme, but with a normal keyboard, the choice of inflection being made automatically while the performer plays as on a conventional instrument… Groven's work has made just intonation practicable on keyboard instruments that are no more difficult to play than ordinary ones'. Mark Lindley, 'Just intonation', *The New Grove Dictionary of Musical Instruments*, MacMillan, p. 339. For more recent work on the notion of adaptive instruments, see: H. M. Wagge, 'The intelligent keyboard', 1/1 (4):1, pp. 12-13; Clarence Barlow, 'Two essays on theory', *Computer Music Journal*, 11 (1), 1987, p. 53; Larry Polansky, 'Paratactical tuning: An agenda for the use of computers in experimental just intonation', *Computer Music Journal*, Vol. 11, No. 1, Spring 1987, pp. 61-8; William A. Sethares, 'Adaptive Tunings for Musical Scales', *Journal of the Acoustical Society of America*, 96 (1), July, 1994, pp. 10-18; Andrew Horner and Lydia Ayers, 'Common tone adaptive tuning using genetic algorithms', *Journal of the Acoustical Society of America*, 100 (1), July, 1996, pp. 630-40; William A. Sethares, *Tuning, Timbre, Spectrum, Scale*, Springer Verlag, 1997, Chapter 7.

[97] Sometimes known as EWI's and EVI's - 'electronic woodwind instrument' and 'electronic valve instrument', after model names used by Akai in the mid-1980's.

[98] For example, see: http://www.mit.edu/afs/athena/user/s/t/stratton/www/digtrump.html, for a report of Christopher Stratton's experimental 'acoustic-electronic trumpet'.

‘logical brass’ and other ‘logical’ instruments is too hypothetical to speculate about cost. While an adaptive acoustic piano (as suggested later) is not theoretically impossible, it would certainly be exceptionally expensive to create.

An alternative approach to reducing costs might be to decide to create three or four ‘families’ of instruments, each for an individual tuning system, without great deliberation as to which these ATS are. To some extent these might satisfy known musical preferences, reduce research costs, and allow more resources to be channelled directly into creating the instruments themselves. These ‘families’ might be, for example:

- some multiple of 12-ET - such as 24-ET or 36-ET;
- a system of extended Just Intonation;
- one of the more ‘radical’ ET systems - say 31, 34 or 41-ET;
- some kind of ‘quasi-continuum’ system - to complement the ‘electroacoustic’ model.

Nevertheless, in what follows, the value of substantial research regarding each of the above criteria is assumed.

Feasibility: Instruments and Performance

As mentioned, a survey of *instrumental feasibility* for ATS is presented in Section 5: *Preliminary thoughts regarding New Instruments* (pp. 62 - 105).

Ideally, the evaluation of the criteria of *timbre, audibility, intonational integration, reference tones, notation* and *intonational accuracy* each require considerable research and experimentation. In addition to consensus about which systems ‘sound good’, both theoretical and empirical results are needed to justify choices for research prototypes. Computer simulation and physical modelling techniques would also be appropriate.

The criterion of *audibility* is not critical in terms of distinguishing pitches at the limits of the musical range, but the practicality of distinguishing very small intervals in complex harmonic passages is an important issue. *Audibility* and *notation* are directly related to the issue of *manageability.*

The criterion of *notation* refers to the need for an intuitive and immediate representation of the pitch system itself, which consolidates the (good) habits of current notational practice. This is not, it seems, equally achievable in all alternative systems, but is unlikely to be a critical factor, especially if a *single* ATS is chosen, because performers can adapt quickly to well thought out notation.[99]

The criterion of *intonational integration* may be less of a limiting factor than an opportunity for creative rethinking. Mixed instrumental combinations for music in 12-ET which include the piano or organ pose well known intonational problems.[100] Whichever system(s) are chosen for development, they should engender new solo repertoire for fixed and non-fixed intonation instruments, and successful combinations in mixed ensembles.

Pitched sounds will function most effectively *reference tones* not only if an instrument facilitates secure pitch, but also if the instrumental *timbre* aids the perceptibility of that pitch. This is because pitch definition, and the immediacy with which a heard pitch can be identified or acted on, depends on timbral properties. Generally speaking, strictly harmonic timbres whose lower partials are not omitted or suppressed are most likely to do this

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[99] It is more difficult, for example, to define a ‘group’ of notational systems which will interchangeably serve a variety of tuning systems, in which a given pitch is notated by identical or functionally equivalent signs. No attempt is made to cover these notational issues here. For a compendious overview of the notation of ATS, see: Gardner Read, *20th Century Microtonal Notation*, Greenwood Publishing, 1990. However, see also Rudolf Rasch’s highly critical review of the same volume: Rudolf Rasch, ‘Review of Gardner Read, *20th-Century Microtonal Notation*’, *Perspectives of New Music*, Vol. 39, No. 1, 1992, pp. 258-262.

[100] Lewis Jones has pointed out that these problems occur in at least two situations: (i) the tendency of players of instruments of non-fixed intonation to seek better approximations of just consonances, and other local inflections; and (ii) absolute stability of pitch - since instruments of non-fixed intonation may wander together, for example, while a keyboard instrument is silent. Personal communication, 1997.

best.[101] In addition, the importance of providing reliable pitch references reinforces the advantages of developing instruments of fixed as well as non-fixed intonation.[102]

The criterion of *timbre* is of special importance. It is argued below, particularly following Sethares' work on tuning and timbre, that the consonance or dissonance of two (or more) simultaneous complex tones should not be considered purely in terms of the mathematical ratio of the frequencies of the fundamentals which form the interval(s) between those tones, but rather as the result of the specific relationships between the spectral structures of two (or more) complex sounds. In this sense, the notion of a 'just interval' may be interpreted as referring to relations between component harmonics of the sounds which form an interval. This has implications for the relation between tuning theory and instrument design.

Many studies of *intonational accuracy* have been carried out, with reference to a variety of instruments and tunings.[103] To my knowledge none have addressed this question specifically in terms of attempting to predict the feasibility of performance of ATS on new instruments. The subject would benefit from considerable experimental evidence. It is not dealt with here, except for the general remarks in the '*Overview*' at the beginning of the paper, and in the discussion of intonational tolerance (pp. 51 - 55).

Some Basic Terms - Just Intonation and Temperament

It is impossible to consider here all the possible kinds (let alone all the instances) of alternative tuning systems and scales.[104] Two broad categories which dominate contemporary discussion and composition are *Just Intonation* (JI) and *Equal-Temperament* (ET). Many other basic concepts, such as the *Pythagorean Scale*, *meantone-temperament* (MT), *Well-Temperament* (WT), 'non-octave' temperaments, and 'stretched' scales, are also important. The purpose here is merely to present an introduction to some essential questions of tuning theory - particularly for those to whom it is unfamiliar. In the following elementary descriptions no special knowledge of alternative tunings is assumed, nor is it the intention to evaluate the options.

Just Intonation

'Just intonation' has been described as 'the hypothesis that the ear hears rationally'.[105] In a strict (and conventional) just intonation system, each note of a scale is (ideally) tuned so that the frequency ratio between the 'tonic' pitch and each other degree of the scale forms to a 'small number ratio' (or 'low integer ratio'). Following Partch, each interval (and note) of a scale is therefore named by this ratio rather than as a scale-degree (or as C, D Eb, etc). The 'tonic' itself is denoted by '1/1' since this ratio describes the frequency ratio between a note and itself.[106] The interval of a just fifth is denoted by '3/2'[107] because that is the frequency ratio between the pure fifth and 1/1. Thus if 1/1 is 'A 440 Hz' then the fifth will be 'E 660 Hz', and this interval is equivalent to 701.9 cents.[108] 4/3 describes the just fourth (498.1 cents); 5/4 denotes the just major third (386.3

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[101] 'The cues to virtual pitch are primarily provided by harmonics below the eighth… harmonics 3 through 6 are the most essential.' Ernst Terhardt, 'The concept of musical consonance: a link between music and psychoacoustics', *Music Perception*, Spring 1984, Vol. 1, No. 3, p. 290. See also: Ritsma, R, 'Frequencies dominant in the perception of the pitch of complex sounds', *Journal of the Acoustical Society of America*, Vol. 42, No. 1, 1967, pp. 191-98. However, it would appear that these components do not have to be perfectly harmonic: see for example, Elizabeth Cohen, 'Some effects of inharmonic partials on interval perception', *Music Perception*, Spring 1984, Vol. 1, No. 3, pp. 323-349.

[102] Electronic instruments are also used to provide reference pitches, in rehearsal or performance.

[103] See for example, Franz Loosen, 'Intonation of solo violin performance with reference to equally tempered, Pythagorean, and just intonations', *Journal of the Acoustical Society of America*, 93 (1), January 1993, pp. 525-39.

[104] A list of 2000 alternative scales can be found at: http://www.tiac.net/users/xen/scala/downloads.html. A description of each of these scales in cents is given at: http://www.tiac.net/users/xen/scala/scales.html. These were compiled by John Chalmers and Manuel Op de Coul.

[105] Vogel, op. cit., p. 11.

[106] The term 'tonic' is only used by analogy: 1/1 need not function as a tonic in a traditional sense.

[107] David Doty has introduced the following distinction: *interval* relations are represented with a colon (eg., 3:2 denotes the interval of a fifth), whereas a degree of a scale is represented by using a slash (eg., 3/2 denotes the note G if 1/1 is C). Not all just intonationists adhere to this convention (including Partch whose *Genesis of a Music* was written much earlier, and in which the notation of just intervals in terms of ratios was coined). For simplicity, in this text intervals and pitches are both notated as *a*/*b*; purely mathematical ratios are notated as *a*:*b*. Doty, op. cit., p. 23.

[108] As opposed to the 'fifth' in 12-ET which is 700 cents. There are 1200 cents to the octave, and therefore 100 cents to a semitone in 12-ET. Cents are calculated using the following formula: c=(1200/log 2) x log r, where c is the interval in cents, r is the ratio, and

cents); 6/5 denotes the just minor third (315.6 cents); and so on. When low integer ratio intervals are employed using sounds which have (more or less) harmonic spectra - especially wind, brass and strings - the resulting sounds are relatively beat free, and their character is rather smooth and pure. Harmonic sounds employed in just intonation have a remarkable propensity to blend tonally.[109]

The acoustic reasoning behind JI stems from the harmonic series. Sustaining acoustic instruments produce tones which are harmonic to a remarkably accurate degree; and true harmonics stand in low integer ratio to the fundamental. The ratios which describe consecutive harmonics relative to the fundamental (in octave reduced form) are: 1:1, 2:1, 3:2, 4:2 (or 2:1), 5:4 (or $5:2^2$), 3:2, 7:4 (or $7:2^2$), 2:1, 9:8 (or $3^2:2^3$), 5:4, 11:8 (or $11:2^3$), and so on. In a modern explanation, adopting a JI system in harmonic music ensures that, either the optimal matching of lower harmonics (which are often the most prominent in instrumental sounds), or that the harmonics of simultaneities are separated by an intervallic distance which is greater than the 'critical band'[110]. In both cases this results, at least in fairly simple harmonic contexts, in smooth and resonant harmony. There is a natural tendency to JI harmony in many vocal and instrumental contexts - particularly in choirs, wind and brass ensembles (especially they are not mixed) - although this is seldom 'strict' or conscious.

Composers and theorists deliberately working with JI have adopted the notion of a 'Limit' to describe the highest number (or factor) which occurs in any interval ratio within a given system.[111] Commonly, '5-Limit' JI refers to any scale created on the principles outlined in the previous paragraph, in which the ratios which describe its intervals contain no number greater than five (or some product of numbers less than five).[112] Thus a composition or a compositional system may be said to be in 5, 7 (or higher)-Limit JI. Although (for obvious reasons) a 'Limit' is normally prefixed by a *prime* number, this is not necessarily the case because a composer may adapt freely their conception of JI for the purpose at hand. The term 'extended JI' does not have a fixed meaning, but may refer to systems of JI including 11 or 13-Limit intervals (or above), or other systems based on more radical adaptations of JI principles. The use of 11 and 13-Limit JI is posited on the audibility of the 11th and 13th harmonics, yet many listeners feel that while the octave reduced versions of 11th and 13th harmonics are themselves consonant, other 'primary' or 'secondary' intervals belonging within 11 and 13-Limit JI are not.[113]

(1200/log 2) is the constant value 3986.313714. For example, the ratio 16/15 gives the interval in cents (3986.313714 x log (16/15)), which equals 111.731 cents.

[109] The theory of Just Intonation is based on the notion that just ratios (3/2, 4/3, 5/4 etc.) define just intervals. This is may legitimately be taken to mean that such ratios define points on the intervallic continuum for the greatest 'smoothness' of simultaneous dyads. It is important to distinguish this from the related but distinct notions that - just ratios define 'primary' intervals, or that a just ratio locates the intervallic 'centre' of certain primary intervals, or even that just ratios define 'natural intervals'. While the 'smoothness' concept is true for harmonic tones, the latter notions are by no means obvious, especially considering melodic as opposed to harmonic intervals. Burns and Ward argue that 'there is no evidence for the existence of natural perceptual boundaries for melodic musical intervals' and that this lack of evidence 'implies that individuals in a given culture learn the scales of their culture from experience, not because of any innate propensity of the auditory system for specific intervals'. Edward M. Burns & W. Dixon Ward, 'Categorical perception - phenomenon or epiphenomenon: evidence from experiments in the perception of melodic musical intervals', *Journal of the Acoustical Society of America*, 63 (2), Feb. 1978, p. 466. Similarly, Parncutt and Strasburger suggest that 'The role played by frequency ratios *per se* in interval perception is doubtful… psychoacoustic research suggests that intervals are perceived not as frequency ratios but as pitch distances.' Richard Parncutt & Hans Strasburger, 'Applying psychoacoustics in composition: "harmonic" progressions of "nonharmonic" sonorities', *Perspectives of New Music*, Vol. 32, No. 2, Summer 1994, p. 92. We return to these controversial issues later.

[110] The critical band is 'the frequency range within which two signals compete for the same receptor cells on the basilar membrane'. See David Butler, *The Musician's Guide to Perception and Cognition*, Schirmer, 1992, pp. 83 & 224. The significance of this is explained later.

[111] The notion of a 'Limit' was first coined by Harry Partch.

[112] In this discussion, the *primary 5-limit* (comprising the set of intervals: (2/1), 3/2 (4/3), 5/3 (6/5), 5/4 (8/5)), is distinguished from the *secondary 5-limit* (9/5 (10/9), 9/8 (16/9), 16/15 (15/8), 25/16 (32/25), 25/18 (36/25), 25/24 (48/25)). The latter is the set of all possible combinations of the former (but not including the primary 5-limit intervals). Note that 6/5 and 8/5 are included in the former because they are the inversions of 5/3 and 5/4 - for example, in a system based on the octave, including 5/3 (just major 6th) automatically implies the inclusion of 6/5 (just minor 3rd).

[113] The notion of a 'limit' should not be equated with the highest 'harmonic' which the system allows, since a 7-Limit JI scale will normally include (for example) the 8th, 9th and 10th harmonics (i.e. the notes 1/1, 9/8, 5/4, (C, D and E if C is 1/1)). In fact, on this description, any higher harmonics may be included in the scale except those whose harmonic numbers are primes or multiples of primes greater than 7. Normally, in 'extended JI' more pitches are added to a system, often in terms of adding the harmonics of harmonics. For example, if the seventh harmonic (7/4) of the seventh harmonic (7/4) is added, the new scale degree will form the ratio of 49/32 with 1/1. A considerable number of composers, especially in the USA, have developed alternative systems along such lines, and some have created acoustic instruments (most often fixed intonation percussion) to realise them.

Five JI systems are listed in APPENDIX II - the JI major and JI minor scales, both of which are examples of 5-Limit JI (Tables 1 and 2); Erv Wilson's (2/4) 1 3 5 7 Hexany, and the (3/6) 1 3 5 7 9 11 Eikosany - 7-Limit and 11-Limit JI respectively (Tables 42 (a) and (b)); and Harry Partch's 43-division JI scale - 11-Limit JI (Table 43).

While the rudimentary purpose of JI is to obtain purity and resonance of melody and harmony, other reasons for adopting a JI system might be to obtain a unity[114] of sound world, and to obtain a fresh and intriguing harmonic palette. Further, David Doty argues that:

> Just Intonation provides a greater variety and superior quality of consonances than [12 division] equal-temperament, but its resources are my no means limited to unrelieved consonance. Just Intonation also has the potential to provide more varied and powerful dissonances… the simple consonant intervals can be compounded in a great many ways to yield more complex dissonant intervals, and in part because the consonant intervals are truly consonant, the dissonances are rendered that much sharper in contrast.[115]

JI is also advocated on the basis that the human voice (and indeed mind) are naturally predisposed to 'simple' intervals, but as we saw in note 109 this is debatable. Similarly, JI need not be restricted to 'unrelieved consonance' if inharmonic timbres are used, although, again, the rationale for using a JI scale in the first place might then be misleading. But this should not stop composers writing the music they want to hear.

In JI the intervals cannot be equal between every scale degree, but there will almost always be a number of scale steps which are equal, and many symmetries and other relationships to be drawn upon in composition. On instruments of fixed intonation exact transposition of melodic fragments (without accepting a 'modal' change) is therefore limited, depending on the system, as is modulation. This is often seen as a drawback to JI as compared to equal-temperaments, but advocates of JI argue that 'modulations' in a JI system provide characterful variation, and that in JI systems with many divisions, modulations are practical and may retain the desired 'purity' of harmony. In that case, the original reasoning for choosing a JI system rather than an ET (with a similar number of octave divisions) may not be very clear. However, it does seem that, unless a flexible system of JI is used, there is a tendency for compositions not to modulate *adventurously* - this is probably a function of the aesthetics of composers working with JI, and the tuning systems themselves.

As explained, in both traditional and Partchian conceptions of JI, the intervals of a scale are chosen for the consonance of their relationship to 1/1. Erv Wilson, an American tuning theorist whose published writings largely consist of extremely concise geometric diagrams, has revolutionised the possibilities of JI by using combinatoric sets to formulate pitch structures (scales) which dispense with an unambiguous pitch centre.[116] To take the simplest example: 1, 3, 5 and 7 are combined in six possible pairs: (1 x 3), (1 x 5), (1 x 7), (3 x 5), (3 x 7), and (5 x 7). A simple scale is derived from this series by considering each product as a relationship to 1/1, and bringing this relationship within the octave (by introducing powers of 2). This is shown in *Figure 1*.

	5 x 7 x 2^{-5}	**1 x 5 x 2^{-2}**	**3 x 7 x 2^{-4}**	**1 x 3 x 2^{-1}**	**1 x 7 x 2^{-2}**	**3 x 5 x 2^{-3}**
Scale	35/32	5/4	21/16	3/2	7/4	15/8
(Cents)	155.14	386.31	470.78	701.96	968.83	1088.27
Intervals	8/7	21/20	8/7	7/6	15/14	7/6
(Cents)	231.17	84.47	231.17	266.87	119.44	266.87

Figure 1: Erv Wilson's '2/4 1 3 5 7 hexany', showing scale points relative to a 'virtual' 1/1, and the resulting steps of the scale.

114 Clearly, this is not the same level or kind 'unity' which analysts look for in motivic and harmonic compositional technique. For example, advocates of JI sometimes argue that '3-ness', '5-ness', or '7-ness' etc., are audible properties of intervals, but this is strongly disputed.
115 David Doty, op. cit., p. 1.
116 See Erv Wilson, 'D'allessandro, like a hurricane', *Xenharmonikôn,* XII, Spring 1989, pp. 1-39; Kraig Grady, 'Combination-product set patterns', *Xenharmonikôn*, IX, 1986, unpaged; Paul Rapoport, 'Just shape, nothing central', Musicworks 60, Fall, 1994, pp. 42-9.

The generic term 'hexany' is given by Wilson to six-note sets generated in this way. This particular example is the '2/4 (2 out of 4) 1 3 5 7 hexany'. Chalmers has described this form of scale derivation as follows:

> combination product sets are harmonically symmetrical, polytonal sets with virtual or implicit tonics which are not tones of the scale. Although [for example] the hexany is partitionable into a set of rooted triads…, the global 1/1 for the whole set is not a note of the scale. In this sense, combination product sets are a type of atonal or non-centric musical structure in just intonation.[117]

An extended scale, for example, may be generated by extrapolating the same or a related principle outward from the elements of this kernel, or by initially choosing a larger number of elements. In Wilson's schemata, a scale is typically generated such that no prime factor occurs more than once in any ratio (scale degree). This approach lends weight to less familiar but intriguing intervals, and also suggests ways in which composers might explore new kinds of melodic and harmonic world. Some of Wilson's systems have a haunting and enigmatic aural beauty - while also being resourceful and systematic.

APPENDIX II Table 42 (a - d) show the scalic permutations of the '1 3 5 7 Hexany' and the '1 3 5 7 9 11 Eikosany'. The latter is a 20 note scale (derived from the combination product as shown). The labyrinthine nature of the Eikosany is obvious, although its structure is best understood from Wilson's diagrams (not reproduced here). Careful examination of Table 42 (d) shows many intervals in the scale which differ only by very small amounts, suggesting potential for musical 'punning' between alternative relationships. It also suggests simplified representations of the scale, and that adopting a scale apparently as 'complex' as this for an alternative instrumental manufacturing standard should not be dismissed totally out of hand. Comprehending the totality of relationships implicit in the scale would initially be difficult for composer and performer, but the reward is a system of considerable resources.

Temperament

The history of Western tuning is largely a chronicle of attempts to devise a *fixed* tuning system which is practicable on a keyboard, and which resolves two apparently irreconcilable ideals. The first is to retain just intervals, especially the fifth (3:2) and the major third (5:4); the second is to achieve exact and unlimited transposition and modulation. To this end, a *temperament* is a system typically designed to achieve a compromise by subtly 'tempering' (adjusting) intervals of the scale.[118] Historically, at its most simple, some temperaments were devised to favour the accuracy of the fifth, others to favour the major third; in equal-temperaments the accuracy of the fifth, thirds and other intervals is simply the result of mathematical contingency. In general, as Mark Lindley has put it, temperaments are:

> Tunings of the scale in which some or all of the concords are made slightly impure in order that few or none will be left distastefully so.[119]

It is well known that a 'circle' of just fifths does not close, and that four just fifths minus two octaves do not equal a just major third. Similarly, neither three just major thirds nor four just minor thirds equal an octave. The resulting 'anomalies' (the small differences which result from each mismatch) are respectively known as the *pythagorean comma* (23.5 cents), the *syntonic comma* (21.5 cents), the *lesser diesis* (41.1 cents), and the *greater diesis* (62.6 cents). These and other anomalies are important for an understanding of temperament, but only the first two are discussed here. Historically, the agreement between octave, fifth and major third is of primary importance; the minor third is also important, but to a slightly lesser degree.[120]

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[117] John H. Chalmers, *Divisions of the Tetrachord*, Frog Peak Music, 1993, p. 116.

[118] J. Murray Barbour defines temperaments as tuning systems 'some or all of whose intervals cannot be expressed in rational numbers'. J. Murray Barbour, op. cit., p. xii.

[119] Mark Lindley, 'Temperaments', *The New Grove Dictionary of Musical Instruments*, MacMillan.

[120] There are many theoretical factors against which instances of Meantone-Temperaments, Well-Temperaments, and Equal-Temperaments have been traditionally evaluated. Of these the most deep rooted is the comparison of individual scale steps to small (or smallest) just ratios, known as the 'goodness of fit to low integer ratios'. This is easier to *quantify* than the potential for modulation implicit in a scale, and thus the analysis of competing scales is often made with reference to specific musical *objects* or *relationships* (for example - major and minor triads, diatonic scales, the division of the major third and of the tone, and the proximity of the 'leading note'

Pythagoras devised what is known as the *Pythagorean Scale* by projecting a series of successive just fifths above (or below) a starting note, transposing each new pitch back into the original octave as necessary. To calculate the ratio of each successive fifth is very simple: multiply the ratio which defines the previous scale degree by 3/2; then, if the resulting ratio is greater than 2/1, multiply by 1/2 until the ratio falls within the octave. Suppose the 'tonic' (1/1) is C. In this case G is 3/2 (approx. 701.9 cents above C), D is 9/8 (203.9), A is 27/16 (905.9), E is 81/64 (407.8), B is 243/128 (1109.8), and so on.[121] Projecting this series of fifths in both directions, gives (C, G, D, A, E, B, F#, C#) and (C, F, Bb, Eb, Ab, Db). But C# and Db differ - C# is 113.7 cents sharper than C, while Db is only 90.2 cents sharper than C. In fact, no matter how far a series of just fifths is extended (in one or both directions), no two members of the series are ever exactly equal in mathematical ratio. The interval between C# and Db in this example (approx. 23.5 cents) is identical for any two notes separated by a cycle of 12 just fifths, and is known as the *pythagorean comma*. As a consequence, in the Pythagorean scale (with 12 scale degrees), every fifth is just except one, which is a pythagorean comma flat and is known as the 'wolf'. (In APPENDIX II, Table 3, this is the interval F# to C#). Another significant feature of the Pythagorean scale is that the 'major third', for example the interval from C to E (81:64, or 407.8 cents), is approx. 21.5 cents wider than the just major third (5:4, or 386.3 cents). This anomaly (the syntonic comma) crops up with startling regularity in tuning theory and is equal to the interval 81/80.

Meantone Temperaments

Following Pythagoras' model, the generation of a scale from a single interval - especially the fifth, as the 'simplest' interval next to the octave - is thought to give the scale coherence. In regular meantone temperaments, for example, the scale is generated by a sequence of identical *tempered* fifths. In the Pythagorean scale, the size of the generating (pure) fifth determined the size of the major third. Thus a tuning system is sometimes evaluated in terms of the relative proportion in which the fifth and major third are tempered. At one extreme, the fifth of the Pythagorean scale is just (wholly untempered), but the third is a full syntonic comma (21.5 cents) wider than just; at the other extreme, in what is called 'strict' (or ¼ comma) meantone-temperament, the major third is just and the fifth tempered by 5.38 cents.[122] Outside these extremes the overall approximation of fifth and major third is worsened unless we consider systems with more than 12 divisions to the octave. If, however, the *minor* third which results from a meantone cycle is also taken into account, the opposite (useable) extreme to the Pythagorean scale is traditionally considered to be 1/3 comma Meantone.[123] Many such meantone-temperaments were devised and employed during the Renaissance and early Baroque periods.

In ¼ comma meantone-temperament, each fifth is narrowed to 696.6 cents (5.38 cents narrower than just), so that five successive fifths (C, G, D, A, E) result in a just major third (5:4, 386.3 cents).[124] The temperament is called *¼ comma* meantone because 5.38 cents is exactly a ¼ of the *syntonic* comma; and *meantone* because D subdivides the interval between C and E exactly.[125] More generally, the term 'meantone' is used to refer to any temperament in which the interval of a 'tone' subdivides the 'major third' (whether slightly narrow or wide from just) into two equal parts.[126] *Figure 2* shows a selection of meantone systems compared to the Pythagorean scale. The right hand column shows the sum of deviations for each temperament from the just fifth, and major and minor thirds.

Temperament	**Fifth**	**Deviation from Just 5th**	**Major Third**	**Deviation from Just M3**	**Minor Third**	**Deviation from Just m3**	**Sum of the three shown deviations**

relative to the octave), rather than *processes*, which becomes exceptionally complex. See, for example, J. Murray Barbour, op. cit.; Wendy Carlos, 'Tuning at the cross-roads', *Computer Music Journal*, Vol. 11, No1, Spring 1987, pp. 29-43; Darreg and McLaren, op. cit.; Scott R Wilkinson, *Tuning In*, Hal Leonard Books, 1988, pp. 81-5.

121 For successive fourths (that is, the inverted series of fifths) multiply by the inversion of 3/2 - that is, 4/3.

122 That is, a quarter of a the syntonic comma.

123 Thanks to Paul Erlich for comments on this.

124 See, for example, J. Murray Barbour, op. cit., pp. 26-7.

125 That is, C to D forms a tone which is exactly half way, or the 'mean' distance, to E.

126 In fact, by projecting a fifth of fixed size through successive degrees, if the staring point is C, than D automatically subdivides the distance to E, so long as the size of the 'fifth' is within certain limits (or else the notes are no longer 'D' and 'E').

Pythagorean Scale	701.955	0.00	407.82	21.51	294.14	21.51	43.01
12-ET	700.000	1.96	400.00	13.69	300.00	15.64	31.28
1/6 Comma Meantone	698.371	3.58	393.48	7.17	304.89	10.75	21.51
1/5 Comma Meantone	697.654	4.30	390.62	4.30	307.04	8.60	17.20
1/4 Comma Meantone	696.579	5.38	386.31	0.00	310.26	5.38	10.75
2/7 Comma Meantone	695.810	6.15	383.24	3.07	312.57	3.07	12.29
1/3 Comma Meantone	694.786	7.17	379.15	7.17	315.64	0.00	14.34
4/9 Comma Meantone	692.397	9.56	369.59	16.73	322.81	7.17	33.45

Figure 2: Variations of meantone temperament, comparing sizes of fifth and resulting deviations in the thirds to the Pythagorean scale and 12-ET.

However the ‘anomalies’ are not distributed in the meantone-temperaments so that they go unnoticed. Just as the cycle of twelve pure fifths lies sharp of the octave, in ¼ comma Meantone, for example, twelve fifths of 696.6 cents falls flat of the octave: the ‘wolf fifth’ is therefore 41 cents wider than just.[127] While meantone systems in general achieve good purity of the thirds and triads in ‘near’ tonalities, 12-ET is unique in *closing* a cycle of 12 equally-sized fifths, and thus might be called ‘1/12 comma Meantone’.[128] 12-ET was not adopted in the Renaissance and Baroque periods, not because it was unknown, but in large part because the major thirds are 13.7 cents wider than just, and the minor thirds 15.6 cents narrower (with inverse results for the sixths). The timbre of the keyboard instrument on which a temperament was realised was also probably important. Lindley, for example, has argued that:

> Not all timbres… are equally conducive to temperament. In general, it is only when the component overtones of the timbre (together with the fundamental tone of the note) form a virtually pure harmonic series that consonant intervals will sound sufficiently different in quality from dissonant ones for the need of tempering the concords to arise. A pronounced degree of inharmonicity in the timbre, as in a set of chimes or a xylophone, eliminates the qualitative difference, except in the case of the unison or octave, between the sound of a pure concord and that of a slightly impure or tempered one.[129]

Irregular Temperaments

In the irregular Baroque temperaments (also known as ‘circulating’ or ‘well’ temperaments) the wolf fifth is avoided by distributing fractions of the Pythagorean comma amongst various of the fifths. In a comparison of twelve irregular schemes (mainly authentic Baroque schemes but including two 20th Century schemes devised for performing J.S. Bach), Lindley has demonstrated the variety of such possibilities. For example, in a temperament devised by Werckmeister (1681) eight fifths are just, while 4 have a ¼ of the Pythagorean comma subtracted; in one of Sorge’s (1744) temperaments, four fifths are just, four are flattened by 1/6th of the comma, and four are flattened by 1/12th of a comma; and so on.[130] Each scheme is devised with a preference for particular melodic and harmonic colours, eliminates the wolf, and makes a good proportion of the thirds acceptable.[131]

Equal Temperaments

In an *equal-temperament* every adjacent step of the scale is theoretically equal, and exact transposition on fixed pitch instruments is not restricted to certain degrees of the scale. An octave-based *n*-division equal-temperament is generated by any cycle of *n* identical intervals which reaches an exact octave. 12-ET, for

[127] Note that Lindley, in Table 1 of the entry on mean-tone says that ¼ comma meantone will have a wolf of 36 not 41 cents. Is this because here the comma is the *pythagorean* comma? Barbour says the meantone wolf is 35 cents sharp?

[128] 12-ET is thus a special case of meantone, as are all equal-temperaments with a recognisable and equally bisected major third, although they are seldom referred to as such. See Mark Lindley & Ronald Turner-Smith, *Mathematical Models of Musical Scales*, Verlag für systematische Musikwissenschaft, 1993, p. 52.

[129] Mark Lindley, ‘Temperaments’, 1, *The New Grove Dictionary of Musical Instruments*, MacMillan.

[130] Mark Lindley, ‘J.S. Bach’s tunings’, *Musical Times*, 1986, Table 2, p. 723.

[131] For further information on Irregular -Temperaments see: J. Murray Barbour, op. cit., Chapter VII; see also: Owen H. Jorgensen, *Tuning*, Michigan State University Press, East Lansing, 1991.

example, is generated by two cycles of twelve steps - the fifth at 700 cents and the semitone at 100 cents. 13-ET may be generated by thirteen separate cycles each of thirteen steps.[132] However, in 13-ET there is no 'cycle of fifths', because the closest interval to the fifth is 36.5 cents wider than just - that is, too wide to be considered a fifth (see APPENDIX II, Table 13, and APPENDIX I (b)). 19-ET, however, can be generated by nineteen separate cycles of nineteen steps, and one of these cycles *is* considered a cycle of fifths, since the 'fifth', at 695.7 cents, is only 7.2 cents narrower than just, and functions as an acceptable fifth.[133]

24 and 36-ET, for example, have two and three cycles of fifths respectively, but in these systems the cycles are of twelve steps each and are mutually exclusive; alternatively, 24 and 36-ET may be generated from 24 or 36 step cycles of other intervals - 1/24th, 5/24ths, 7/24ths … or 1/36th, 5/36ths, 7/36ths… of the octave. In general, any equal-temperament in which the number of divisions is a prime number will have as many 'cycles' as it has divisions; others will have less. Only certain equal-tempered systems result in 'fifths', 'thirds', 'sixths' and other intervals which may be considered acceptably close to just. Historically, 19-ET and 31-ET have been favoured amongst alternative equal-tempered systems, because of the advantageous number, degree and distribution of reasonable approximations to just ratios, considering the number of divisions and their manageability. Both systems may be thought of as a single cycle of fifths (of 19 or 31 steps) which generate a species of meantone temperament.[134]

We saw that if the spiral of fifths in the Pythagorean scale is not 'closed' by a wolf fifth, then an infinite array of pitches may theoretically be generated. For example, all 88 keys of the piano are might be tuned to an 'open cycle' of just fifths and fourths in which no exact octaves occur. Taking this one stage further, *non-octave* temperaments are scales in which the octave is not the basic subdividend.[135] I have chosen not to discuss this category of scales because to design acoustic woodwind and brass so that they would each reliably overblow at the same interval, say, slightly above or below the octave, and at other appropriate intervals, would pose special problems. Similarly I do not discuss the prevalent idea of stretching (or contracting) a scale by a small amount, ostensibly to achieve slightly better consonances throughout the scale.[136] This idea is familiar from the stretching of the piano scale, but instruments of non-fixed intonation would probably not benefit from this due to their intrinsic intonational flexibility.[137]

APPENDIX I (a) provides a table of interval zones and rational intervals which are chosen to show 'principal' intervallic 'zones' and 'nodes' between 1/1 and 2/1 for harmonic tones. The method of choosing these intervals was simple but somewhat subjective: starting from the simplest and most well defined 'nodes' - 3/2 (4/3), 5/4 (8/5), 6/5 (5/3), 8/7 (7/4), 9/8 (16/9) - other 'nodes' were gradually added, ultimately describing the interval

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[132] Every interval of the scale will generate the complete scale, a fact which holds in any *n*-ET where *n* is a prime number.

[133] Note how close the fifth in 19-ET is to that of 2/7 Meantone-Temperament (as shown in the table above).

[134] Note that the fifth in 19-ET (694.74 cents) is almost identical to that in 1/3 comma meantone (694.79); the fifth in 31-ET (696.77) is very close to 1/4 comma meantone (696.58). It is perhaps due to aesthetic preferences which go beyond the (traditional) rationale of just intonation that these systems have been accorded a privileged status compared to other systems in their respective ranges (22- and 34-ET especially).

[135] For more on non-octave tunings see: Enrique Moreno, *Expanded Tunings in Contemporary Music : Theoretical Innovations and Practical Applications*, Edwin Mellen Press, 1992; B. McLaren, 'The uses and characteristics of non-octave scales', *Xenharmonikôn*, 14, Spring 1993, pp. 12-22; B. McLaren, 'The uses and characteristics of non-just non-equal-tempered scales', *Xenharmonikôn*, 15, Autumn 1993, pp. 27-41; Wendy Carlos, 'Tuning at the cross-roads', *Computer Music Journal*, Vol. 11, No1, Spring 1987, pp. 29-43; M. V. Matthews and J.R. Pierce, 'The Bohlen-Pierce scale', *Current Directions in Computer Music Research*, ed., M. V. Matthews and J.R. Pierce, MIT Press, 1991.

[136] Mark Lindley, for example, finds reference to 'the principle of imperceptibly stretched octaves' as far back as 1753, although whether this refers to stretching the whole scale is unclear. Mark Lindley, 'J.S. Bach's tunings', *Musical Times*, 1986, p. 723. Note that if a slight sharpening (or flattening) of the octave in an ET system is made (and the scale remains equally tempered), the change of approximation to just ratios (*above* 1/1) is greater in the upper half of the scale than in the lower. For example, if the octave is stretched by 4 cents for a 29-ET scale, the 9th degree of the scale (at 372 cents a somewhat flat 'just major third') will be sharpened by 1.24 cents, but its octave complement the 20th degree of the scale (at 828 cents a sharp 'just minor sixth') will be sharpened by 2.76 cents. The inverse is true considering intervals *below* 1/1.

[137] Two further reasons for not considering non-octave scales are: (a) it seems unlikely that each woodwind and brass instrument could be manufactured to overblow consistently at the *same* slightly stretched or contracted octave; (b) even if this was achieved, both the timbre and playability of these instruments could be significantly destabilised by the absence of the second harmonic. Conceivably, as suggested above, the 'logical' instruments fitted with a corrective pitch sensor might be able to achieve this, but it is perhaps unlikely that the effort would be worth the trouble.

between 1/1 and 2/1 in terms (more or less) of discrete (sometimes very fine) zones, each distinguishable from the next in terms of the 'aural character' of each zone.[138] (APPENDIX I (b) and (c) are discussed later).

In APPENDIX II, Tables 1 and 2 show the standard 'major' and 'minor' Just Intonation scales; Tables 3 and 4 show the Pythagorean scale and Werckmeister III - a characteristic Baroque 'circulating' temperament; Tables 5 to 41 show the equal-temperaments from 5- to 41-ET.[139] An explanation of the tables is given at the beginning of the APPENDIX - as well as showing approximations to small integer ratios, the tables display the number and distribution pattern of relatively good approximations. Regular distribution of near approximations across the scale is normally considered advantageous.

A few comparisons of some *n*-ET systems presented in APPENDIX II may be useful. Consider for example 19-ET and 20-ET (Tables 19 and 20):

- *Purity*: 19-ET approximates small ratio intervals better than 20-ET; the approximations in 19-ET lie within the 5 and 7-Limits, and no interval of 19-ET is 'compared' to intervals in the 11-Limit or above. In 20-ET eight degrees of the scale fall closest to 11-Limit intervals or above, and the overall accuracy of approximation is poorer than in 19-ET.

- *Number*: The number of 'familiarly close' approximations to small ratios is greater in 19-ET, and the *distribution* of these in 19-ET is spread quite evenly; the equivalent distribution in 20-ET shows a large gap between the Pythagorean minor third and the Pythagorean major sixth.

- *Primeness*: 19-ET is clearly more 'prime' than 20-ET - that is, the smallest ratios are approximated with greater accuracy (the perfect 5th and 4th in 20-ET do not exist as such).

- *Flexibility*: 19-ET contains a single cycle of narrow fifths which affords a variety of meantone temperament; 20-ET comprises two independent cycles of rather wide fifths.[140]

A variety of 'compelling' intervals in the available scalic and chordal formations are rudimentary to our expectation of instrumental music.[141] Taken together these factors predict that 19-ET is a more consonant

[138] Originally, the intervals chosen to form APPENDIX I (ie., the intervals against which the Tables of ATS are compared in APPENDIX II) were simply the nine most commonly accepted Just Ratios and their inversions: {1/1 (2/1), 3/2 (4/3), 5/4 (8/5), 6/5 (5/3), 7/4 (8/7), 7/6 (12/7), 9/8 (16/9), 10/9 (9/5), 15/8 (16/15)}, but this soon seemed inadequate. This was because (at least to my ear, which is very much conditioned by 12-ET) it is quite natural to distinguish some kind of 'minor third' that lies *between* 7/6 and 6/5; and a 'neutral third' which lies *between* 6/5 and 5/4; and so on. Thus the system was gradually extended to include more complex ratios up to the 11-Limit. More recently, this was again refined, following discussions with Joseph Sanger, who collaborated on and helped to type (and retype) these tables. Obviously, many very subtly different intervals can be distinguished from those given in APPENDIX I, yet it is fair to say that for practical, instrumental purposes, the chosen intervals are a good representation of the most useful and distinguishable '*zones*' for (harmonic) tones. In fact, some of these distinctions are perhaps more subtle than is useful; the inclusion of complex pythagorean intervals could also be dispensed with. Moreover, it becomes increasingly unclear to what extent frequency ratios play any part in defining recognisable zones for intervals less than the small semitone, or larger than the large major seventh. Interestingly, the zones themselves have some correspondence with Partch's 43-division JI scale, which is listed in Table 43. This aspect of the paper remains provisional, for reasons explained in the note to APPENDIX II.

[139] For a concise diagrammatic comparison of 'the adequacy with which the simplest equal-tempered tunings approximate all justly tuned intervals of the 3- through 15-limits', see Paul Erlich, 'Tuning, tonality and twenty-two-tone temperament', *Xenharmonikôn* 17, Spring 1998, pp. 13-17.

[140] The criterion of *flexibility* addresses whether a system provides the approximations necessary to achieve an extended set of existing scales and harmonies with instruments of both fixed and non-fixed intonation. A full discussion of this is unfortunately beyond the scope of this paper, and is addressed here in somewhat abstract terms. The scalic properties typically desired in general only belong to those ATS which maximise approximations to certain intervals or 'intervallic zones'- but this requires a much more detailed study than is possible here - see, for example, Paul Erlich, ibid., pp. 12-40. The most common distinction regarding the scalic properties of ETs is that between systems which provide a species of meantone temperament (such as 12-, 19- and 31-ET) and those which do not (for example, 22-, 34- and 41-ET) - given that all of these systems possess good harmonic intervals. It is sometimes argued that a newly adopted 'quasi-universal' ATS would have to support old as well as new forms of diatonic music, and that such a system must therefore be 'meantone'; this is a similar argument to that advocating that an adopted ATS should retain 12-ET as a subset. My own view is that new instruments should essentially be made for new music - if special new instruments are needed to realise (for example) baroque or renaissance music, then appropriate instruments should be made for that purpose. My thanks to Paul Erlich for comments on this.

system than 20-ET. Leaving other criteria aside, this would suggest that 19-ET is potentially a more flexible and compelling musical system than 20-ET - or at least that 19-ET is likely to be useful to more composers.

Similarly, compare 24 and 27-ET (Tables 24 and 27):

- *Purity*: 24-ET contains all the familiar intervals of 12-ET, with 3/2 (and 4/3) very close to just. However, in 24-ET a large proportion of the remaining intervals fall closest to intervals in the 11-Limit - the 11th harmonic is particularly closely approximated. In 27-ET only six scale degrees approximate 11-Limit ratios or above, and for the rest there is an even spread across the 3, 5 and 7-Limit intervals. While 3/2 (and 4/3) in 27-ET are not so close to just as in 24-ET, they border (just on the outside?) of acceptability for the 5th and 4th, but a good proportion of the smaller ratios in the 5 and 7-Limits are well approximated.

- *Number*: The proportion of 'familiar' approximations is between the two systems is marginally biased toward 24 rather than 27-ET and the *distribution* of these is fairly even between the two systems.

- *Primeness*: It is difficult to judge whether 24 or 27-ET is the more 'prime'. Considering 3/2 (and 4/3), 24-ET is more prime. But both systems have an identical major third of 400 cents (considered here as belonging to the zone of the Pythagorean major third rather than the Just major third), and in terms of the next smallest ratios 27-ET clearly approximates the series of superparticular ratios 6/5, 7/6, 8/7 (and their inversions) better than 24-ET.

Likewise, compare 31 and 36-ET (Tables 31 and 36):

- *Purity*: 31-ET is immediately striking for the exceptional closeness of a large proportion of its scale degrees to small number ratios. The 5th and 4th is fractionally less close to just than 36-ET, but the accuracy of approximations to the series 3/2, 4/3, 5/4, 6/5, 7/6, 8/7, 9/8 in 31-ET is only rivalled by 41-ET (in the range of *n*-ETs considered here). Even the intervals in the area of the tritone fall relatively close to just ratios. Again, 36-ET has close approximations of 3/2 (and 4/3) and all the intervals of 12-ET are included, so at least every third interval (excepting the tritone) is familiar. 36-ET also contains some good approximations of septimal ratios (especially 7/6, 8/7 and 9/7 and their inversions).

- *Number*: the relative proportion of 'familiar' approximations is biased toward 31 rather than 36-ET, and their *distribution* more or less equivalent between the two systems.

- *Primeness*: Again, it is difficult to judge whether 31 or 36-ET is the more 'prime'. Considering 3/2 (and 4/3), 36-ET is more prime. But the accuracy to the nearest subsequent small ratios in 31-ET perhaps outweighs the difference between the 5th and 4th in the two systems.

- *Flexibility*: 31-ET (like 19-ET) comprises a single cycle of narrow fifths which affords a variety of meantone; 36-ET comprises three independent cycles of slightly purer fifths - that is, three parallel cycles 12-ET.

At this point a caution should be suggested. In my own compositional work, using electronic simulations of acoustic instruments, one of the ET systems to which I have been particularly drawn is 32-ET. A quick glance at APPENDIX II Table 32 shows that 32-ET does not satisfy particularly well the criteria which are being suggested here as advantageous. I have found its exotic and expressive palette of tones, including the wide fifth and the narrow major third, rather interesting, and that particular timbres (such as a very bright piano) give especially attractive consonances, and bright sharp dissonances.

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[141] This statement may rightly be contradicted, on the grounds that any scale can be used to compose convincing music of one sort or another. However, the implications of the remark should be taken in the context of discussing large-scale future investment, and the search for a 'quasi-universal' system.

A recent study of 'intonation sensitivity' concludes that, broadly speaking, there are two kinds of listeners - those who prefer just intervals and those who prefer tempered intervals.[142] On the face of it, I might appear to be someone who prefers tempered intervals - but in fact I enjoy both for what they are. It seems likely that this 'split' is a gross simplification of musical discrimination, and that a majority of listeners learn to appreciate and enjoy both types of sonority, without necessarily being able in the normal course of events to distinguish them. It seems that good music might be composed in many (possibly any) of the ATS under discussion.

But our aim is to identify tuning system(s) which will enable the successful expansion of the melodic and harmonic possibilities of contemporary instrumental language in the most *compelling*, *productive* and *adaptable* ways. Individual composers will want to write extremely consonant or extremely dissonant instrumental music, or both. They will write music which is fast and slow, simple and complex music, tonal and atonal; most importantly, they will write music which is currently unimaginable. In the context of creating a new acoustic instrumental system, the task is to find tuning system(s) and instrumental technologies which will best encourage all of this, minimising the degree of compromise for each.

Beyond the Traditional Criteria for Evaluating Alternative Tuning Systems

Most discussions of 'which tuning system is best?' involve comparison of tuning systems along the lines described above. They try to answer the following questions: is Just Intonation or Temperament best and in which of their many forms? If Temperament is preferred, then which *n*-ET or *n*-WT best approximates just intervals (or small number ratios, or the harmonic series, or fusion in harmonic tones)? If a JI system is preferred, then which prime 'limit' is the 'natural' limit of intervallic resources? These questions are the tiniest tips of enormous icebergs. The implications which ensue - which scales and harmonies are available and effective, whether a system may be considered coherent from diverse points of view, whether the smallest interval of a system adequately distinguishes each degree of the scale from the next, issues of transposition and modulation, etc., - are something of a labyrinth.[143]

For new acoustic instruments, the relation between tuning and timbre, and the criteria which acoustic instruments themselves impose are also of importance.[144] What makes a given scale or range effective for a certain instrument (for example, the intervals of over-blowing on woodwind and brass) is not something that can necessarily be altered at will. In addition, the relationship between timbre and tuning may also be important for new developments, although the relation between an instrument's design and its timbre is highly complex. The following discussion merely scratches the surface of these relationships, but it is suggested that the integration of tuning, timbre and instrumental design may be a key to finding a way through that labyrinth.

There therefore follows a brief discussion of the notions of consonance and dissonance. This is followed by an examination of the relation of tuning and timbre; a brief look at Terhardt's 'two-component theory of consonance'; and the relation between intervallic character and intervallic zones - before returning to some more sophisticated ways of comparing ATS. Then, in Section 5, new instruments and instrumental feasibility are considered.

Consonance and Dissonance

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[142] Or to be more precise, those who prefer particular triads containing intervals which deviate from just by ± 15 cents. See Linda A. Roberts and Max V. Mathews, 'Intonation sensitivity for traditional and non-traditional chords', *Journal of the Acoustical Society of America*, 75, (3), March 1984, pp. 952-9.

[143] For those who wish to visit infinity, and I recommend the journey, the spaceship timetable will include: Hermann Helmholtz, *On the Sensations of Tone,* Dover, 1985; Harry Partch, *Genesis of a Music*, Da Capo, 1979; John H. Chalmers, *Divisions of the Tetrachord*, Frog Peak Music, 1993; Mark Lindley & Ronald Turner-Smith, *Mathematical Models of Musical Scales*, Verlag für systematische Musikwissenschaft, 1993. Many useful mathematical analyses can also be found for example in articles by John Chalmers, Ervin Wilson, Brian Mclaren and others in various issues of *Xenharmonikôn* (available from Frog Peak Music, Box 1052, Lebanon NH 03766, USA; or http://www.sover.net/~frogpeak/).

[144] Enrique Moreno has commented that the question of new instruments for ATS is 'an engineering problem', but for acoutic instruments it is not merely this. Moreno is not exclusively concerned with acoustic instruments - a proportion of recent theory is based on the assumption the realisation of ATS will probably be electronic. Enrique Moreno, *Expanded Tunings in Contemporary Music: Theoretical Innovations and Practical Applications*, Edwin Mellen Press, 1992, p. 94.

'Consonance' and 'dissonance' have been distinguished in many ways, and judgements consonance and dissonance have changed over the centuries. James Tenney's *History of 'Consonance' and 'Dissonance'* distinguishes at least five versions of the distinction:[145]

- *Melodic* consonance/dissonance - implicit in successive melodic intervals;
- *Polyphonic* consonance/dissonance - resulting from simultaneity in polyphonic music;
- *Contrapuntal* consonance/dissonance - as described or defined by rules of counterpoint;
- *Functional* consonance/dissonance - the tendency of dissonance to imply movement, particularly towards consonance or resolution;
- *Sensory* consonance/dissonance - which 'equates dissonance with roughness and the presence of beats, and consonance with smoothness and the absence of beats'.[146]

The range of distinctions corresponds to a range of phenomena, and emphasises the fact that, in the previous discussion, the focus on distinguishing scales containing a high proportion of 'consonant' intervals is not necessarily due to a preference for aural 'smoothness' *per se*. The consonance/dissonance opposition is rather an umbrella term which, historically, has described a variety of musical phenomena: 'consonance' provides points of rest and stability, melodic and harmonic; musicians tend to hear and adjust intervals as if there were a kind of gravitational pull towards 'consonant' intervals;[147] and interval/sound relations display a degree of 'rightness' (or 'compellingness') as they relate to a continuum of consonance and dissonance, relative to musical context.

The following two sections comprise a simplified account of the theory of '*sensory* consonance'.[148] 'Sensory consonance' refers to phenomena of smoothness and roughness which are perceived when sounds are heard together. Considered as a property of intervals, sensory consonance is the

> 'typical sensorial phenomenon… related to simple integer frequency ratios… holding… for subjects without any experience in musical harmony'[149]

and normally refers to the aural 'smoothness' which results when a 'consonant' interval is played perfectly or almost perfectly in tune. It is suggested that while sensory consonance is not necessarily the 'root' cause of all aspects of 'consonance' here described, it plays a highly influential role.[150]

Relating Tuning and Timbre - from Helmholtz to Sethares (1)

As described in *On the Sensations of Tone*, Hermann Helmholtz (1821-84) investigated the continuum of intervals between unison and the octave (and above), by comparing violin tones sounded together, one held at a constant pitch and the other being varied on the continuum.[151] Helmholtz calculated the resulting consonance or dissonance of intervals on that continuum by considering 'roughness' as a function of beats. He considered that roughness is at a maximum when there are 33 beats per second, and decreases to zero when the beating slows so much that there remains no roughness, or when beats become inaudible. An outline of the graph that Helmholtz drew as a result of this experiment can be seen in ***Figure 3***.

[145] Unfortunately I have been unable to obtain Tenney's book (J. Tenney, *A History of 'Consonance' and 'Dissonance'*, Excelsior Music Publishing Co., New York, 1988). This summary is based on Sethares' summary of Tenney in Chapter 4 of *Tuning, Timbre, Spectrum Scale*.
[146] Sethares, op. cit., p. 75.
[147] As we will see, following Sethares, we should perhaps say 'consonant combinations of sounds'.
[148] The mathematical aspects of these researches have been omitted here.
[149] R. Plomp and W.J.M. Levelt, 'Tonal consonance and critical bandwidth', *Journal of the Acoustic Society of America*, 1965, Vol. 38, p. 551.
[150] Each of these tendencies are to some extent culturally determined; and each phenomenon is extremely commonly recognised, although not universally agreed upon. The terms 'consonance' and 'dissonance' therefore refer to continua of smoothness and roughness, stability and instability, gravitation and weightlessness, 'rightness' and 'awkwardness'. It is beyond the scope of this discussion to attempt to explain each of these phenomena. But it is reasonable to suppose that, even if there is not a single cause for all of them, the cause of one aspect of consonance/dissonance will also be responsible in part for other aspects.
[151] Benade has stated that 'one does not hear clear-cut beats between mistuned violin tones…' Arthur H. Benade, *Fundamentals of Musical Acoustics*, Dover, 1990, p. 549. This issue arises a couple of times in this paper - to see the full quote, see below, note 194.

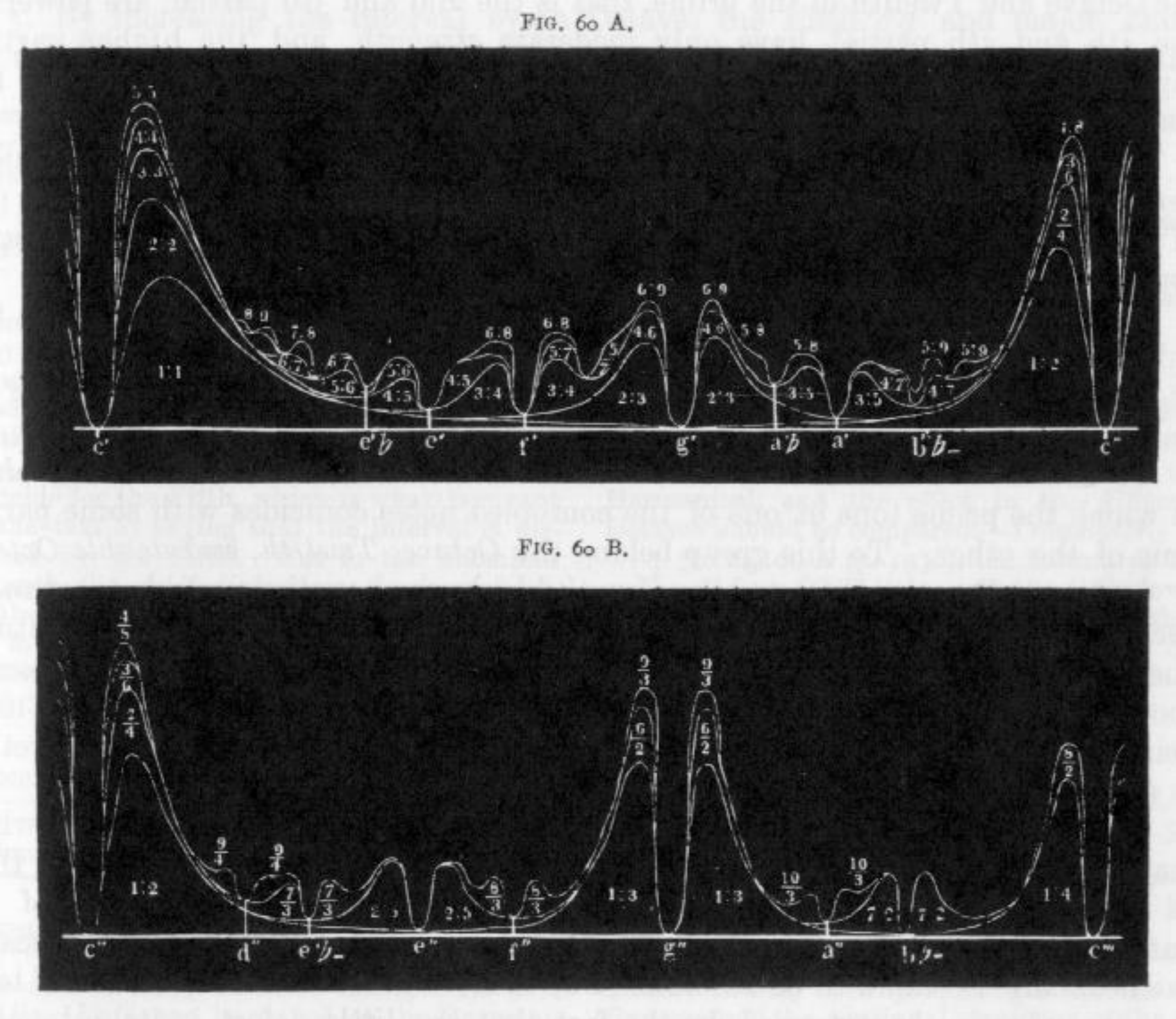

Figure 3 : Hermann Helmholtz, *On the Sensations of Tone*, Dover, 1985, p. 193. Fig. 60a shows a dissonance curve for one octave; Fig. 60b includes a second octave. (The illustration is reproduced with the permission of Dover Books).

This kind of graph, - that is, one which plots dissonance (or consonance) against interval - has since come to be known as a 'dissonance curve'. Helmholtz's graphs show maximal consonance for the unison and the octave, followed (in order of consonance) by the 5^{th}, 4^{th}, major 6^{th} and major 3^{rd}, and so on, confirming traditional expectations of a hierarchy of consonant and dissonant intervals. The exact points of maximal consonance correspond to the just fifth, the just fourth etc.; but the curves may also be taken to describe 'zones' rather than 'points' of consonance - that is, a small portion either side of the apex of a curve shows an intervallic 'zone' or band which is 'close to maximally consonant' (or 'maximal' relative to that part of the curve).

Figure 4 shows Harry Partch's (1901-74) 'One-Footed Bride' (*Genesis of a Music*, 1949), which is a similar kind of graph - although unfortunately I do not know the exact means by which Partch arrived at it.[152] The consonance maxima correspond broadly to those found by Helmholtz.

[152] Paul Erlich has pointed out that the relative height of the maxima of the curve is related to the relative simplicity of the just ratios to which selected points on the curve correspond - and that between these points the curve is simply 'joined up' arbitrarily. Email correspondence from Paul Erlich.

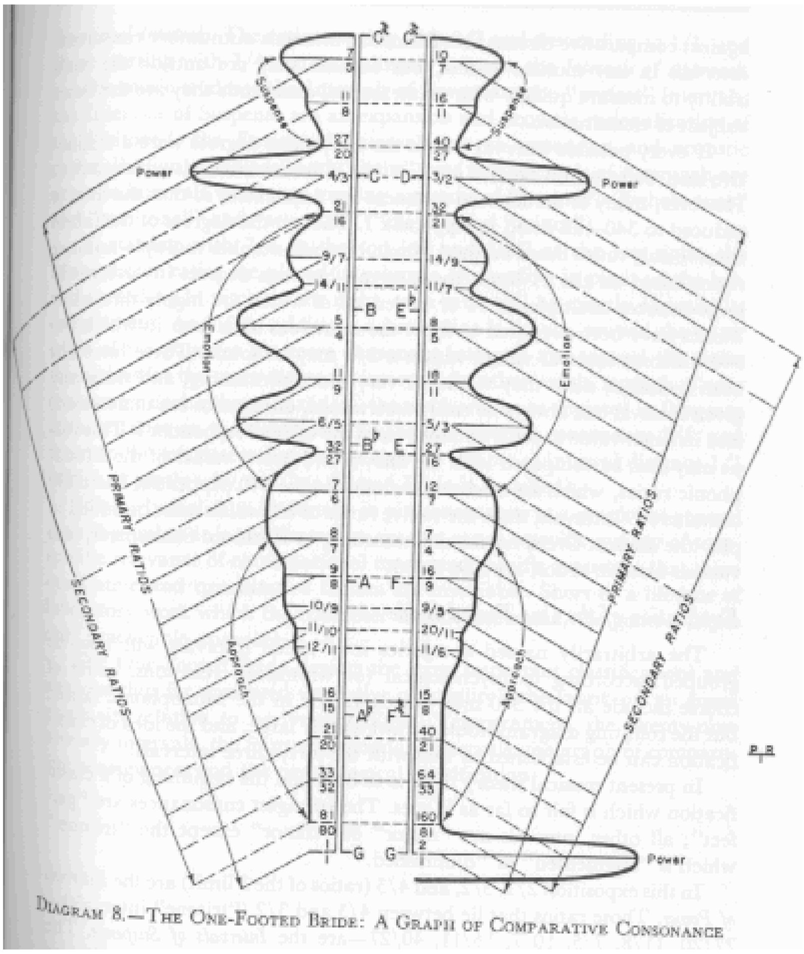

Figure 4 : Harry Partch, Genesis of a Music, p. 155, Da Capo, 1979. (The illustration is reproduced with the permission of Da Capo Press).

However, in a famous and controversial experiment conducted in 1965, Plomp and Levelt showed that if two *pure sine tones* are compared in a similar manner, the dissonance curve that results does not have the same shape as that derived in Helmholtz's experiment.[153] Here, a maximum dissonance (or 'roughness') is reached at an interval just less than a semitone, but dissonance diminishes steadily as the interval increases beyond this, and no deviation in the curve is noticed in relation to the traditional just ratios (3/2, 4/3, 5/4 etc.). Nor do any of the intervals which are normally thought dissonant cause deviations in the curve. For example, the level of 'dissonance' of the tritone is similar to that of the fifth, and the dissonance of the major seventh and minor ninth is similar to that of the octave. However, if harmonic partials are added one by one to the sine tones, forming complex harmonic tones, step by step the curve discovered by Helmholtz is recovered. In addition, Plomp and Levelt showed that although sensory consonance is relative to the absolute frequency of the two tones, the interval of maximum sensory dissonance remains (fairly) constant relative to the critical band. The experiment was controversial in its day, but today is commonly accepted as part of the groundwork of modern acoustics.

Plomp and Levelt's results were confirmed in experiments carried out by Kameoka and Kuriyagawa, who extended and refined their work.[154] Kameoka and Kuriyagawa proposed a mathematical model of calculating the 'absolute dissonance' of complex tones, and of chords comprising two or more complex tones. Simplifying greatly, this calculation sums the resultant 'dissonance' between each and every pair of harmonics which

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[153] R. Plomp & W.J.M. Levelt, 'Tonal consonance and critical bandwidth', *Journal of the Acoustical Society of America*, 1965, Vol. 38, pp. 548-60.

[154] A. Kameoka and M. Kuriyagawa, 'Consonance Theory Part I: consonance of dyads', *Journal of the Acoustical Society of America*, Vol. 45, No. 6, 1969, pp. 1452-9; A. Kameoka and M. Kuriyagawa, 'Consonance theory part II: consonance of complex tones and its calculation method', *Journal of the Acoustical Society of America*, Vol. 45, No. 6, 1969, pp. 1460-9. The validity of the experiments was strengthened because the experiments gave similar results for Japanese subjects.

comprise the two (or more) complex tones. Experimental tests were first carried out to compare the experience of subjects listening to sound signals presented through loudspeakers with the predictions made using the model. Different kinds of signals were presented: (i) complex tones comprising two to five components, each with an arbitrary spectral structure; (ii) complex tones comprising up to twelve components, again with varying spectral structures; and (iii) chords comprising two complex tones, each tone being of the same harmonic spectral structure. In all these cases, results showed good correspondence between mathematical predictions and experimental findings. Thus, Kameoka and Kuriyagawa claimed to prove 'the validity of the dissonance calculation theory to chords of harmonic complex tones'.[155]

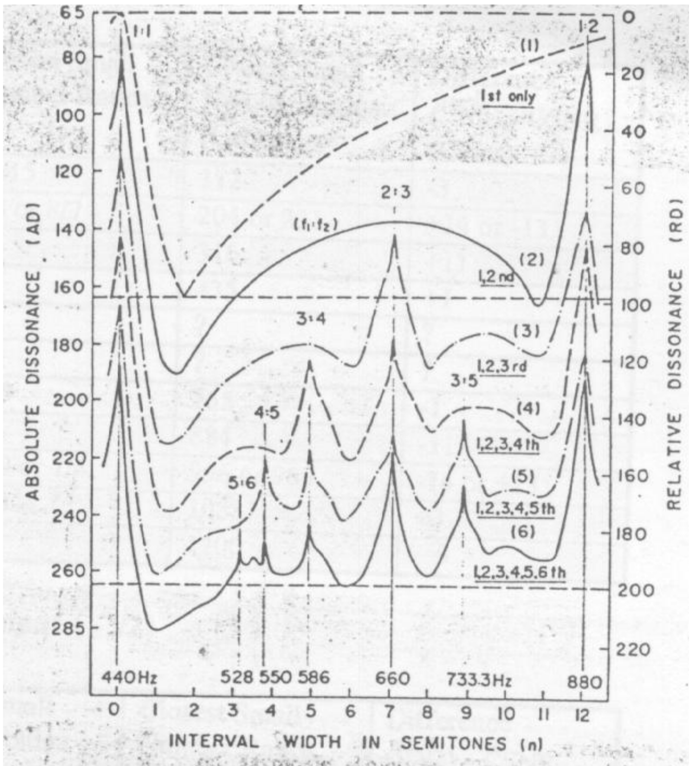

FIG. 8. Consonance characteristics for chords of two harmonic complex tones. Chord 1 consists of two tones that include the fundamental only. Chords 2–6 consist of two harmonic complex tones including the fundamental components and their harmonics from the second to the sixth, respectively. All the components have an equal sound pressure of 57 dB SPL.

Figure 5 : A. Kameoka and M. Kuriyagawa, 'Consonance theory part II: consonance of complex tones and its calculation method', *Journal of the Acoustical Society of America*, Vol. 45, No. 6, 1969, p. 1465. (The illustration is reproduced with the permission of the *Journal of the Acoustical Society of America*).

Their conclusion was that:

> The consonance characteristics of chords of two *harmonic* complex tones showed peaks at simple frequency ratios. This was due to coincidence of harmonics…the consonance of chords depends greatly on the harmonic structures. The conventional theory of harmony on the other hand, classifies consonant tones by musical intervals based on the fundamental frequencies, regardless of their harmonics. Many musical tones produced by natural instruments include both even and odd harmonics, and the variation of harmonic structure is rather limited, so their consonance characteristics may not greatly differ from the results shown in graph iii). Accordingly, the conventional theory of harmony may not cause great errors as far as natural instruments are concerned…[but]…Even for natural musical tones, the consonance

[155] Kameoka and Kuriyagawa, ibid., p. 1465.

> characteristics must be different by instruments if described in detail.[156] ['graph iii' is reproduced in *Figure* 5].

Kameoka and Kuriyagawa continue:

> It became clear that the fifth (2:3) is not always a consonant interval. A chord of two tones that consists of only odd harmonics, for example, shows much worse consonance at the fifth (2:3) than at the major sixth (3:5) or some other frequency ratios.[157]

Plomp and Levelt's and Kameoka and Kuriyagawa's models would seem to suggest an interesting way of investigating an ideal extended tuning system for existing acoustic instruments - that is, by applying these models to detailed mathematical descriptions of the spectra of those instruments. But there are at least two reasons why it is unlikely (or unclear) that experiments in this direction would lead to a radical discovery. The first and most important reason is that recent research confirms that the spectral components of the 'normal' tones of sustained acoustic instruments (for example - woodwind, brass and bowed strings) are, as predicted by the theory of 'self-sustaining' oscillations,[158] in exact (or extremely close to) integer relationship.

> A recently developed high resolution frequency tracker… has made it possible to measure the ratios of the frequencies of the upper harmonics of a sound with respect to its fundamental frequency with high accuracy. Calculations were carried out on digitised sounds produced by a clarinet, alto flute, voice, piano, violin, viola and cello…[159]

For the sustained instruments in this group the frequency ratios of harmonics (in this study the number of harmonics considered for each instrumental sound varied from 5 to 25) were in what Brown calls 'exact' integer relationship - that is, as exact as the accuracy which the measuring process itself allowed. In effect, it was shown that spectral components will normally deviate from an ideal harmonic series by no more than about 2-3 cents, but a smaller limit of mean deviation (which might be expected) could not be verified within the accuracy of the calculation.[160]

The second reason is that even if systematic patterns or similarities of inharmonicity were discernible in existing instruments, it is unlikely that these would prove significant in comparison to their *harmonic* similarities, that is, in terms of implying that one tuning system might be preferable to another. Rather, conventional instrumental spectra will 'imply' the consonant intervallic ratios with which we are familiar. Of course, orchestral instruments which have inharmonic spectra (the piano, percussion) complicate this, hence our question of which ATS will best encourage timbral integration. [161]

Relating Tuning and Timbre - from Helmholtz to Sethares (2)

The most important work on the relationship between timbre and scale has been carried out by William Sethares. His book - *Tuning, Timbre, Spectrum, Scale* (1997) - must rank amongst the most significant

[156] Kameoka and Kuriyagawa, ibid., p. 1468.
[157] Kameoka and Kuriyagawa, ibid., p. 1460.
[158] This is described below on pp. 44 - 45.
[159] Judith C. Brown, 'Frequency ratios of spectral components of musical sounds', *Journal of the Acoustical Society of America*, Vol. 99, No. 2, 1996, p. 1210.
[160] 'Continuously driven instruments such as the bowed strings, winds, and voice have phase-locked frequency components with frequencies in the ratio of integers to within the currently achievable measurement accuracy of about 0.2%. Since frequency fluctuations greater than the measurement accuracy are inherent in any sound produced by a human performer [on such an instrument], improvement of the measurements is unnecessary.' Judith C. Brown, ibid., p. 1218.
[161] A comprehensive study of spectral components for the full range of sustained (and impulsively driven) orchestral instruments would be useful - firstly to establish beyond doubt whether or not there are any significant patterns of inharmonicity for conventional sustained instruments; if there are, then secondly, although the implications for harmonic theory would be minor, these phenomena might suggest ways in which (significant, controllable) inharmonicity could be encouraged in new instruments. However, the variation of the *amplitude* of spectral components of existing instruments is relevant to preferred ATS; and especially if particular upper components can be deliberately limited or suppressed. Timbre varies from instrument to instrument, from player to player, and from note to note; and, as is well known, initial transients are important in the identification of sounds. If the steady-state element of sustained instrumental sounds is extremely close to (or exactly) harmonic, the amplitude of harmonics perhaps remains the most significant factor for relating 'timbre and scale' in conventional sustaining instruments. This is discussed further in '*An Inharmonic Instrumentarium*?' (pp. 102 - 105).

contributions to recent music theory, and is of intrinsic theoretical importance for this study. Sethares takes up the theory of sensory consonance and develops it in some highly imaginative and practical ways. This work may be especially useful to composers working with electronics but is not limited to this. In particular, it is clearly applicable to modifications of impulsively driven instruments, and also, if more tentatively, to sustained acoustic instruments.

The 'key idea of the book' is that:

> [C]onsonance and dissonance are not inherent qualities of intervals, but are dependent on the spectrum, timbre, or tonal quality of the sound.[162]

Sethares begins by showing that if the partials of a sound with a harmonic spectrum are electronically manipulated in a specific way, then the octave (2/1), which is traditionally the most consonant interval, can be made dissonant, and similarly, a traditionally dissonant interval such as an augmented octave can be made consonant.[163] To do this, the intervals which the partials make relative to the fundamental are stretched or contracted. When two sounds with altered spectra are heard together, the intervals formed between harmonics of the sounds therefore stand in a new relationship to each other. They will be most consonant when their nearest combined harmonics lie outside the critical band, and most dissonant when they do not. In general, the spectra of any two sounds may be altered so that our experience of them when they are sounded together is one of 'sensory smoothness' or 'sensory roughness'. Changing the spectrum of a sound of course changes its timbre, but Sethares also shows that even when spectra of instrumental sounds are manipulated quite radically, with care the character of an instrumental sound can be maintained. The implications of these phenomena are developed in a number of directions.[164]

The most central of these to the present subject is Sethares' use of dissonance curves to predict which scale would be most suitable for a given timbre (or any combination of timbres), and which timbre or timbres would be most suitable for a given scale. In the first case, it is shown that for any given spectrum (or spectra), there exists one or more scales which correspond to the minima of the dissonance curve which is generated by those spectra. The resulting scale(s), for example, may turn out equal-tempered or irregular, depending on the spectra. In the second case it is shown that given any particular scale, it is possible to derive from the scale a spectrum (or spectra), the minima of which, if plotted on a dissonance curve, will correspond to that scale.[165] Sethares offers the definition that:

> A spectrum and a scale are said to be *related* if the dissonance curve for the spectrum has minima at scale positions.[166]

In short, Sethares is saying that if you want to write music using a particular scale or tuning system, then certain specifiable timbres will sound particularly consonant in that scale, and may contribute to its musical effectiveness. Alternatively, if there is a certain sound which is attractive to you, then by analysing the spectrum of that sound it is possible to predict a scale that will be musically useful. It is important to note that the theory is not biased towards consonance - since, as Sethares points out, the technique will predict scales and timbres for any degree of consonance or dissonance a composer chooses.[167]

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[162] Sethares, op. cit., p. xii.

[163] A CD accompanies *Tuning, Timbre, Spectrum, Scale,* providing audio demonstrations which give excellent support to the theoretical thesis of the book.

[164] Earlier, pioneering experiments in the relation between timbre and harmony may be familiar to some readers, but I think it is fair to say that they are much less convincing or powerful in scope than Sethares' recent work. See, for example, John R. Pierce, 'Attaining consonance in arbitrary scales', *Journal of the Acoustical Society of America*, 1966, 40, p. 249; J.M. Geary, 'Consonance and dissonance of inharmonic tones', Journal of the Acoustical Society of America, 67 (5), May 1980, pp. 1785-9; J. Sundberg, ed., *Harmony and Tonality*, Royal Swedish Academy of Music, No. 54, 1987; John R. Pierce, *The Science of Musical Sound*, W.H. Freeman and Co., 1992, pp. 87-92; M. V. Matthews and J.R. Pierce, 'The Bohlen-Pierce scale', *Current Directions in Computer Music Research*, ed., M. V. Matthews and J.R. Pierce, MIT Press, 1991.

[165] The techniques used to accomplish this are left for the reader to investigate directly in Sethares' own writings.

[166] Sethares, op. cit., p. 89.

[167] Sethares stresses that the purpose of dissonance curve analysis is not necessarily to find a way of making consonant music, but rather to help the composer, tuning theorist or instrument designer to *control* the degree of consonance or dissonance required.

In APPENDIX II Table 11 it may be seen, for example, how few close approximations to just ratios there are in 11-ET. The octave (2/1, 1200 cents) is of course just. Few of the other intervals of 11-ET would normally be considered acceptably close to just. The 'just minor third' (327 cents) is slightly sharp, at 11 cents wider than 6/5 (316 cents), and the 'just major sixth' is therefore correspondingly narrow relative to 5/3; but both intervals deviate less from the nearest *just* (as opposed to Pythagorean) ratios than do the minor third (300 cents) and major sixth (900 cents) in 12-ET. The closest intervals to either a just fifth (3/2, 702 cents) or a just major third (5/4, 386 cents) are roughly a quarter-tone sharp or flat. For the latter reasons particularly, 11-ET (like 13 and 23-ET) is normally thought to be amongst the most dissonant and awkward of ATS in which to compose.

But Sethares has composed a catchy and convincing 'alternative pop' style composition in 11-ET, *The Turquoise Dabo Girl*, which is included on the CD which accompanies the book. For this electronically realised piece, the partials of harmonic sounds are manipulated so that dissonance curves drawn for each individual timbre show minima at the scale steps of 11-ET.[168] An example of this dissonance curve is shown below in *Figure 6.*

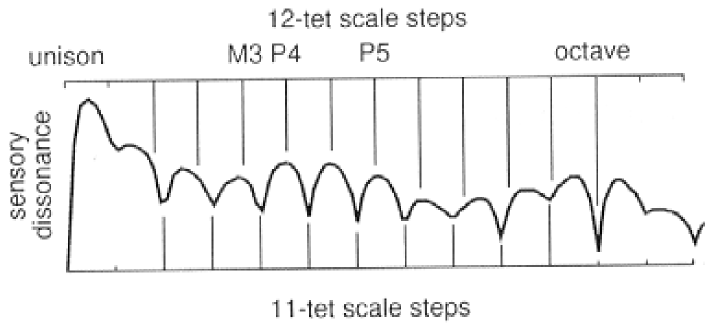

Figure 11.1. Dissonance curve for the spectrum with equal amplitude partials at $[1\ a^{11}\ a^{17}\ a^{22}\ a^{26}\ a^{28}\ a^{31}\ a^{33}\ a^{35}\ a^{37}\ a^{38}]$ where $a = \sqrt[11]{2}$. The minima of this dissonance curve occur at many of the 11-tet scale steps (bottom axis) and not at the 12-tet scale steps (top axis).

Figure 6 : William A. Sethares, *Tuning Timbre, Spectrum, Scale*, Springer Verlag, 1977, p. 236. (The illustration is reproduced with the permission of Bill Sethares).

Two versions of *The Turquoise Dabo Girl* are included on the CD - one with conventional harmonic timbres, the other with the spectra deliberately 'related' to 11-ET. Although the 'notes' are identical, the difference between the two versions is clearly audible - and remarkable. The version with conventional spectra sounds awkward and somewhat 'whiney', and there is a kind of fog of dissonance through which it is difficult to hear things clearly. In the version with related spectra the harmonic content sounds more clearly, and the melody speaks more directly; there is much less 'whineyness', and the unusual melodic and harmonic structure are easier to identify.

Three further points are also striking about the version with related spectra. Firstly, although the underlying scale structure and harmony is intriguingly strange and 'xenharmonic', the 'awkwardness' is *reduced* in the version using related spectra because the 'whineyness' is reduced. Secondly, Sethares also presents the individual sounds of each of the (electronic) instruments on a separate track on the CD, and the degree to which the changes in the spectra affect the timbres of *individual* instruments differs markedly in the way one timbre appears to be changed more than another - yet in *ensemble* this is much less noticeable. Thirdly, in the version with related spectra it feels as if the harmony and sounds really fit together, and the music takes on a 'singing', resonant quality, a feeling somewhat akin to the difference between playing an acoustic instrument which has been tuned and set up well, and one that has not.

The CD demonstrates that Sethares' technique might be applied successfully to any tuning system, and the fact that Sethares has generalised these results makes the theory particularly compelling. Computer code and step

[168] Sethares, op. cit., p. 236.

by step instructions are provided for making the calculations needed to relate tuning and timbre. These are not entirely automatic routines, and require a degree of choice and second guessing from the user. The results are therefore not absolute, but from the examples given it is clear that Sethares' technique successfully 'relates' tuning and timbre.

The theory is also a powerful predictive hypothesis about musical universals. As he suggests, one way of asking whether the hypothesis is true is to examine the relationship between scale and timbre in more than one culture. What, for example, is the related scale for the timbre of the sitar? In *Tuning, Timbre, Spectrum, Scale* Sethares devotes a chapter to comparing both the pelog and slendro scales of specific Indonesian Gamelans to the sounds which their particular instruments make. He concludes that:

> The slendro scale can be viewed as a result of the spectrum of the bonang [one of the instruments of the gamelan] in combination with a harmonic sound, while the pelog scale can be (slightly less surely) viewed as resulting from a combination of the saron and a harmonic sound. Thus, gamelan scales exploit the unique features of the spectra of the nonharmonic instruments of which they are composed, yet retain a basic compatibility with harmonic sounds like the voice.[169]

A comparative study investigating how tuning systems from various cultures 'relate' to the timbres of indigenous instruments on which those scales are realised would be extremely interesting.[170] Another approach might be to analyse the relation of indigenous scales to the spectra resulting from indigenous vocal technique. For example, are the scales and harmony of Bulgarian folk music 'related' to the spectra of characteristic singing styles of Bulgarian men and women? Could we assess how or whether different cultures display different levels of dissonance tolerance? Certainly, since harmonic spectra are a intrinsic feature of Western orchestral instruments (excluding percussion etc.), the theory of the relation between timbre and tuning is highly suggestive of why Western music is the way it is - and perhaps why the musics of other cultures are the way they are.

It is also remarkable that the dissonance curves for harmonic sounds - as arrived at by Helmholtz, Partch, Plomp and Levelt, Kameoka and Kuriyagawa, and Sethares - are similar and correspond to just ratios. This would seem to be evidence that scales comprised of just ratios do form the 'related' scale for harmonic spectra; it also makes clear that 12-ET corresponds closely to the dissonance curve for harmonic spectra, and therefore might be considered the nearest ET which is close to being 'related'.

The theory of sensory consonance has far-reaching implications for the development of ATS, since it implies that the notion of a 'just' interval is relative to or dependent upon the spectra of the sounds which form the interval, in effect, redefining the notion of a 'just interval'. To an extent it supports the arguments of just intonationists who base the theory of JI on the harmonic series, but suggests that if the sounds of a composition are not harmonic, then the justification (in itself) is misleading. This has interesting implications:

> an alternative to playing in a just intonation scale using harmonic tones is to manipulate the spectra of the sounds so as to increase their consonance in 12-[ET]. …Both approaches eliminate the disparity between 12-tet and harmonic tones, one by changing the related scale, and the other by changing the related spectra.[171]

To put this another way, if the timbres of conventional instruments could be modified so that their spectra are 'related' to 12-ET, then music might be performed (in effect) in 'just intonation' while the scale itself remains 12-ET. More radically, if it were at all possible to modify instrumental timbres so that they formed the related spectra for (say) 24 or 36-ET, then instrumental music in quarter-tones or sixth-tones could be performed in 'just intonation'.

[169] Sethares, op. cit., p. 187.
[170] Sethares also mentions this in: William A. Sethares, 'Local consonance and the relationship between timbre and scale', *Journal of the Acoustic Society of America*, 94 (3), Sept. 1993, p. 1227.
[171] Sethares, *Tuning, Timbre, Spectrum, Scale*, Springer Verlag, 1997, p. 217.

However, whether it is *possible* to create deliberately controlled inharmonic timbres on *sustaining* acoustic instruments is controversial. At present, it would seem there are no known methods for enabling woodwind, brass or strings to produce intentionally inharmonic spectra without becoming 'non-sustaining' and/or losing the crucial property of producing tones which are heard as 'fused'.

> Musical instruments capable of producing a sustained tone… consist essentially of one or more resonant systems (air columns, cavities, strings) with very nearly linear acoustic behaviour, excited by a non-linear source (lips, reed, air-jet, bow) with which they are coupled to form a regenerative feedback loop… [T]he natural modes of any real acoustical resonant system are never in exact harmonic relationship, because of second-order effects like end-corrections and string stiffness. It is common experience that non-harmonically related sounds ("multiphonics" or "burbles") can be produced on most wind instruments, though in normal tone production the overtones are accurately harmonic and locked in both phase and frequency to the fundamental.[172]

The 'regenerative feedback loop' is the basis on which the normal sustained tone of the instrument depends. For the loop to operate effectively, spectral components must be in exact or near exact integer ratios (enabling what Benade has called 'multi-mode co-operation') otherwise they will (a) fail to 'co-operate' with each other, and (b) fail to provide a 'fused' tone (the more the sound is harmonic the more the ear tends to hear the tone as a single fused pitch). Although woodwind and brass, for example, might be constructed so as to have a series of intentionally inharmonic *resonances*, the feedback loop determines that a fundamental tone will only actually engage 'resonances' of the instrument which match the integer (or very near integer) *harmonics* of that fundamental - that is, where the player is trying to create a normal sustained tone.[173]

Benade has described graphically the relationship between resonant frequencies (those intrinsic to the acoustic system) and the spectral component frequencies (those which are actually brought to life under normal playing), as here, in reference to the saxophone:

> slight imperfections in the air column design may displace (misalign) the resonance frequencies from the strictly harmonic relationships that are characteristic of the complete simple cone. During start-up, the not-quite-harmonic "original" components quickly synchronise with one another to produce ultimately a *strictly harmonic* regime of oscillation… all of the generated harmonics in the perturbed system are running slightly off resonance relative to their corresponding air column modes so that they generate slightly less energy and the regime is slightly less stable. Under these conditions, the oscillation starts up more slowly and irregularly, and the players says that the instrument lacks a clear tone and responsive behaviour. As a practical matter - in order to ensure good playability for the musician - it is sufficient to arrange for the first two or three resonant frequencies of the air column to be aligned so they are harmonic relative to one another.[174] [Benade's italics]

Thus, according to this description, even if higher resonances were made to agree with an alternative 'related' scale, they will still not *sound* in resultant spectra. Further, they may make the instrument less stable, and/or less 'perfect' in tone.

Sethares and Fletcher have suggested that to create an acoustic instrument with sustained inharmonic spectra it would be necessary to drive an inharmonic system (in this case a specially made instrument bore or string) in such a way that 'mode-locking' does not occur - at the same time, the system must be both non-linear and

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[172] N. H. Fletcher, 'Mode locking in nonlinearly excited inharmonic musical oscillators', *Journal of the Acoustic Society of America*, 64 (6), Dec. 1978, p. 1566.

[173] I am especially grateful to Paul Erlich for his comments on this question in an earlier draft. For an advanced study of the physics of sustaining instruments, see J. Woodhouse, 'Self-sustained musical oscillators', in *Mechanics of Musical Instruments*, Ed. A. Hirschberg, J. Kergomard & G. Weinrich, Springer Verlag, 1995, pp. 185-228.

[174] A. H. Benade and S. J. Lutgen, 'The saxophone spectrum', *Journal of the Acoustic Society of America*, 83 (5), May 1988, p. 1900.

capable of producing consistent spectra across its range. This is a great challenge, but not necessarily impossible. Further discussion is given below in '*An inharmonic instrumentarium?*'.[175]

Despite these qualifications, it is no exaggeration to say that the theory of sensory consonance/dissonance, and Sethares' work in particular, may inaugurate a new era in the theory of tuning systems. It is no longer tenable to view competing theories of just intonation and equal-temperament as inevitably irreconcilable - as has seemed the case for centuries. It also identifies a number of alternative approaches to new instruments for ATS:

- to adapt existing instruments so that they provide a new scale, and retain (or improve) their existing timbral characteristics;
- to adapt existing instruments so that their scale and timbre are in some way 'related' - in terms of partial frequencies or amplitudes, or both;[176]
- to analyse the timbres of existing acoustic instruments, singly and in combination, to establish their specific effectiveness and 'relatedness' for particular tuning systems;
- to speculate using electronic simulations about the kind of tuning and timbral combinations which might be effective and practical for acoustic instruments.

Terhardt's 'Two-Component' Theory of Consonance

According therefore to the theory of sensory consonance, the minima of a 'dissonance curve' define the points of 'greatest smoothness' for simultaneous sounds with specific spectra. The question remains - is the cause of 'greatest smoothness' for simultaneous sounds also the cause of other phenomena of 'consonance', such as 'stability', 'gravitation' and 'rightness' etc.?

Ernst Terhardt has proposed a 'two-component theory of musical consonance', the two components being 'sensory consonance' (as has been described) and 'harmony'.[177] Terhardt's concept of 'harmony' comprises two interrelated parts: (i) the perception and recognition of harmonic complex tones as arising out of the human acquisition of speech; and (ii) 'virtual pitch' - the phenomenon of hearing a harmonic complex tone as a single fused 'fundamental' - that is, whether or not the fundamental is actually present in the spectrum.

> The whole learning process… in which an individual acquires familiarity with the basic musical intervals by aural analysis of complex harmonic tones, will with highest probability take place as an essential part of the perception of speech, as speech is the most significant auditory signal to humans. The purpose of that learning process, according to the virtual-pitch theory, is to enable the auditory system to extract virtual pitch from any complex tone, even when some harmonics (in particular the fundamental) are not present. It is thus a sort of by-product that in that learning process a sense of harmonic intervals (octave, fifth, etc.) is acquired.[178]

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175 Bart Hopkin too has commented that the 're-tuning of spectra is standard for marimbas and large bells… Wind instrument makers work hard to make their instruments [perfectly] harmonic - a subtle process very much tied to what has been learned historically by trial and error… I suppose they could work just as hard to achieve specific non-harmonic spectra, but it would be subtle and fraught with difficulties'. Personal communication, March 1998.

176 An initial experiment, for example, might attempt to refine the timbres of some existing '12-ET' acoustic instruments, to see whether their spectra can be more closely 'related' to 12-ET, and to see if this results in something akin to 'just intonation'. One might think of this as the 'reverse' of stretching the scale of the piano when it is tuned.

177 Terhardt writes that 'We consider the term musical consonance to be subsuming the principles that are regarded as governing tonal music'. While 'atonal' music is seldom absolutely 'non-tonal', for present purposes we must be wary of a theory of consonance and dissonance which takes tonal practice as its theoretical basis. The important point here is that Terhardt argues that the theory of sensory consonance cannot be the sole explanation of the multiple phenomena of *musical* consonance. It should be pointed out that Terhardt's seminal work in this area preceded that of Sethares, so there is no implication that Terhardt was 'responding' to Sethares. See Ernst Terhardt, 'The concept of musical consonance: a link between music and psychoacoustics', *Music Perception*, Spring 1984, Vol. 1, No. 3, pp. 276-295.

178 Ernst Terhardt, op. cit., p. 288. Terhardt's theory is very much based on that of Helmholtz, although Terhardt argues that '[a]pparently it had escaped Helmholtz's attention that the principles of 'auditory pattern recognition' with harmonic complex tones… primarily apply to natural human speech, namely the voiced speech sounds'. However, Helmholtz did remark on the recognition of human speech in the most strikingly relevant way: 'we have all our lives remarked and observed the tones of the human voice more than any other, and *always with the sole object of grasping it as a whole* and obtaining a clear knowledge and perception of its manifold

Thus, in terms of ‘harmony’, the privileged status of intervals formed by just ratios follows from the fact that the simplest and most basic pattern which a child learns to recognise is, in the first place, the harmonic complex tone. Just ratios are already *part of* that special pattern, which from the earliest age has to be learnt in order to hear, understand, and speak - and thus, before the effect of different dyadic complex tones ever becomes an issue. Terhardt sees the two phenomena as interdependent: firstly, the phenomenon of virtual pitch - the ‘abstraction’ of a basic pattern as a fundamental, biological function of hearing; and secondly, the particular intervallic structures which are more or less ‘built-in’ to (or acquired by) the human hearing function.[179]

The musical point of this is that, as Terhardt puts it:

> The fact that a pitch can be perceived although there does not exist any spectral component at the frequency corresponding to that pitch provides the key to understanding the whole harmony phenomenon, that is, tonal affinity, compatibility, and fundamental note relation...[180]

‘Tonal affinity’ is the perceived ‘similarity that exists between harmonic tones an octave, a fifth, or a fourth apart’; the ‘compatibility of chords and/or melodic segments… is manifested in the invertibility of chords’;[181] and the ‘fundamental note relation’ refers to the tendency of chords to imply a root or fundamental tone, although ‘there seems to exist only one type of chord that possesses a completely unambiguous fundamental note: the major triad’.[182] If these hypotheses are accepted, they have striking implications for the importance of good approximations to the octave, fifth, fourth (and perhaps major third), and for the presence of a single cycle of fifths, in any ‘quasi-universal’ ATS.

However, Terhardt argues that the two principles - ‘sensory consonance’ and ‘harmony’ - do not necessarily have exactly corresponding implications for scale intonation. According to Terhardt the former imply just intonation whereas the latter do not. ‘[h]armony’ - in Terhardt’s special sense -

> is governed by pitch relationships (as opposed to frequency relationships), and musical intervals, in terms of pitch, do in many cases not exactly correspond to ‘just’ intonation of frequencies. A well-known example of this discrepancy is the octave enlargement phenomenon… thus the concept of musical consonance reveals that scale intonation necessarily is a compromise between conflicting requirements.[183]

Not only is it the case that judgements of intervals do not always correspond to small integer ratios, but that typically, the octave is commonly judged as being wider than a frequency ratio of 2:1, and in particular, ascending intervals of other types are thought to be ‘in tune’ when stretched relative to just ratios. These variations are thought to occur in situations where tuning constraints are minimal or maximal. The number of factors which have a bearing on this - musical learning, language acquisition, the structure (non-linearity) of the ear, the timbre of an instrument or voice, the instrument a performer typically plays (especially the piano) - pose a labyrinthine problem.[184] However, it seems extremely unlikely that small integer ratios do *not* have an important influence in determining, in Terhardt’s sense, ‘musical intervals, in terms of pitch’.[185]

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changes of quality’. Helmholtz, op. cit., p. 104, (my italics). Helmholtz provides a detailed discussion of vowel resonances - and a perceived relationship between vowel harmony (different vowels of speech are formed by different harmonic intervals) and auditory fusion seems to be latent in the above statement. See Helmholtz, op. cit., pp. 103-119.

179 ‘[Terhardt’s] concept of harmony may be interpreted as involving familiarity with patterns of pitch within musical chords or sonorities, on various cognitive levels. Complex tones are perceived within a chord by harmonic pitch-pattern recognition, just as pitch is perceived in speech. In general, the easier it is for a complex tone to be perceived in this way within a chord, the greater is the consonance of the chord… At higher, more musical levels of familiarity, specific chords or chord progressions may be perceived as consonant if they occur often in music, or have particular musical functions’. Richard Parncutt & Hans Strasburger, op. cit., p. 92.

180 Terhardt, ibid., pp. 287-8.

181 Richard Parncutt has developed a related notion - ‘the pitch commonality of two sonorities may be defined as the degree to which they evoke, or are perceived to have, common pitches’. Richard Parncutt & Hans Strasburger, op. cit. p. 94; also Richard Parncutt, *Harmony: A Psychoacoustical Approach*, Springer Verlag, Berlin, 1989.

182 Terhardt, ibid., p. 279.

183 Terhardt, ibid., p. 294.

184 In the present discussion this is left to further debate, but it must be noted that if interval identities are not determined by frequency ratios, this throws into question the analysis given in the tables of APPENDICES I, II etc., and other statements in this paper. It is, however, reasonable to suppose, following Terhardt’s own argument, that frequency ratios do play a part in determining intervallic

Terhardt distinguishes in this theory those elements which belong to psychoacoustic phenomena (being to some extent universal) and those which are the product of learning (being the product of culture and training). Interestingly, virtual pitch (which is a psychoacoustic fact) and the 'built in' recognition 'template' of the harmonic series (which is thought to be learned) are both thought to be universal.[186] Thus, if the perception of *harmonic* complex tones is in some way 'built-in' to aural perception, what are the consequences for the perception of *inharmonic* timbres or a scale 'related' to inharmonic spectra? And does the fact that it is 'built-in' mean it is permanent, and immutable?

The extent to which a sound is heard as 'fused' depends on the degree of inharmonicity - but also familiarity. The inharmonic timbres used by Sethares in *The Turquoise Dabo Girl* and other demonstration examples mentioned, are certainly heard as fused, and imply a 'fundamental'.[187] As discussed below, Sethares has carefully chosen a partial structure which mimics the harmonic series (see below, *Footnote 382*). This does not contradict Terhardt's view:

> According to the virtual pitch theory… it is supposed that harmony is not entirely 'hard-wired' in the auditory system but rather is heavily affected by, if not entirely dependent upon, a learning process which is performed by every individual in the speech-acquisition phase of his/her life. Hence, according to our concept learning actually *is* involved in musical consonance. Therefore it follows that by use of suitable sound generating facilities (eg., computers) new tonal systems may eventually be aurally acquired…[188]

In the 20th Century, musical appreciation has already undergone a process of 'transcending' the conventional 'harmonic' basis of music, a process which has been on-going for many years. The 'liberation of sound'[189] is almost total in 'academic' electroacoustic music, and is becoming increasingly orthodox in much popular music (even if it is usually heard in a harmonic context); the enjoyment of atonal contemporary works similarly depends on learning and familiarity - as does the appreciation of music of unfamiliar cultures. This is however not to deny the existence of 'basic perceptual criteria of harmony'.[190] Rather, it is to accept both the psychoacoustic (or 'natural') and its learned aspects, and points to their *mutual* importance in the future of music.[191]

Intonational Tolerance and 'Zonality'

In the 'two component' theory of consonance, two related explanations of intervallic 'nodes' are posited - sensory consonance, and Terhardt's concept of 'harmony' - of which the latter encompasses a variety of factors - cultural, learnt, and psychoacoustic. The location of each node is influenced by the 'built-in harmonic template' of the ear (Terhardt); this 'template' is not entirely fixed, but is learnt, and to some extent can be 're-learnt'; and the relationship between sound *spectra* is also important, so that a 'related scale' may be formed from the intervals which correspond to minima bands on a 'dissonance curve' for a given timbre or combination of timbres (Sethares).

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identities (and a major part in determining sensory consonance) - so analysis in terms of ratios may serve at least as a kind of first approximation.

185 This is why, to my way of thinking, arguments of this kind do not invalidate the usefulness of (for example) the tables of APPENDIX II - that is, given its qualification in terms of 'intervallic zones'. See the explanation at the beginning of APPENDIX II.

186 If a child is exposed to music whilst in the womb (or from the earliest age) does this merely complement the more fundamental need of the child to acquire speech? A child learns to identify other kinds of sounds and to mimic them - does the need to interpret and copy speech entirely override the importance of these other interactions?

187 In rare cases this may vary from listener to listener, and in certain situations a listener may choose to 'hear-out' certain partials. These special cases do not contradict the main point.

188 Terhardt, ibid., p. 294.

189 "It must be quite widely known by this time, since I have been boring people on the subject for almost half a century, that my aim has always been the liberation of sound…". From Varèse's lecture in Princeton in 1959. Fernand Ouellette, op. cit, p. 47.

190 Terhardt, ibid., p. 294.

191 As an addendum to this section I have included APPENDIX V: *A Fusion Model of Melodic Harmonicity*, which contains a brief account of the idea that melodic 'rightness' may derive from the implicit structure of a harmonic tone, in which the octave-adjusted 'just' intervals (2/1, 3/2, and 5/4) predominate in the 'fabric' of fusion. The significance of this is left open to debate.

In this section, the notion of *intervallic character* is introduced without attempting to base it directly on either of these explanations, although it may coincide with aspects of both. 'Intervallic character' is the subjective identity of an interval which differentiates it from other intervals, and corresponds to an intervallic 'zone' rather than to an exact interval. Of itself, this phenomena does not seem to be determined solely by frequency ratio, timbre, or learning.

To the composer interested in ATS, a crucial question is whether there are 'other', 'new' intervallic characters, which have unique identities and are yet as compelling as the familiar intervallic characters or 'zones' of 12-ET. The search for the ideal, quasi-universal system is the search for which ATS best provides both the 'new' and 'old' intervals, and extends the palette of tones in the most coherent way. APPENDICES I (b) and (c) are, in a sense, an attempt to define the 'primary' *available possibilities* of unique intervallic characters for harmonic tones, being also practical for acoustic instrumental composition.

Consider, for example, the opening four notes of Beethoven's *C# minor String Quartet Op. 131* - a slow and exposed ascending major third, G# to B#, rising to C#, and which then falls to an accented A. This phrase has an extraordinary other-worldly character, which might equally have been the opening of an atonal work. How wide or narrow can the first major third be before it sounds 'out-of-tune'? At what point is that interval no longer a 'major third', or no longer acceptable?[192]

Judging from various famous recordings, the ascending third is normally taken nearer to the tempered third (400 cents) than a pure 5/4 (386 cents). To remain a 'major third' appropriate to Beethoven (if the concept makes sense) it seems it may be as wide as 408 cents, but not much narrower than 384 cents. This may seem to give exaggerated leeway, but the melodic line is unaccompanied, and its very strangeness suggests (even to contemporary listeners) a special freedom. This is curious from the point of view of JI, because one might expect that an exposed, unaccompanied major third, right at the beginning of a work, would tend towards just, rather than centring on the tempered major third. This may be a result of conditioning. As the fugue develops, intonation depends very much on the interaction between the further entries of the subject and the surrounding harmony, so perhaps this also influences the intonation of the opening bars.

Although the listener is not necessarily aware of it at this stage, B# is of course the leading note, which is normally sharpened to lead to the tonic. It would seem, in general, that in tonal music the intonational tolerance of an interval depends on harmonic context and/or the scale degrees from which it is formed. To illustrate this, consider some simple examples. In the English folk melody *Greensleeves* the contemporary musician is certainly used to hearing (or singing) the opening minor third as something in the region between 295 and 310 cents - roughly speaking the minor third of 12-ET (300 cents). This minor third does not correspond exactly to either 6/5 (316 cents) or 7/6 (267 cents), but is often described as a depressed approximation of the former. In JI terms, it might be described in various ways, for example as the nineteenth harmonic (19/16, 297.5 cents), the 'pythagorean' minor third (32/27, 294 cents), or 25/21 (301.8 cents).[193]

A similar example is the beginning of the saxophone theme from the first movement of Rachmaninoff's *Symphonic Dances Op 45* (a melodic ascending minor triad). Nothing, I think, would induce a good performer to play the initial minor third either as 6/5 (which does not have the necessary flavour), or worse, as 7/6 (which is too depressed). Even so, in both examples the intonational leeway of the mediant note is similar, and is determined not simply by tonic minor harmony, but by the fact that the two notes are tonic and mediant. Returning to another example in Beethoven, at the beginning of the fourth movement of the *Bb Major String Quartet Op 130*, the intonational leeway of the minor third figure D - B - D is noticeably stricter. Here, the harmony is G major, and the degree of leeway in this minor third is less (it is taken closer to 6/5). Perhaps this is because the sonority desired of the tonic major triad requires it to be tuned strictly (in JI), to a degree that is not typically demanded of the minor triad (possibly due to the special correspondence between the major triad

[192] Harry Partch expressed disinterest (as will many Just Intonationists perhaps) in the idea that a mistuned fifth or third is still 'heard as' a fifth or third, because he wanted to hear just ratios. In a sense he was right - composers should certainly specify and compose what they want to hear. See Harry Partch, op. cit., pp. 414-7. But a theory of intonational tolerance is certainly pertinent if a new system of instruments were to be made for a unique tuning system which is Equal- or Well-Tempered.
[193] Each of these assignations is shown in APPENDICES I and II etc. The ambiguous definition of common intervals compared to their equivalents in terms of JI ratios, except under special quasi-laboratory conditions, further supports the idea that ratios are not the sole determinants of intervallic identity.

and the harmonic series). Again, other classes of minor third (supertonic/subdominant, leading-note/supertonic, etc.) seem to display differing but characteristic degrees intonational tolerance. It would be convenient to explain this in term of ratios, but, due firstly to ambiguities regarding which ratio a particular interval is actually an instance of, this is by no means obvious; secondly, it is highly unlikely that frequency ratios are the sole determinants of preferred intervals, or of 'intervallic character'.

APPENDIX I (b) and (c) shows informal, subjective observations of the limits within which intervals belong to one 'intervallic character' or another. These limits were not determined in terms of roughness or beating, but rather in terms of where the identifiable character or identity of an interval undergoes a change - and takes on a new character. To obtain this chart, recorded samples of (mainly) bowed violin tones[194] were used (with and without vibrato);[195] the samples were not intonationally perfect, and the results are highly provisional.[196] Two columns either side of the 'rational interval' show the limits within which an interval retains its 'identity' - abstracted from an harmonic context, so far as this is possible.[197] This was judged, first by listening to the character of intervals alone (and amongst other intervals), and secondly, by guessing the intonational limit within which practical music making would be considered 'in tune' for individual intervals.[198] Note that the purpose here was to describe an *ideal* limit rather than a 'workable' one - the zones proposed are much *stricter* than intonational limits actually occurring in performance.

It is normally simple to distinguish whether an interval is of a familiar type, because relatively large intervallic areas which are considered 'out of tune' lie between the familiar nodes. It is however much more difficult to state exactly where one interval becomes another. This becomes even *more* difficult when we consider much finer divisions. For example, the transitions between the three 'minor thirds' (7/6, 19/16 (or the pythagorean minor third), and 6/5) are very smooth, and there are no obvious 'steps' which clearly differentiate them.[199] The column marked 'grey area' shows ambiguous regions where it is unclear to which category an interval belongs; it is even unclear where the 'grey areas' begin and end. It would be highly useful, for any theory of ATS, to complete a more scientific study of how relatively naïve listeners and experienced musicians distinguish different intervals on the continuum, and to know to what extent listening habits, cultural factors etc., influence this.

In the cases of the major and minor thirds, the fourth and the fifth, upper and lower limits are marked harmonic (h) or melodic (m), and show that intonational tolerance is greater for unaccompanied melodic intervals than for intervals resulting from simultaneities. Obviously the table shows tolerance only for dyads - no attempt is made to consider tolerance in triads, other chords, or chord progressions. However, three points are worth noting: (1) the asymmetry of intonational tolerance between intervals and their inversions; (2) the intervallic zone within which an interval retains its character is not always 'centred' about the relevant just ratio;[200] (3)

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[194] Benade has pointed out that 'one does not hear clear-cut beats between mistuned violin tones of the sort that painfully advertise slight errors between two wind instrument sounds. It was not difficult for me to recognise… that the weakness of the beats implied unsteadiness in the sticking and slipping of the rosined bow on the string… The fact that each partial of a string tone is spread over a bandwidth of about 20 cents means that there is a diffuseness to the string tone which has enormous implications for the musician. On the one hand it allows larger tuning errors to be made in ensemble playing before the discrepancies become unacceptable, and on the other it permits the composer to write a wide variety of chords having many degrees of consonance and dissonance…' Arthur H. Benade, *Fundamentals of Musical Acoustics*, Dover, 1990, p. 549. Sampled tones do not simulate this phenomenon since there is no bow in operation. The absence of synchrony of vibrato, normally occurring due to movement of the hand, may also be a factor in these results. But these experiments need to be done with the full range of orchestral instruments.

[195] Since the purpose here is to identify 'aural character' rather than properties of consonance or dissonance - vibrato tones were used to simulate (so far as that makes sense given the experiment) a 'musical' rather than a laboratory situation.

[196] Parncutt and Strasburger have said that 'there may be a psychoacoustically determined limit to the density of harmonically distinct steps in a scale…. By 'harmonically distinct' we mean contributing to different virtual pitches… The psychoacoustic data suggest that the maximum possible density of harmonically distinct scale steps lies somewhere in the range nine to twenty-four per octave'. But they also go on to suggest that '[i]f the width of pitch categories are indeed dependent on learning, then it should be possible to reduce their width by repeated exposure to appropriate patterns of sound - for example, microtonal music.' Richard Parncutt & Hans Strasburger, op. cit., pp. 119-20.

[197] There was however a tendency to hear each interval as if its lower tone were 1/1.

[198] Again, considered as far as possible in isolation.

[199] It might be argued that judging the 19th Harmonic or 'quasi-tempered' minor third as an independent zone is the result of conditioning by 12-ET; alternatively, one might say that 12-ET is possible *because* this intermediary zone is in itself compelling, and is not 'subsidiary' to either 6/5 or 7/6.

[200] As for example is suggested by the slope of Helmholtz's dissonance curve for violin tones (see *Figure 3*).

different limits of tolerance were discerned for different instruments - for example, in general, trumpet tones are much less forgiving of intonational deviation than violin tones (this is not shown in the table).[201]

As we saw, in APPENDIX II the equal-temperaments between 5- and 41-ET are compared to the integer ratios of APPENDIX I (a), the purpose of which was to help the reader to compare for themselves, in a relatively simple way, the 'harmonicity' of those systems.

In APPENDIX II it was also intended to quantify the 'efficiency' with which the intervals of each temperament fall within the 'microtonal' interval *zones* as set out in APPENDIX I (b) and the somewhat less microtonal zones in APPENDIX I (c). The idea was to compare the relative 'zonality' of these systems - that is, the extent to which each of these *n*-ETs provide approximations, not to the low integer ratios themselves, but to the 'centres' of the (nominal) intervallic zones. The 'nominal centre' has no acoustic significance in itself, but enables a quantification of the xenharmonic 'efficiency' (or redundancy) of each temperament in terms of the proportion of intervals which fall within the boundaries of a zone, and by how much. If there exist only a limited number of effective, distinguishable 'zones' between 1/1 and 2/1 (for harmonic tones), then the 'quasi-universal' xenharmonic system is one in which provides an optimal correspondence (no omissions, no redundancy) between scale steps and 'zones' - at the same time as maximising harmonicity. Various computer programs were written to analyse this,[202] but unfortunately it proved impossible to present these results here, partly due to the provisional and complex nature of APPENDICES I (b) and (c).[203]

For evaluating a preferred ATS for acoustic instruments, suitable for a wide variety of music, three points therefore seem evident:

- thorough investigation of the relationship between intonational tolerance, 'primeness' and harmony, is required;
- *harmonic* limits of tolerance are more significant for our purpose (because they are more narrow) than melodic limits;
- it may be useful to consider instrumental timbres which display the narrowest intonational tolerance to predict a system of the greatest 'universality'.

Ideally, then, harmonicity functions might take into account the relative tolerance of different intervals, and rigorous research on this topic would be useful. Lindley and Turner-Smith for example remark that:

> the range of quantities for tempering overlaps that for out-of-tune-ness, so there have been disagreements as to whether certain temperaments were acceptable. Among the many which have been proposed, however, none has ever gained much acceptance in which any 5^th^ is tempered by as much as 2/3^rds^ of 1% of an octave.[204] [ie., 8 cents]

It is often commented that for microtonal systems the smaller an interval (or interval range) the more likely we are to be able to pitch (or identify) an interval accurately, although this must be subject to familiarity.[205] But I have yet to discover a study of intonational tolerance which distinguishes (for example) categorical perception, 'out-of-tune-ness' and the relevance of timbre. In some analyses of ATS the differing degrees of accuracy appropriate to approximating different intervals within a scale are incorporated on a more or less intuitive basis, or relative to the harmonic series, rather than on empirical results.

It would also be interesting to relate intonational tolerance to dissonance curve analysis, perhaps by examining the limits of intonational tolerance for various *inharmonic* tones. Are there, for example, constants common to the intonational tolerance of both harmonic and inharmonic tones? Is it possible to define intonational

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201 See note 194 .
202 These programs were written by Joseph Sanger.
203 Further explanation of this experiment is given at the head of APPENDIX II.
204 Mark Lindley & Ronald Turner-Smith, *Mathematical Models of Musical Scales*, Verlag für systematische Musikwissenschaft, 1993, p. 37.
205 Edward M. Burns & W. Dixon Ward, 'Categorical perception - phenomenon or epiphenomenon: evidence from experiments in the perception of melodic musical intervals', *Journal of the Acoustical Society of America*, 63 (2), Feb. 1978, pp. 466; see also, Pierre Boulez, 'At the edge of fertile land', *Stocktakings from an Apprenticeship*, Clarendon Press, Oxford, 1991, pp. 162-4.

tolerance (so far considered a more or less subjective phenomenon) as a mathematical function of spectral structure? A definitive notion of the limits within which any interval remains itself and 'in-tune' would provide a more precise guide as to which ATS will really offer the best opportunities. In addition, Sethares has demonstrated that a 'node' is associated with the *minima* of a dissonance curve for given spectra, so it would follow that no zone of intervallic stability can exist at or near the dissonance *maxima* of the curve. For example, a number of the 'quarter-tone' intervals of 24-ET and other systems correspond loosely to the dissonance maxima of the curve for spectra having six harmonic partials.[206] For harmonic tones there are limitations to the existence or number of effective 'new' nodes. There would certainly be value in bringing a number of these interrelated analyses and criteria to bear on the evaluation of large-*n*-division pitch systems.

Large n-Division Temperaments

In Western art music, it is likely that composers, performers and audiences would only seriously consider adopting a new system of instruments and tuning (as I have said, not necessarily as a replacement, but as a parallel system), if, in general, it retains important features of 12-ET, and in addition gives significant *improvements.* From the broadest perspective, the key relevant properties of 12-ET are:

- 'unlimited' exact transposition and modulation;
- good approximations to 'just' intervals (12-ET being the nearest 'related' equal-temperament for harmonic spectra);
- manageability.

Systems of just intonation only offer 'unlimited' transposition and modulation when there are very many divisions of the scale. But a very large number of scale degrees is difficult to manage, and seemingly impossible for building new acoustic instruments. The composer Ezra Sims, for example, has attempted to transcend this question by adopting a 72-division system, of which it might be said that it is neither a 'fixed' system of JI, nor is it 72-ET. As Sims describes it, this is a kind of 72-JI that modulates (potentially) through 72-ET:

> I am still prepared for my music to be played in equal-temperament… But I now think in terms of Just ratios, and, where instruments of fixed pitch are not involved, I really expect the older practice of tuning the current key in something like Just, but adjusting the relations between keys to something like equal-temperament in order to avoid going off the instruments.[207]

The difficulties of building a system of acoustic instruments with (for example) 72 deliberate and discrete divisions might be overcome by building some of them for 24 or 36 divisions, players inflecting third-divisions of quarter-tones, or 'semi-1/36ths'. While such a system might provide a remarkable degree of universality, and not prove impossible in performance (witness Sims' music on CD using conventional instruments),[208] it is almost certain that most composers would consider a 72-division system unmanageable or unnecessary.

Let us suppose, for example, that 36-ET was adopted as a standard. To the Just Intonationist, the drawback of adopting 36-ET would be that, on instruments of fixed intonation, the fifth (3/2), fourth (4/3), major and minor thirds (5/4 and 6/5), major and minor seconds (9/8 and 16/15), would be no nearer JI than is 12-ET. But a piano which had 36 keys to the octave could be tuned at different times to JI or other alternative temperaments; a variety of interchangeable bars might be made for various tuned percussion; organ pipes often have movable collars which can be used to tune individual pipes within narrow limits. On the other hand, intonational fine detail is largely determined by the performer on instruments of non-fixed intonation, so a flute or oboe successfully designed for 36-ET would certainly suffice for many forms of extended JI. Is it therefore the case that Just Intonationists and 'Temperamentists' could agree about what new instruments would be 'universally' useful? Or that a reluctance to agree would rest merely on the fact that some instruments of fixed intonation are not easily re-tuned, or on the unwillingness or expense of re-tuning fixed intonation instruments for individual

[206] For example, the intervals of 1, 23, 13, 15, 7 and 9 quarter-tones. Sethares, *Tuning, Timbre, Spectrum, Scale,* p. 92, Figure 5.1.
[207] Ezra Sims, 'Yet another 72-Noter', *Computer Music Journal*, Vol. 12, No. 4, Winter 1988, p. 31.
[208] For example: 'The Microtonal Music of Ezra Sims', CRI CD 643.

concerts? Put like this, from the point of view of a new instrumental system, the divide hardly seems insurmountable.

To the 'Temperamentist', 36-ET would provide an extended system in which 12-ET is preserved as a subset. We might therefore ask: is it crucial that instruments of *fixed* intonation be tuned (theoretically) to equal-temperament, as opposed to some form of 36-division 'Well-Temperament'? Strictly speaking, of course, there is no 'cycle' of 36 adjusted-fifths, but a 36-division 'Unequal-Temperament' may be devised to afford better harmonicity than 12-ET; and while it would not offer perfect transposition, it could equal or better the number of (reasonably strict) transpositional scale degrees available. Another radical alternative would be to incorporate (for example) three semi-independent tuning systems into one 36-division gamut. This could be three interlocking systems of 12-WT, or a hybrid in which one cycle of fifths gives 12-ET, another cycle gives some form of 12-WT, while the remaining 12 pitches form a system of Just Intonation - each system mutually interlocking with and extending the other. Such a system might support a large number of aesthetics.

However, even if an unusual system such as this was adopted as a 'standard' for new instruments, should makers of instruments of *fixed-but-variable* intonation aim to construct them for 36-ET or '36-UT'? From the point of view of flexibility, and performing multiple systems, the answer is probably 36-ET; from the point of view of commitment to a single system, the answer is - as close as possible to the exact system itself. A 36-division system is not being advocated here, but is used only as an example: in terms of unequal systems much of the above argument applies to systems in the region of 29 to 41 divisions. However, if one of the chosen 'multiple' systems is to be *n*-ET, then properties of *n*-ET are critical, as its intrinsic flexibility and its alignment to a maximum number of 'intervallic zones'. Moreover, if timbral adaptations can be made to give smoothness effects emulating just intonation, an equal-tempered system may give the best of both worlds.[209]

APPENDIX III (a) shows the order and degree to which ET systems (from 5 and 53-ET inclusive) approximate the following intervals: 3/2 (4/3), 5/4 (8/5), 7/4 (8/7), 9/8 (16/9), 6/5 (5/3), 7/6 (12/7), 11/8 (16/11), 7/5 (10/7), 10/9 (9/5), 9/7 (14/9) and 16/15 (15/8).[210] This selection of 22 'primary' intervals is somewhat arbitrary, but arguably represents a logical starting point for the expansion of tonal resources. For each of these intervals, APPENDIX III (a) lists the *n*-ET systems in the order in which a scale member most nearly approximates the given ratio.[211]

APPENDIX III (b) shows, for 5- to 41-ET (and to 53-ET), the greatest deviation of any approximation to the 22 intervals considered for each temperament.

APPENDIX III (c) lists *n*-ETs in order of the *summation* of the deviations from *all* the 11 chosen intervals.[212] Since the larger the number of divisions the more likely the approximations will be small, in APPENDIX III (d) the same summations are *multiplied* by *n* (the number of divisions in the relevant *n*-ET), and re-ordered accordingly. The intention is to quantify harmonicity relative to manageability. In this simple derivation the five 'best' systems in the range 5 to 41-ET, are (in order): 41, 31, 22, 19 and 12-ET.

Note that intervallic zones or differing levels of intonational tolerance (of each interval) are not taken into account in APPENDICES III (c) or (d).[213]

[209] If it is far fetched to create deliberately inharmonic acoustic sustaining instruments, it is at least reasonable to suppose *n*-ET systems with good (traditional) consonance characteristics (for example 19, 31 or 34-ET) would be susceptible to the minimal adaptations required. Before the 20th Century, and especially before electroacoustics, instrumental music depended on the fusion of individual tones, and on the fusion of their combination. The *raison d'être* of alternative scales would perhaps be lost without tonal fusion, and it would seem that inharmonicity will only be taken so far for acoustic instruments.

[210] Not dissimilar tables can be found in: Ivor Darreg, 'Table of errors in equally-tempered intervals', *Xenharmonikôn* 10, 1987; and Mark Lindley & Ronald Turner-Smith, *Mathematical Models of Musical Scales*, Verlag für systematische Musikwissenschaft, 1993, Table 17, pp., 116-19.

[211] Here, only eleven of these intervals are considered - rather than twenty-two - since every octave-based *n*-ET system is symmetrical about the tritone. Ideally, if intonational tolerance is taken into account, all intervals and inversions should be considered.

[212] Shown for 5 to 53-ET, and for immediacy, also for 5 to 41-ET.

[213] In a more sophisticated analysis, a system might be penalised if a specific interval does not fall within an intonational tolerance limit. The limits suggested in APPENDIX I (b) are too provisional for it to be useful to do this here. Another refinement is to weight the relative importance of approximating individual intervals. This is controversial because some theorists hold that the more complex the interval the more important it is to approximate it accurately, while others hold that it is more important to approximate simpler intervals,

In this paper I have assumed 41-ET (arbitrarily) to be the largest possible division of the octave for the construction of new orchestral instruments. 41-ET provides, with various other systems, exceptional melodic and harmonic variety.[214] It should be noted, however, that in a large division such as 41-ET, a single 'interval zone' may be approximated by more than one scale degree.[215] For example, in 41-ET, the 13th and 14th scale steps (380.5, 409.8 cents) both fall within the 'zone' we would 'normally' call the major third;[216] their inversions (790.2, 819.5) fall within the zone of the minor sixth. Due to the ambiguity for both performer and listener, it is sometimes difficult to assess whether this feature in a system is a drawback or a benefit. In 41-ET, however, the lower of these two thirds is on the limit of acceptability for an ideal just major third (5/4, 386.3 cents), and the upper of the two closely approximates the Pythagorean major third (81/64, 407.8) - there is therefore an aurally distinguishable difference of character, albeit rather subtle. While the absence of an ideal 'pure' third may not be ideal for 'harmonic' purposes, it does mean that the two intervals can be made distinguishable in composition.

An intonational pitch adjustment comprising the smallest interval of 41-ET (29.3 cents) will not, in various circumstances, change the perceived interval *class* - at least this seems to be the case for most listeners. Nevertheless, a change of interval *character* may be perceived.[217] In either case, however. the interval of 29.3 cents can certainly be used melodically, and much depends on the kind of music and the composer's ability to *make* such distinctions audible and meaningful. 41-ET (and other large division systems) offer a hugely expanded harmonic system, although, as Lindley and Turner-Smith point out, in such a system performers may be:

> unable to project the content of a chromatic composition, because their intonation is not exact enough to do so.[218]

The overriding considerations are, however, whether it is practical to make special instruments, and whether composers and performers will find it manageable - or desirable - to adopt a large division system.

Mathematical and Analytic Approaches

Many mathematical strategies and functions for evaluating alternative systems have been devised. An excellent account of some of these approaches can be found in John Chalmers' book *Divisions of the Tetrachord*.[219] Chalmers considers, amongst others, functions proposed by Euler (*gradus suavitatis*), Tenney (*pitch distance*, *harmonic distance*), Wilson (*complexity function*) and Barlow (*indigestibility function*). For example, the latter starts out by formulating classical intuitions about how to calibrate the relative 'simplicity' of ratios in general, and derives functions of 'harmonicity', aiming towards a general index of consonance against which the goodness of fit of *less* small ratios may be compared.[220] A general discussion of these theories is beyond the

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holding that it is especially important for the tuning of intervals such as the fifth and major third to be optimised. These views might seem to cancel each other out - either way, defining the limits of intonational tolerance is a key issue.

214 J. Murray Barbour provides a useful historical survey of what he calls 'multiple division systems' - ie., systems with more than 12 divisions. He mentions, for example, that Paul von Jankó 'set himself the task of ascertaining the best system between 12 and 53 divisions, and chose the 41-division'. J. Murray Barbour, op. cit., p. 122.

215 Erlich's comparison of ETs in the quoted article is restricted to discussion of systems no larger than 34-ET for this reason: Paul Erlich, op. cit. , p.13.

216 In APPENDIX I (a) it can be seen that both intervals actually fall toward the outer edges of the 'ideal' major third zone, showing the inadequacy of treating the low integer ratio (5/4) as the 'centre' of that zone.

217 For a discussion of 'categorical perception' and the perception of interval class, see: Edward M. Burns & W. Dixon Ward, 'Categorical perception - phenomenon or epiphenomenon: evidence from experiments in the perception of melodic musical intervals', *Journal of the Acoustical Society of America*, 63 (2), Feb. 1978, pp. 456-68; Richard Parncutt & Hans Strasburger, 'Applying psychoacoustics in composition: "harmonic" progressions of "nonharmonic" sonorities', Perspectives of New Music, Vol. 32, No. 2, Summer 1994, pp. 88-129; John A. Sloboda, *The Musical Mind : The cognitive psychology of music*, Oxford, pp. 23-8.

218 Mark Lindley & Ronald Turner-Smith, *Mathematical Models of Musical Scales*, Verlag für systematische Musikwissenschaft, 1993, p. 31.

219 John Chalmers, *Divisions of the Tetrachord*, Chapter 5, Frog Peak Music, 1993. This intensely detailed theoretical syllabus of ATS is more disinterested than any exposition I could hope to give. The work is an astonishing antidote to intellectual complacency regarding ATS, in its thoroughness, and in its ability to inspire wonder not only at the number of alternatives before us, but at the strange modernity which moves so many composers, including myself, to be drawn toward equal-tempered systems.

220 Clarence Barlow, 'Two essays on theory', *Computer Music Journal*, 11 (1), 1987, pp. 44-55.

limits of this paper, but one particularly useful approach is outlined below - the analysis of equal-temperaments in terms of 'consistency'.

Consistency

The comparison of equal-tempered systems in terms of 'the goodness of fit to just ratios' shows the accuracy of each *individual* interval of the scale relative to an 'ideal' tuning or ratio. However, this approach does not address whether, when three or more degrees of the scale are sounded together (or played consecutively), the *combinations* of smaller intervals are 'consistent' with the larger intervals formed thereby. The theory of 'consistency' was first developed by Paul Erlich, and both Paul Hahn and Manuel Op de Coul have also contributed to elaborating it.[221]

Erlich based his analysis, following Partch and others, on the idea that the history of Western music has been characterised by a gradual acceptance of higher consonance *limits*. In short,

> Western modal and tonal music generally conforms to what is known as the 5-Limit… ratios involving numbers no higher than 5, aside from factors of 2 arising from inversion and extension, are treated as consonances in this music, and thus need to be tuned with some degree of accuracy… Medieval music conformed to the 3-Limit, the only recognised consonance being the octave (2:1) and the perfect fifth (3:2)… By the end of the 15th Century, the 5-Limit had eclipsed the 3-Limit as the standard of consonance and two new consonant intervals were recognised: the major 3rd (5:4) and the minor 3rd (6/5).[222]

Obviously, other intervallic ratios occur in these musics, but according to this theory, intervals which are treated as consonances correspond to intervals found within a given *m*-limit. In Erlich's approach to consistency, only intervals within the (primary) *m*-Limit are relevant - meaning only intervals formed by ratios containing numbers equal or less than *m* or their octave equivalents.[223]

In the simple goodness-of-fit-to-just-ratios-analysis of equal-temperaments the relationship of each individual scale degree is considered relative to 1/1. But if a scale is 'inconsistent' at a certain limit, then it is not possible (in fixed intonation) to take advantage of the implied 'nearest' scale degree when forming consonant chords (or particular melodic figures) - that is, within the specified *m*-limit. For example, 'goodness-of-fit' shows that 24-ET has a better approximation of the harmonic seventh than 12-ET. However, suppose that we want to form a root position 'just intonation dominant seventh' chord, approximating 7-Limit just intonation, in 24-ET.[224] The just chord is described by the ratios '4:5:6:7' - that is, the notes 1/1, 5/4, 6/5 and 7/4. There are thus *six* ratios implicit in this chord: 5/4, 6/4 (3/2), 7/4, 6/5, 7/5 and 7/6. In 24-ET the closest approximation to 5/4 (major third, 386 cents) is 400 cents (8 quarter-tones), and the closest approximation to 7/5 (septimal augmented fourth, 583 cents) is 600 cents (12 quarter-tones). If the two *just* intervals are added the result is equal to 7/4 (harmonic seventh, 969 cents). But the closest approximation to 7/4 in 24-ET is 950 cents (19 quarter-tones), and $8 + 12 \neq 19$. And although the same chord (C E G Bb) can be formed by exactly the same intervals in 24-ET as in 12-ET, there is no *single* optimal approximation of the just dominant seventh chord in 24-ET, since, whichever way the chord is tuned, 7/4 and 7/5 cannot both be 'optimally approximated' at the same time within the temperament. Therefore, 24-ET is 'inconsistent' at the (primary) 7-Limit.

In 12-ET the nearest realisation of this chord is of course C E G Bb (when 1/1 is C). And it can be shown that *all* of the six ratios implicit in this chord are given the best possible approximation in 12-ET by C E G Bb (or 4, 3 and 3 semitones). So although the chord can in fact be tuned better in 24-ET than in 12-ET, 12-ET has the virtue of being consistent, whereas 24-ET does not. In fact, it can be shown that chord formation in 12-ET is

[221] A more formal explanation of consistency and some related ideas is given in APPENDIX IV.
[222] Paul Erlich, 'Tuning, tonality and twenty-two-tone temperament', *Xenharmonikôn* 17, Spring 1998, pp. 12-40.
[223] These terms are explained more fully below, but see Appendix IV for a more formal definition of *primary m*-limit, *secondary m*-limit, level -l, level -2, and so on.
[224] It is by no means obvious that the 4:5:6:7 (JI) version of the dominant seventh chord is necessarily its ideal realisation. As I understand it, the value of the theory of consistency does not rest on this premise, but rather on a more general notion of the effectiveness of intervals and interval combinations, for which the JI approach provides the simplest available working hypothesis. Some extensions to this approach to 'consistency' are mentioned in APPENDIX IV.

actually consistent to the primary *9*-Limit, whereas 24-ET is consistent only to the 5-limit. Erlich has provided the following table, which shows consistency for equal-temperaments between 2 and 41-ET:[225]

***n*-ET**	**Consistent to *m*-Limit**	***n*-ET**	**Consistent to *m*-Limit**	***n*-ET**	**Consistent to *m*-Limit**	***n*-ET**	**Consistent to *m*-Limit**
2	*3*	12	*9*	22	*11*	32	*3*
3	*5*	13	*3*	23	*5*	33	*3*
4	*7*	14	*3*	24	*5*	34	*5*
5	*9*	15	*7*	25	*5*	35	*7*
6	*7*	16	*7*	26	*13*	36	*7*
7	*5*	17	*3*	27	*9*	37	*7*
8	*5*	18	*7*	28	*5*	38	*5*
9	*7*	19	*9*	29	*15*	39	*5*
10	*7*	20	*3*	30	*5*	40	*3*
11	*3*	21	*3*	31	*11*	41	*15*

Figure 7 : Level-1 Consistency for equal temperaments between 2 and 41-ET as provided by Paul Erlich.

On page 38 above, 24 and 27-ET were briefly compared, and it was unclear which system was likely to be the most productive basis for composition. The consistency function perhaps suggests that since 27-ET is consistent to the 9-Limit it may be more productive than 24-ET, which is consistent only to the 5-Limit. Further, if musicians will only consider adopting a new tuning system if it gives improvements over 12-ET, then, are *n*-ET systems which are consistent to the 9-Limit (or greater) the only ones worthy of consideration?

A further aspect of consistency is what has been called consistency '*level*', an idea developed particularly by Paul Hahn. Consistency relative to the *primary m-limit* is known as level 1 consistency (or just plain 'consistency'): if each of *a*, *b* and *c* represent primary *m*-limit intervals, and ~*a*, ~*b* and ~*c* are their nearest approximations in *n*-ET, then *n*-ET is *m*-Level consistent at level 1 if for all (*a* + *b*) = *c* then (~*a* + ~*b*) = ~*c*. Similarly, if each of *a*, *b* and *c* represent secondary *m*-limit intervals, and ~*a*, ~*b* and ~*c* are their nearest approximations in *n*-ET, then *n*-ET is *m*-Level consistent at level 2 if for all (*a* + *b*) = *c* then (~*a* + ~*b*) = ~*c*. This is explained in more detail in APPENDIX IV, where Table I shows that 12-ET is not only level-1 consistent at the 9-Limit, but also level-25 consistent at the 3-limit, and level-3 consistent at the 5-limit. On the one hand, being 'level-25 consistent in the 3-limit' simply means that the approximation of the 5th in 12-ET is very close - but it also implies the 'consistency' of *combinations* of 5ths in 12-ET, as occurs both melodically and in more complex harmonic progressions. On the other hand, if the fifth is not well approximated in an equal temperament, then realisations of familiar scales occurring in that system will not correspond to a cycle of fifths (or will not exist).

Level 1 consistency essentially measures the relative coherence of chords formed within the *m*-limit, and the coherence of melodic figures comprising consonant ratios. Levels 2 and above indicate the extent to which chord progressions or somewhat more complex combinations of intervals remain consistent within a temperament, in terms of the coherence of the individual steps which comprise them. The theory of consistency (especially level 2 and above) is of most practical relevance to performance on instruments of fixed intonation. The *combined* criteria of just ratio approximations and consistency ratings are probably important indicators (along with others) for choosing an ET which will provide a quasi-universal system. Again, the value of collaborative researches between the mathematical, acoustic and instrumental aspects of the subject is a further reason for establishing a collaborative network and Centre(s).

Provisional Conclusions

The compositional decision process which was described above - of choosing one ATS rather than another on the basis of what a work itself implies - has suggested many of the themes introduced in this section. Each of them may be elaborated substantially. Although I have strong personal preferences for certain ATS, it would be

[225] I am grateful to Paul Erlich for providing this table together with considerable help and advice in preparing this section of the paper.

preferable not to express these here. Nevertheless, it seems important to present in very general terms my own personal and provisional conclusions about a preferred tuning system for new acoustic instruments. The following remarks assume that it is probably not feasible to create a range of new instruments which each provide (optimally and individually) for a number of tunings, and that a decision regarding a unique system is necessary. As with everything else in this paper, these comments are merely offered up for debate.

- I would favour an *Equal-Tempered* system for *instrument design* - some of the reasons are given above.

- Assuming existing instrumental timbres, I would favour a system which divides the octave into some system between 27 and 41 (inclusive) divisions to the octave but excluding 28, 30, 35-ET due to the sizes of the fifth. Personally, I enjoy and have used 19-ET and 24-ET especially, and various other systems with a smaller number of divisions than 27, but to my mind they are too restrictive to be 'universally' useful. This is especially the case for the smaller division systems because with these it is difficult to compose effective music which is fast moving, particularly if it is chromatic. For slower music the smaller division systems have intrinsically 'different' harmonic and melodic soundworld, which is valuable, but these interesting 'awkwardnesses' can be explored in the larger division systems anyway. Systems *larger* than 27- to 41-divisions, however, are difficult to manage (the systems in the suggested range may already seem problematic, yet I think this is sometimes exaggerated). In my opinion, larger divisions (above 41) give diminishing returns relative to manageability.

- Greater divisions of the octave (than 41) are likely to be impractical for specially built new acoustic instruments (41 seems difficult enough!) especially if a complete and integrated system of instruments of fixed and non-fixed intonation is to be considered.

- Woodwind and brass will be disadvantaged for systems which give less good approximations of at least the 3^{rd}, 5^{th} and 7^{th} harmonics, that is, unless the modes of overblowing of every instrument of a new instrumental system can be adapted appropriately. Otherwise, for example, although it is a valuable temperament in itself, since 29-ET has no good major third it could not be a prime candidate for a 'universal system'.

- I am undecided about the value of retaining 12-ET as a 'subset' (the only possibility in the range specified being 36-ET). Two reasons are sometimes given for thinking that this is important: (i) to continue to perform in 12-ET on new instruments, and (ii) to encourage the integration of old and new musical languages. These are powerful arguments. However, if, for example, 31, 34 or 41-ET proved exceptionally compelling to a wide range of composers, then it would be strange to prefer 36-ET for the above reasons, since works in 12-ET can be performed on existing instruments anyway, and it is conceivable that performers might double on existing and new instruments in an individual work.

- 31, 34, 36, 38, and 41-ET all give exceptional overall harmonicity, and would each allow for exploratory systems of tonality *and* atonality. However, from my personal experience of using simulations for composition in these systems, harmonicity *in itself* does not imply an *overriding* preference for these over other systems in the suggested range. In addition, 27 to 41-ET all provide adequate opportunity for effective dissonance.

- It is highly important to choose a provisional ATS standard in which the contemporary bifurcation of music into tonal *and* atonal (which is available in 12-ET) is not stunted: it is *not* the case that the majority of ATS offer this an effective possibility. 19-ET, for example, has such strong harmonic implications that (to my ear at least) it is difficult to compose effective *atonal* music using sustaining instrument (very harmonic) timbres. At the same time, we need to be open to previously unimagined musics, so far as this is possible.

- Each equal-tempered ATS has an individual ‘xenharmonic’ character of its own. This is pronounced for smaller division systems but gradually becomes less so as the number of divisions becomes greater.[226] For the purposes of choosing a single system which will be useful to a great many composers, it is not necessarily an advantage that a system has a strong ‘in-built character’. Systems in the range 27 to 41 seem to me a practical compromise in this respect - systems with greater divisions being preferable. The question of the individual melodic and harmonic character of each system is important, but systems with greater divisions are generally more versatile.

- If it proved possible to systematically adapt the spectra of acoustic instruments so they ‘relate’ to an alternative scale, then the drawbacks of smaller division equal-tempered systems would be lessened. Possibly, the least radical instrumental adaptations would be necessary for 24 and 36-ET, and perhaps systems such as 19 and 22-ET.

[226] Obviously this is most true for music which uses the complete gamut of the scale.

5. Preliminary thoughts regarding new instruments

A brief survey of some mechanisms enabling conventional instruments to be built or adapted for ATS is given below. We have already identified four related areas of investigation:

- instruments with conventional timbres which are modified for new scales;
- instruments in which tuning and timbre are deliberately 'related';
- the analysis of conventional instrumental timbre to gauge their effectiveness for certain ATS;
- electronic simulation as a speculative tool for the instrument builder.

The bulk of what follows concerns the first of these, but may also be relevant to the second. The idea of '*An Inharmonic Instrumentarium?*' is also discussed below. The analysis of instrumental timbre is omitted; some relevant applications of electronics are given in Section 6.

General principles

We have seen that systems such as 31-, 34-, 36-, or 41-ET, or other many-division just or well-tempered systems, may provide the variety of harmonicity that could be productive in new music, and satisfy a variety of aesthetics. So it would help us to know how great a number of divisions of the octave could be practical, effective and affordable, for every orchestral instrument. We also want to know, if possible, whether the nature of the instruments themselves implies something about which ATS would suit them best, individually and in consort. The latter question is relevant both in terms of acoustic properties (such as how overblowing, or how the shape of the bell, or other factors affect intonation etc.), and in terms of the 'correspondence' between instrumental timbres and particular tuning systems.

The design of new instruments for ATS will surely strive for familiar objectives, maximising:

- quality and uniformity of tone;
- dynamic potential and responsiveness;
- ease of performance;
- the available 'chromatic' notes of the scale;
- accuracy of pitch, in respect of the intended tuning system;
- facility for reference tones (minimising dependence on subjectivity for non-fixed intonation instruments);
- control (as far as is practical and desired) of relations of tuning system and timbre, for individual instruments and for interrelations between instrumental sonorities.

These ideals present conflicting demands. Anthony Pay, writing about the conventional clarinet, suggests we should not be

> too demanding of the instrument in terms of its intonation. An instrument with a good sound is more flexible with regard to intonation anyway - we may need to bend notes, but they bend more easily and the sound remains acceptable. [227]

Problems of tuning, and the balance between quality of sound and accuracy of intonation, involve many variables. For the clarinet, the modal ratios (the pitch relationships between registers) are related to the diameter and configuration of the bore, and the size and placement of tone-holes and speaker-hole(s). If there is only one speaker-hole (there are a number on *Oehler* and *Marchi* systems), it must not only effect a compromise (intonationally) for overblown notes but also serves as a tone-hole in its own right. The fraising of tone-holes and other perturbations of the bore, the shape of the bell, and the density, weight and thickness of the barrel, each have an effect. But each of these factors also contributes to the overall quality of timbre, and it is axiomatic that tone quality in new instruments is not compromised. To make a fine instrument which accurately

[227] Anthony Pay, 'The mechanics of playing the Clarinet', in *The Cambridge Companion to the Clarinet*, Cambridge University Press, 1997, p. 121.

provides a much larger number of notes per octave, the precise balance of these factors may need radical and/or very sensitive modification.

Woodwinds and brass are probably the greatest challenge to the instrument designer interested in ATS. However, as suggested below, some alternative tuning systems may coincidentally bring improvements of tone and/or technique to particular instruments; and improvements for instruments in 12-ET, perhaps using electronic interfaces, could be a by-product of developing new versions of them.

Leaving aside (for the moment) the possibility of 'relating' tuning and timbre, the idea that the acoustic properties of an instrument will influence the effectiveness of certain ATS can be illustrated with the following example. In his book *Clarinet Acoustics*, O. Lee Gibson suggests that

> The central, most important principle for the design of wind instruments is the inverse relationship between the size of the bore and the size of its first-mode to overblown-mode ratios.[228]

More specifically,

> If one considers clarinets of different bore diameters which have similar placements of tone holes, similar fraising, and similar departures from a cylinder, one can quite accurately predict the size of at least one twelfth which will be encountered on each instrument, B^b-F… the sizes of each of the remaining twelfths may vary considerably with different placements and sizes of the speaker vent.[229]

Comparing various models, Gibson shows that the larger the size of the bore, the narrower the result (for B^b-F).[230] He also claims that the volume of the central third of the bore has a determining relationship to the frequency ratio of the instrument's first and second modes,[231] and that the diameter of the bore also plays an important part in determining the spectral structure (timbral quality) and responsiveness of the instrument.

This suggests that the relationship between certain basic dimensions of an instrument and its intended tuning system may have a special relevance. To build an effective 17-ET or 19-ET clarinet (for example), the (average) twelfth would ideally be (approximately) 4 cents wider or 7 cents narrower, respectively, than just. In this case, the relationship between the preferred tone and responsiveness of a clarinet with a certain bore diameter and configuration might be matched to a particular tuning system. If a smaller bore clarinet is generally thought to be more responsive and effective (as Gibson suggests), then 17-ET might be preferred for its wider fifth; or similarly, 29-ET preferred to 31-ET. Perhaps the variability of such results, between twelfths or between instruments, would negate this conclusion. Yet, as we have seen from Sethares' work, certain timbres privilege certain tuning systems.[232] Perhaps these factors, considered in combination, would provide criteria for surmising the effectiveness of a tuning system for a given instrument.

This is a small example, which arises because the clarinet, exceptionally, overblows at the octave and a fifth. But it would also imply, for example, that in an ATS which has no good approximation of the fifth there may be limits to the usefulness of the clarinet.[233] Similarly, in the topmost range of the flute, for example, most of the scale is based on the 4th harmonic, but D5 is conventionally taken as the 3rd harmonic (of G4), and A6 to C7

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228 O. Lee Gibson, *Clarinet Acoustics*, Indiana University Press, 1994, p. 29.
229 Gibson, ibid., p. 33.
230 Gibson, ibid., p. 34. I am unable to work out whether Gibson's citations of divergent 12ths are + or - '*x*' cents from the just or equal-tempered interval.
231 Gibson, ibid., p. 35.
232 As Kameoka and Kuriyagawa noted, the odd-harmonic structure of the lower range of the clarinet spectrum is again significant in this respect.
233 A point about which I am unclear is whether the 'octave-enlargement phenomenon' discussed by Terhardt and others has implications for the overblowing of 'ideal' woodwinds and brass. See Ernst Terhardt, 'Pitch shifts of harmonics, an explanation of the octave enlargement phenomenon', Proceedings of the 7th ICA, Budapest, 3, pp. 621-4; J.E.F. Sundberg & J. Lindqvist, 'Musical Octaves and Pitch', *Journal of the Acoustical Society of America*, Vol. 54, No. 4, 1973, pp. 922-9; William Morris Hartmann, 'On the origin of the enlarged melodic octave', *Journal of the Acoustical Society of America*, 93 (6), June 1993, pp. 3400-9.

may be taken as the 5th harmonic (of F4 to Ab4).[234] Of course, a variety of harmonics are employed by flautists for ease of fingering or special timbral results.[235] In general, agreement between scale points and harmonics 2, 3, 5 (and above) is clearly advantageous. Composers might take exception to the idea that a preference for one ATS over another might be based on acoustic instrumental contingencies. But the idea here is simply to suggest that, together with other examples, there may be considerable benefits to be gained from research into 'which ATS are favoured by which instruments', and, considered together, by a system of instruments. While it might take a life's work to give empirical support to this claim, it would appear that the 'effectiveness' of *a combined system of tuning and instruments* (which is largely if not entirely evidenced by 12-ET) depends on:

- coincidental agreement between the inherent acoustical properties of different instruments; and
- the coincidence of those properties with the attributes of a given tuning system.

The development of computer modelling software for instrument design (for example, for woodwind tone-holes),[236] Sethares' work on timbre, and research focusing on physical modelling technologies[237] also support the idea of bringing together work from each of these areas.

234 Murray Campbell & Clive Greated, *The Musician's Guide to Acoustics*, Dent, 1987, pp. 283-4

235 See, for example, the table of natural harmonics in: Robert Dick, *The Other Flute - A Performance Manual of Contemporary Techniques*, Multiple Breath Music Company, 1989, pp. 10-11.

236 'Resonans', for example, a software package developed at IRCAM, 'allows, among other things, the calculation of the resonance frequency of any acoustic duct from its geometry. The apparatus is generally composed of several tubes, open or closed, correctly placed on the instrument'. *Systèmes Micro-Intervalles* brochure. See also: P. Bolton, 'Resonans', *Fellowship of Makers of Historical Instruments Quarterly*, April 1995.

237 As one example amongst many, see the latest publication list for the *Center for Computer Research in Music and Acoustics* (CCRMA) at Stanford University (http://www-ccrma.stanford.edu).

Woodwind

A variety of design approaches for fully 'chromatic' and intonationally accurate conventional woodwinds in ATS are described below. These include:

- 'Logical woodwind' - software configurable electronic keywork;
- adaptations of tone-hole configuration and keywork;
- multiple bore woodwind;
- valve-controlled length corrections using 'branched tubes';
- alternative tone-hole closure mechanisms;
- single or multiple slide (or 'telescopic') mechanisms;
- 'Logical feedback' woodwind: adaptive, fixed & multiple ATS; the 'networked orchestra';
- alternative woodwind technologies in combination.

'Logical Woodwind' - software configurable electronic keywork

In 1967 Professor Giles Brindley built what he called the 'Logical Bassoon'. The brilliant ideas behind this design have not been developed commercially because, although the new instrument is apparently easier to play than the conventional bassoon, the challenges the latter poses can be overcome anyway, and because the market is in any case conservative. But the advantages of logical keywork for serious contemporary music, particularly for ATS, are almost overwhelming.

The Logical Bassoon is a fully acoustic instrument, its prime innovation being that the mechanical keywork which acts as the connecting mechanism between fingers and tone-hole pads is replaced by an electrical circuit board. Hoping to improve the conventional bassoon, Brindley aimed

> to preserve the timbre of the best notes, improve [the timbre] of inferior notes, and make the player's task easier.[238]

The result is

> an instrument in which each octave could be fingered in the same way, with the addition of the relevant octave key… The fingering thus remains constant, apart from the use of the octave keys, and the logic circuits determine which holes should be opened or closed. The instrument…has a sound which is indistinguishable from a normal German instrument…[239]

Brindley adapted terminology from propositional calculus to describe the logical relationships between the positions of keys and tone-holes in terms of 'closing formulae'.[240] A closing formula is simple or complex, depending on the number of logical operators that are needed to describe it. For practical reasons, mechanical keywork is limited in the extent to which complex combinations can be realised; a logic circuit is unlimited. Describing the development of conventional keywork, Brindley describes how

> No mechanical device making the state of a hole depend on the states of more than three finger-plates has achieved lasting popularity. Some have been tried, but players… have returned to older and simpler mechanisms.[241]

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[238] Giles Brindley, 'The Logical Bassoon', *The Galpin Society Journal*, XXI, 1968, p. 152. In a personal communication, Brindley has explained that four Logical Bassoons were completed (of which versions one and four survive), and a highly successful Logical Contra Bassoon. Brindley also began to build a Logical Bass Clarinet for Alan Hacker, but unfortunately this was abandoned - not, apparently, due to any intrinsic impossibility of design. Professor Brindley also mentioned that he continues to play one of these logical bassoons every week in an amateur orchestra in London.

[239] Jeremy Montagu, *The World of Romantic and Modern Musical Instruments*, David and Charles, London, 1981, pp. 70-3. Close examination of Brindley's fingering charts show it is not strictly true that, in the case of the Logical Bassoon, the fingering is the same for each octave. However, it is certainly simpler than that of conventional bassoons; and, in principle, further work would almost certainly be able to achieve this objective.

[240] G.S. Brindley, 'Logical woodwind instruments', *Proceedings of the Royal Institution of Great Britain*, Vol. 48, 1975, pp. 207-9.

[241] Brindley, ibid., p. 208.

On the other hand, the logic circuit is therefore

> practically unlimited in the complexity of logical operations that it can perform… The introduction of the logic circuit frees both the fingering and the 'holing'… from all mechanical constraints.[242]

Following principles similar to those of Brindley, in 1973 Edwin Norbeck built a 'Computer-Assisted Flute' - again, an acoustic instrument.[243] In this case the relation between finger-keys and solenoids (the magnetic devices which actually open or close the tone-holes) was controlled by a 'general purpose computer'[244] rather than a special circuit board. An experimental fingering system was used, demonstrating the virtual relation between fingering and tone-hole closure, as well the educational potential of the device:

> The seven notes of the octave were selected by the first three fingers of the right hand. The thumbs were only used to support the instrument. There was a sharp key and a flat key, both operated by the little finger of the right hand. These were only meant to be used for accidentals. The sharps or flats in the key signature were automatically generated by the computer… two other keys which were operated by the left hand designated the octave. [245]

In 1992, exploring the potential of electronic keywork for acoustic woodwind, Howard McGill built a clarinet, in which tone-hole closure is also controlled by logic circuits and solenoids. Again, this is a fully acoustic instrument, not a MIDI wind controller, although McGill has suggested he might in future be attracted to creating a hybrid of both. The instrument has a standard clarinet body, from which the conventional keywork has been removed. In this case, for simplicity, the instrument is not chromatic, and some tone-holes have been filled in, giving scales of Bb and C major only, overblowing to F and G. Eight button-like keys, seven on the top of the instrument and one below, control the opening and closing of key-pads via a computer circuit board also built by McGill.

According to both Brindley and McGill there is little or no loss of 'feel' when performing on a 'logical' woodwind. Where tone-holes are fingered directly there may be a loss of tactile control, and of the ability to 'half-hole' a note. But McGill has said that for covered holes there is virtually no difference between pressing a key-lever and pressing an electronic button - claiming that, if anything, the electronic system is cleaner and feels more 'direct'. In any case, contact between the lips and the reed is more significant to the 'feel' of the instrument. It is also fair to say that each of these experiments is probably somewhat primitive, considering what might be achieved by a team of experienced instrument designers, and computer and electronics experts.

Brindley, Norbeck and McGill have all emphasised the value of logical woodwind in terms of improved tone and intonation, faster action and easier fingering, and in the potential for standardisation - using the same fingering system (or systems) for all woodwind. Describing the potential advantages of electronic keywork, McGill writes:

> Each instrument could be taken to the limit of acoustic perfection now that the mechanical restrictions have been lifted. It would be possible to place a different octave key at the precise pressure node location for all 12 notes of the scale. Various fingerings [encoded in software, could] provide perfect ventings for each note and also standardise harmonic and multiphonic fingerings, which up to now have involved rather contrived finger positions.[246]

Brindley made the same points, and the first version of the logical bassoon was built with eleven speaker holes instead of the usual three (it also has 17 tone-holes). However, Brindley has since said that in practice the

[242] Brindley, ibid., p. 209.
[243] Edwin Norbeck, 'Computer-assisted woodwinds', *Woodwind World*, December 1973, pp. 8-9 & 34.
[244] In 1973 - a computer costing $50,000, Norbeck, ibid., p. 9.
[245] Norbeck, ibid., p. 9.
[246] Howard McGill, *Intelligent Woodwind*, unpublished dissertation, 1992, Cambridge University, p. 13.

degree of improvement which the extra speaker-holes affords, at least on the Logical Bassoon, is not especially significant, and on later versions the number of speaker keys was reduced.[247]

More significantly, the possibility of being able to configure at will the relationship between fingering and holing opens up new conceptions of woodwind. In a modern, 'high-tech' logical woodwind, it is conceivable that any number of fingering/holing configurations could be saved in memory, and chosen at will by the performer via an 'on-board' controller. The immediate technical consequences of this approach would be:

- easier fingering for traditional repertoire;[248] and dynamic fingering configuration for *any* repertoire;
- every possible combination of open and closed holes is fingerable;
- any holing combination is configurable to any key press or combination of presses;
- any desired sequence of holings, including multiphonics etc., is playable;
- the key system is potentially independent of the bore - that is, if desired, keys no longer need to be placed on the body of the instrument.

The immense advantage of logical woodwind for ATS is the possibility of controlling a large number of tone-holes, with relative ease, and with definable, dynamically programmable, logic. Considered together these features suggest many possibilities:

- fully 'chromatic' woodwinds for *n*-ET, where *n* could be relatively large;
- conceivably, 'chromatically transposable' woodwinds for radical *n*-<u>WT</u> or *n*-JI (this might perhaps be facilitated by the use of extremely small tone-holes very high on the bore);[249]
- increased consistency of tone and production;
- logical organisation of fingering patterns to facilitate the 'aural geography' of large *n*-ET etc.;
- conceivably, the ability to switch between a number of conventional, historic or radical tuning systems, each achieved more or less perfectly, *on a single instrument*, using software presets for key assignment and control;
- consistent fingerings from one octave (or other dividend) to the next, for any instrument or tuning system;
- for clarinet-type instruments - fingerings repeating at the octave rather than the 12th;
- simplified (user-definable, dynamically definable) fingering systems;
- standardised fingering systems across all woodwinds (global and personalised);[250]
- all 'logically' available multiphonics, or sequential combinations thereof;
- multiphonics in which individual components may be deliberately tuned - particularly if small tone-holes high on the bore are used;
- software control of user-defined (or conceivably, 'adaptive') tunings;
- extended gestural repertoire using variable (position or pressure sensitive) controllers - e.g. trills and glissandi;
- integration of acoustic and MIDI techniques - e.g. transposition, and sequencer playback (or what McGill calls 'tutoring').
- since finger movements are not directly connected with the tone-hole closure mechanism, it might also be conceivable to implement an internal electronic pitch sensor which will feed back (via software) to corrective venting for any given tuning system in real-time. (This idea is developed in more detail in a separate section (pp. 73 - 74).

Each of these possibilities suggests the value of further research.

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[247] In conversation with the author, October 1997.
[248] Brindley provides a fingering chart for the 'second' Logical Bassoon, showing its relative simplicity. G.S. Brindley, 'Logical woodwind instruments', *Proceedings of the Royal Institution of Great Britain*, Vol. 48, 1975, p. 210.
[249] These very small tone-holes were suggested by Lewis Jones in a private communication.
[250] 'Cross-instrument' fingerings are not unknown on conventional instruments - for example, oboes have been created with saxophone type fingerings (by Loree, Boosey and Hawkes, and others).

Brindley has pointed out that, for instruments with many tone-holes, logical woodwind design must overcome the drawback that

> electromagnets are heavy. The excess in weight over conventional keywork is negligible for a double bassoon and almost negligible for a bassoon or bass clarinet, but serious for a flute or oboe and crippling for a piccolo.[251]

McGill's clarinet is not too heavy, but it is not a chromatic instrument. However, the solenoids used by McGill, for example, were industrial units bought off the shelf. Considering the light force which is required to operate the pads, a custom designed solenoid could almost certainly be made which is lighter and at least as effective;[252] weight savings might also be made in other areas of the instrument. On McGill's clarinet each solenoid sits outside the bore and drives a pad on the end of a piston to and from the tone-hole. Since little pressure is required to hold the pad against the tone-hole, weight might also be saved, for example, by placing the pad on a pivoted lever, using mechanical advantage to reduce either the movement or power required of the solenoid. Alternatively, electromagnetic coils or rare earth magnets might be embedded in the surround of the tone-hole itself, to attract and repel the pads.

Similarly, a rotating shutter or 'iris' diaphragm might be driven by the solenoid, and used to vary closure by different degrees. Advantages might be gained if the shutter were to close with a rotary or sliding action parallel to the bore (extremely fast to eliminate momentary glissando between one note and another). Firstly, this might enable the internal contour of the bore to be made more perfect where the tone-holes are normally placed, by placing the shutters flush with the inner diameter of the bore and minimising the 'chimney' at each tone-hole;[253] secondly, tone-hole closure could be carefully stepped so that the shutter could be used for half (or third)-holing. Another conceivable technique for tone-hole control might be to use piezo-electric plastic, which would be very much lighter and require less electrical energy than solenoids.[254]

The obstacles posed by electronic reliability and responsivity are not serious. The problem of refining pad control, for example, so that unwanted notes do not sound when playing large intervals, was solved by both Brindley and McGill independently. MIDI woodwind controllers have been on the market for over ten years, from which existing features and components - circuitry, interface etc. - might be adapted. It is also likely that the cost of labour and materials to build logical woodwind would be less than for traditional woodwind. Brindley suggested as much in 1975:

> Though mechanical devices to perform complex logical operations are difficult to make, electrical devices which do the same things are cheap and reliable.[255]

Since 1975 such devices have become much simpler to make, and the cost ratio has improved dramatically in favour of logical woodwind. The electronic gadgetry is now cheap, and as Norbeck pointed out, could be pretty much standardised for every woodwind instrument.[256] While logical woodwind are unlikely to be commercially successful in the short term, they are an exciting area for research, and potentially a serious alternative for instrumentalists wanting the best for acoustic music of any type.[257] Their potential for contemporary music in general - and ATS in particular - is compelling.

Adaptations of tone-hole configuration and keywork

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[251] Brindley, ibid., p. 212. In addition to the above references, those interested in Brindley's work might also consult: Leonard Salzedo & Peter E.M. Sharp, 'The logical bassoon', *The Times Educational Supplement,* Friday, 7th February, 1969. My thanks go to William Waterhouse for allowing me access to his archive of material on the Logical Bassoon.

[252] My thanks to Tim Knights for advice and technical information on recent solenoid design.

[253] My thanks to Jerry Vorhees for some useful comments on this topic.

[254] This was suggested by Bill Sethares in a private communication.

[255] Brindley, op. cit., p. 208.

[256] Norbeck, op. cit., p. 34.

[257] It is worth mentioning the composer/sound sculptor Trimpin who has built a 'rational' 24-ET bass clarinet in which solenoid pads are driven by MIDI - as well as many other automatic instruments. Another composer whose work may be relevant here is Matt Heckert, but I have as yet been unable to find out much about it.

Leaving aside electronic mechanisms, conventional keywork can also be successfully adapted for woodwind with greater than twelve divisions to the octave. The *Osten-Brannen Kingma* C flute, for example, is designed for quarter-tones. However, to my knowledge this is the only commercially available *non*-12-ET professional concert woodwind.

> In addition to the standard Böhm mechanism, there are six extra keys; these are used to produce six of the seven quarter-tones and multiphonic vents which are 'missing' on the normal French model (open hole) flute. The seventh 'missing' quarter-tone is achieved by using the C# trill key together with the normal C key.[258]

A fingering chart for 48-ET for this flute has been compiled by the composer and flautist Anne La Berge.

Oskar Kroll mentions two examples of quarter-tone clarinets in his survey of that instrument. The earliest of these was designed by Dr. Richard H. Stein - either in 1906 (according to Alois Haba), or 1911 (built in 1912, according to Kroll).[259] In any case the instrument was built by *Kohlert*. This is a conventional instrument which 'differed from the normal Bb Clarinet only in having additional quarter-tone keys'.[260] Unfortunately Kroll describes neither its precise mechanism, the number or position of extra tone-holes, nor how effective the instrument was. Two other quarter-tone clarinets are mentioned in *Groves Dictionary of Musical Instruments,* both commissioned by Hába. Again, these were built by *Kohlert*, in 1924 and 1931, but as yet I have no further details of them.[261]

An instrument which is extraordinarily suggestive of the power of mechanical keywork is the 'One Hand' saxophone, which was produced by *Conn*. The instrument had the 'seven pearls and all the other keys on the lower half of the body, so that the entire range could be played with the right hand only'.[262] Again, I have no details of this mechanism, nor of its effectiveness.

Traditional mechanical keywork mechanisms are complex and highly refined. Typically, for example, professional flautists prefer the open-hole adaptation of the Böhm flute, which is thought to provide the best tone, and helps players to finger quite easily some unorthodox tunings for various degrees of the scale. To design a woodwind specifically for a large number of octave divisions the most obvious obstacle is the number of keys that can be fingered and the mechanical complexity of providing for very many combinations of tone-hole closure. This is particularly the case for the clarinet, which already has a total of between 23 to 29 tone-holes[263] (depending on make and model) to obtain chromatic 12-ET, requiring the use of 12 keys for each hand.[264] If the number of tone-holes is increased to provide a much finer division of the octave, the mechanism must avoid becoming unworkably complex or heavy. Another question-mark is how to avoid cumulative pad leakage from a very large number of tone-holes, although if the pads are satisfactory and other materials stable this may not be a problem.

It is only possible to place a certain number of tone-holes on the body of an instrument, depending on its length, the diameter of the tone-holes, and their possible configuration - that is, whether the tone-holes are in line (or lines), or are distributed at various angles, or in some kind of spiral. If we set aside questions of how the keywork will sit on the body, and how ten human digits can contend when *n* is large, we might consider the

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258 Osten-Brannen Kingma System Flute brochure.

259 Thanks to Hugh Davies for pointing this out. According to Haba the clarinettist Artur Holas provided advice on keywork and fingering on the basis of which the firm of Kohlert made improvements to Reichard Stein's original design. ['Die von Artur Holas gewonnenen Grifferfahrungen und seine Vorschläge zu einer Klappenanordnung für Vierteltöne ermöglichten es der Firma Kohlert in Graslitz, Richard Steins Vierteltonklarinette vom Jahre 1906 weiter zu verbessern']. From Alois Haba - reference? XXX.

260 Oskar Kroll, *The Clarinet*, translated Hilda Morris (ed. Anthony Baines), Batsford, London, 1968, p. 45.

261 One of these clarinets (or at least, I assume it is one of them) may be heard in Hába's *Suite for Quarter-Tone Clarinet and Quarter-Tone Piano No. 1, Op. 24* on the Alois Hába Centenary 3-CD set released by Supraphon (11 1865-2 913). Unfortunately, no details of the instruments played here are given, although photographs of a quarter-tone piano by Förster are included (Also, see below, note 325). However, I can't help feeling that the music shows off neither the tuning system nor the instruments to best advantage. I have not heard the complete LP recording of Hába's opera *Die Mutter*, but perhaps they may be heard there also, since the work requires two quarter-tone clarinets, as well as two quarter-tone trumpets, and quarter-tone piano and harmonium.

262 Paul Harvey, *Saxophone*, Kahn and Averill, London, 1995, p. 110.

263 For example - the 'standard' Böhm system has 23 or 24, the Oehler system 29.

264 There are, I believe, more than this on Oehler and Marchi models, and less on some student models.

hypothetical limit of '*n*' for 'rational' woodwind - 'rational' in the sense that greatest uniformity of tone is ideally provided by the simplest system of holing.

An example of a 'rational' flute, designed as far as possible to use a 'fingering pattern where the chromatic scale corresponds to the simplest possible progression of fingers, and all cross-fingerings are eliminated', was built by Jim Schmidt.[265] Schmidt argues that this system has advantages over the Böhm system for chromatic contemporary music. Its simplicity suggests that it may not be possible to adopt such a design for larger *n*-ET systems without losing those advantages, but it seems worth mentioning here.

A discussion of the pros and cons of 'rational' tone-hole closure as opposed to more economical systems (in terms of the number of tone-holes) is outside the scope of this discussion. Hypothetically, cross fingerings for large numbers of tone-holes extend microtonal possibilities almost infinitely. I do not attempt to show here the limit of (say) *n*-ET for rational 'Logical' woodwind. Mechanical keywork is less easily adapted to a *range* of alternative tunings on a single instrument (notwithstanding contemporary extended fingering technique); and the design and manufacture of this kind of keywork is expensive, given the laborious adaptations needed for prototyping. But I do not feel qualified to speculate in detail on the possibilities of creating either Böhm or 'rational' system woodwind for large divisions of the octave. However, I would guess that for any large *n*-division system, mechanical woodwind are very likely to cost substantially more than 'Logical' woodwind, both in the prototype stage and in manufacture, due to the great complexity and labour of developing professional quality conventional keywork.

Multiple Bore Woodwind

Kroll's second example of a quarter-tone clarinet is a double-bore instrument, which was patented in 1933 by Fritz Schüller. It is described by Kroll as having

> a switch-valve below the mouthpiece by means of which the air stream could be conducted either through a tube producing normal B^b pitch or into a slightly longer tube pitched a quarter-tone lower. The mechanism functions in such a way that keys and plates serve both bores simultaneously.[266]

Figure 13 of Kroll's book includes a photograph of this interesting but heavy-looking instrument, from which it looks as if the two bores are encased in a single body, and that the two bores arrive and merge into a single bell. Kroll dismisses both the quarter-tone clarinets he mentions (although he may be referring to the music written for them) as 'long since forgotten'. We can only speculate, I think, why this was so.[267]

Weight, and to a lesser extent complexity of keywork, are the obvious hurdles for multiple bore woodwinds. According to their sales brochure, the Finnish flute manufacturer *Matit Flutebrothers* has patented the use of carbon fibre and a magnet spring system (as opposed to needle springs) for all woodwinds. It is argued that the use of carbon fibre considerably reduces the weight of the instrument, resulting in no loss of tone quality:

> Weight for weight, carbon fibre reinforced epoxy is five times stiffer than steel. The body of the MATIT flute weighs only 60 grams, and it is still more rigid than any other flute. The stiffness and lightness of the walls of the body create the best possible conditions for the complex function of the vibrating column of air inside the tube. [268]

Perhaps the obstacles to making double-bore woodwinds are therefore (apart from conservatism) cost and patent rather than intrinsic design? The popular use of double-bore clarinets, hornpipes and to a lesser extent recorders, in many folk musics,[269] suggests that double bore woodwind might have an intriguing future,

[265] Jim Schmidt, 'The custom made flute', *Experimental Musical Instruments*, Vol. III, No. 5, February 1988, pp. 89-91.
[266] Kroll, op. cit., p. 45. This instrument is in the instrument museum in Markneukirchen.
[267] Kroll, op. cit., p. 45. To my knowledge the 24-ET clarinet was not exploited effectively by Hàba. But perhaps 24-ET was the wrong system, or the instruments were awkward, or tonally weak? It is easy to suppose there were less musical reasons too.
[268] Matit Flutebrothers 'Carbonera' brochure.
[269] The double clarinet is thought to be the oldest form of clarinet, dating back to 2700 BC, and remains common throughout the Balkans and the Near East. Many forms of this and the double hornpipe exist. The tone-holes of both pipes are usually stopped by a single finger

especially if, in new versions, the valve which switches between the two bores could also switch to a third position, sounding both tubes.[270]

It seems possible that discrepancies between the temperature of the bores could cause intonational problems. So it is worth mentioning that Professor Brindley introduced a thermostatically controlled heater to maintain a constant temperature in the Logical Bassoon. This seems sensible given that the instrument already requires electrical power for the solenoids.

Valve-Controlled Length Corrections using 'Branched Tubes' (Systèmes Micro-Intervalles *at* IRCAM)

For some years the research group *Systèmes Micro-Intervalles* at *IRCAM*, in collaboration with the acoustical laboratory at the Université du Maine, has been researching instrumental designs for microtones - for the flute, clarinet, basset horn, bass clarinet, and saxophone.

> In the case of wind instruments, musicians have recourse to special fingerings (called 'factices') which do not always produce a stable or homogeneous sonority with other notes, and which sometimes require corrections on the part of players, notably with the lips. The complexity of these fingerings often makes it impossible to play a piece with complete virtuosity…[…]… The original idea of this project was to create a simple system for each type of instrument (all-harmonics or odd-harmonics) to facilitate microtones. Ideally, a mechanism would be placed not on the main body of the instrument but on an easily interchangeable part, such as the head joint of the flute, or the mouth piece or barrel of the clarinet. The mechanism would have to be able to effect the correct adjustment for a given micro-interval, that is, by producing a 'length correction' relative to the note played.[271]

As yet instruments using this mechanism have been realised only in prototype, and research has focused mainly on quarter-tones, but it is claimed that 'the generality of the study can be applied to any micro-interval'. On the flute the mechanism is controlled with the right thumb, but does not cover the complete scale; on the clarinet and saxophone the mechanism sits on the body of the instrument. *Buffet Crampon* has been involved with tests on the clarinet, as has *Selmer* regarding the saxophone. The mechanism itself 'uses branched tubes, open for instruments with all harmonics, closed for instruments with only odd harmonics'; and in their 1988 article Kergomard and Meynial claimed that for 'Böhm flute and clarinet, the success [of this research] is almost complete [regarding the accuracy of] pitch, tone colour and [ease of sound] production.' [272]

As I understand it, in practice 'any micro-interval' means 'any subdivision of the scale of the instrument on which it is fitted' - hence an emphasis, so far, on quarter-tones. In addition to 24-ET, might the mechanism also therefore provide 36 or 48-ET? In theory, with a right to develop this technology, might a woodwind designer apply a similar mechanism to an instrument whose tone-holes and keywork were configured for 17 or 19-ET, thus suggesting a relatively practical way of conceiving woodwind in 34-ET, and 38-ET, or other large, even-numbered *n*-ETs?

Alternative Tone-Hole Closure Mechanisms

The diameter of a tone-hole on a conventional woodwind is, for the most part, fixed - only for certain holes on particular instruments can pitch be directly controlled by movement of the finger changing the open area of the

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- sometimes giving the same note, sometimes a combination of notes. Double flutes, such as the Albanian c*yledyjare* or the Bulgarian *dvoyanki,* are often built to give parallel seconds or thirds, but some players explore a freer polyphony. Double recorders are also fairly common in Eastern European folk music, usually having a single blowhole connected to two pipes - the number of tone-holes on each bore may differ, but usually some coincide and are stopped with a single finger. I am unaware of a folk instrument which is a 'diatonic' parallel of Schüller's clarinet.

[270] The idea of multiple bores also suggests woodwinds in which discrete pitches are selected by valve-operated multiple bores, say one tube for each pitch, or few pitches. The body material would have to be extremely light, or perhaps be supported by a spike.

[271] This text can also be found on the *Systèmes Micro-Intervalles* page at the *IRCAM* website.

[272] J. Kergomard & X. Meynial, 'Systèmes micro-intervalles pour les instruments de musique à vent avec trous latéraux', *Journale Acoustique* 1, 1988, p. 255. Their invention is protected by French Patent No. B8760 GL, and United States Patent 4,714,001.

tone-hole. While certain effects are available when pads are only just open, for ‘clean’ and discrete pitches pad-controlled tone-holes are effectively either open or closed. Other factors being equal, the larger the open area of a tone-hole, the higher the pitch; the smaller the area, the lower the pitch. Three possible mechanisms for controlling this are:

- ‘concentric pads’;
- crescent-shaped sliding covers;
- ‘iris diaphragms’.

Concentric pads would involve two round pads of different diameters, the outer one having a hole in the middle and operable independently, the inner pad fitting together with the outer and closing the hole completely (only being used in conjunction with the outer). The two pads would thus provide stepped venting. The value of this would perhaps be small in comparison to other suggested mechanisms; it seems improbable that more than two ‘steps’ could be made mechanically practical.

The two latter mechanisms could provide stepped or continuous venting. Sliding covers - crescent-shaped to preserve the shape of the tone-hole in semi-closed positions - would probably be housed in an internal slide to provide an air tight seal. However, unless the mechanism were particularly fast, there would be audible pitch sliding when opening and closing holes. (The suggestion was made above to use solenoids to control this kind of mechanism).

The iris diaphragm - a group of three or more ‘yin and yang’, tear-shaped petals, which give a constant shape to the tone hole while opening or closing - also poses problems of sealing. The iris is probably too complex to be actuated other than electronically. Again, if the mechanism were extremely fast, would this eliminate audible glissandi when a shutter moves from one position to another? If not, might it be eliminated by other means?

An American instrument builder by the name of Jim French has apparently built a family of instruments called ‘Frenchophones’ which, as far as I am able to ascertain, are somewhat like a (straight) saxophone, but have tone-holes which are cased in a sliding cover. The tone-holes can thus be shifted to various points on the bore - perhaps while playing, or perhaps they may be set up so that the instrument affords different scales at different times. Exactly how holing is achieved - whether or what kind of a key mechanism is involved - I have as yet been unable to find out. [273]

Single- or Multiple-Slide (or ‘Telescopic’) Mechanisms

The *Vermeulen* flute, which was first developed in the mid-1960’s, comprises two or more cylindrical tubes, one inside the other, and operates on a slide principle. The mouthpiece is attached to one of the tubes, and there are no tone-holes or keys. The player judges discrete pitches by ear, and perhaps by markings on the extending tube. This flute was ‘designed particularly for contemporary music, though according to Hugh Davies the principle resembles that of the *Giorgi* flute devised as long ago as 1888.’[274] Bart Hopkin, for example, also describes some very simple telescoping slide designs for woodwind,[275] and it seems there is no *a priori* reason why something similar might not be incorporated into woodwind designs employing discrete pitch-differentiating techniques.

The sounding length of the clarinet, for example, might be varied to a limited degree by making the top joint so that it is telescopically sprung into the barrel, balance being taken by the right thumb. Such a device would require an extra joint for overall tuning. And of course, the resultant change of pitch for a given telescopic extension would vary for each fingering of the scale.[276] But something of this sort could function, either like a normal trumpet tuning slide, or, for particular results, by restricting its use to specific notes. Arguably, it is not

[273] Thanks to Bart Hopkin and Johnny Reinhard for information on this.
[274] Hugh Davies, ‘Vermeulen flute’, *The New Grove Dictionary of Musical Instruments,* MacMillan.
[275] Bart Hopkin, *Musical Instrument Design - Practical Information for Instrument Design,* See Sharp Press, Tucson, Arizona, 1996, p. 79. This is a valuable and entertaining guide for the would-be instrument maker.
[276] O. Lee Gibson mentions that a 435Hz Clarinet cannot be used successfully at 440Hz because if the length is shortened sufficiently (a mere twenty cents) the scale becomes unmanageable. But this does not invalidate the telescopic slide. Gibson, op. cit., p. 26.

necessary to provide additional *intonational* flexibility for woodwind, and there might be acoustic disadvantages to this idea. It is included here because, to make an instrument which is intonationally successful, it may be a virtue to have additional ways of bending good notes easily without disrupting their tone.[277]

Bart Hopkin's 'magstrip' woodwind designs, while not a 'telescopic' mechanism, also deserve a mention here:

> [M]agstrips incorporate a ribbon-like strip of flexible material held taut over the open slit [length-wise on the woodwind bore] , angling slightly up and away. Using a finger to press this strip down at any point along its length will close the upper part of the tube. The seal of that closure would be problematically leaky were it not for this feature: the instrument is made of steel tubing. The flex-strip over the slit is made of rubberised magnetic material… The magnetism is weak, but it is enough to cause the upper part of the strip to slap down over the slit leaklessly when the finger presses.[278]

The idea behind the magstrip is to make a woodwind which is capable of infinite pitch gradation - like the violin, with 'all the bending and glissing one might wish', as Hopkin puts it - which is equally capable of discrete pitch articulation, made possible by placing tactile (or visual) markers on the magstrip itself.

'Logical Feedback' Woodwind: Adaptive, Fixed & Multiple ATS; the 'Networked Orchestra'

Real-time (instantaneous) digital pitch correction for audio has been available for some years - for example, as part of the *ProTools* digital audio processing suite.[279] A live performance by a singer or instrumentalist can be 'corrected' in real-time so that all intonational inflections are automatically adjusted to correspond to the strict definition of a specified tuning system. The *Antares* allows custom scales, but the typical implementation is *strict* 12-ET. This runs counter to the norm of variable intonation (for singers, wind and brass etc.), but more sophisticated realisations are clearly possible - for example, in its current version the *Antares* is able to 'ignore' vibrato and glissandi.

We may imagine, therefore, a 'logical' woodwind fitted with an internal electronic pitch sensor which provides feedback (via software) to an auto-corrective venting or slide mechanism in real-time, such that the instrument would, in terms of tuning, be self-correcting within configurable limits. For this to work we must assume that the onset of the note is reasonably accurate, and that the pitch correction is small and virtually instantaneous. Since the combination of keys which are pressed at any time 'primes' the mechanism to 'expect' a particular pitch, this too might be used to minimise discrepancies between pitch onset and the target pitch. If this was successful, it could be applied in a number of ways, for example, to achieve strict temperament, or to achieve strict just intonation in any transposition. For *large n*-ETs, for example, the system could ensure that the instrument sounds exactly (or almost exactly) the desired temperament; for JI systems, this would function similarly, but some mechanism would tell the software which pitch is 1/1 at any given time.[280]

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[277] For details of the multiple-telescopic clarinet built by François and Bernard Baschet, which uses five sliding segments, a 'baby gate' mechanism to ensure they extend and contract uniformly, and an ingenious automatic helical system for multiple register holes, see an untitled article by Bart Hopkin, *Experimental Musical Instruments*, Volume VII, #4, January 1992, pp. 4-5. Also worth mentioning here is: Wes Brown, 'An experimental slide bass clarinet', *Experimental Musical Instruments*, Volume VIII, #3, March 1993, pp. 12-14.

[278] Bart Hopkin, *Musical Instrument Design - Practical Information for Instrument Design*, See Sharp Press, Tucson, Arizona, 1996, p. 81. Further remarkable ideas on 'wind instruments with continuously variable pitch control' can also be found in: Bart Hopkin, 'A day in the patent library', *Experimental Musical Instruments*, Vol. VII, #1, June 1991, pp. 16-18.

[279] This utility was recently released in rack-mount form as the *Antares Auto-Tune*.

[280] One reason for thinking this approach might be useful is the relative instability of pitch in woodwinds. In terms of pitch variation due to temperature, woodwinds with many divisions of the octave would not be significantly disadvantaged compared to conventional woodwind, despite what would be a narrower tuning tolerance limit. But there is a tendency for long notes on sustained (feedback) wind instruments to fall in pitch, in extreme cases, by 'more than 20 cents' in the air-column resonance frequency (or fundamental). This is due to changes in gaseous composition - proportions of oxygen and carbon dioxide - within the bore. This is a significant variation, considering that for systems between 27 and 41-ET the basic unit of subdivision of the octave is 44 to 29 cents. See Leonardo Fuks, 'Prediction and measurements of exhaled air effects in the pitch of wind instruments', *Proceedings of the Institute of Acoustics*, Vol. 19, Part 5, (1997), pp. 373-378.

The suggestion here is not to use such a mechanism to translate an acoustically produced sound into a digitally modulated one, but to transmit corrections of the electronically monitored pitch signal to the pitch producing mechanical element of the acoustic instrument. If the mechanical element was tone-hole closure, the special venting arrangement would probably be very small holes high up on the bore; if it were an automatic slide mechanism, it a would have to be higher on the bore than the highest tone-hole. The player would thus be in constant dialogue not only with the vibrations of the instrumental system, but with extremely fine adjustments of pitch being carried out independently by the 'correction' process.[281] This would require the finest possible balance of sensitivity and stability in terms of software interactivity, because unless pitch adjustments were fine, unwanted glissandi and suchlike would result; on the other hand, *extreme* subtleties of pitch inflection need not be overridden, and the player would have to be able to vary the sensitivity of the device (from 0 -100%) at the slightest touch of a key.

As mentioned above (see note 96), 'adaptive tuning' is the term used to describe mechanical or electronic attempts to produce an equivalent to fixed-but-variable intonation. An 'adaptive' instrument automatically adapts the pitch of tones according to the note combinations which are played. This is normally applied to an instrument of fixed intonation, such as a keyboard, and as yet implementations remain experimental. In the 'logical feedback' woodwind described above, the initial idea would be to make a woodwind (or, as we will see later, a brass) which will enable very accurate realisations of radical ATS. But as a by-product, the reverse of 'adaptive tuning' is suggested - that is, enabling an instrument which is normally of fixed-but-variable intonation to be capable (or almost capable) of *fixed* intonation - which at the flick of a switch could revert to fixed-but-variable. It is not difficult to see how this mechanism might also be useful for enabling 'multiple' ATS for individual woodwinds (or brass).

Let us take this one step further. Suppose that two woodwinds each have built-in pitch sensors, of which instrument (A) contains the correction mechanism but instrument (B) only includes the sensor, but that both instruments are electronically 'networked' together. The pitch correction system could be configured so that instrument (A) takes its tuning correction from the current pitch of instrument (B). Alternatively, both instruments might be fitted with the correction mechanism, and each configured to take its pitch correction from an absolute standard (say, A = 440 Hz). In either case, their interaction might be mediated by an 'adaptive' JI algorithm, or by an algorithm that demands strict temperament. Taking this to its logical extreme, complete woodwind and brass sections might be 'networked' - real-time pitch correction might be 'global' or 'per section'. Similarly, pitch control could be configured to respond adaptively, for example, to the lowest current pitch - suggesting an interesting way of creating bass related harmony; or, relative to a central or upper pitch; or even to a moving pitch. Conceivably, a composer could be able to adjust the configuration or algorithm in real-time during performance. Here is a truly explosive conception of purely acoustic 21st Century orchestral harmony.

Alternative Woodwind Technologies in Combination

In any attempt to create orchestral-quality experimental instruments there will be no guarantee that the results will justify the kind of time and effort that routinely goes into learning and practising established instruments. Each of the above mechanisms and approaches has advantages and disadvantages. But perhaps combining them, and/or matching them to existing performance techniques, may suggest a new era of woodwind technology.

For example, research is needed into the implications of combining the logic of existing performance techniques with new design mechanisms. As an illustration, McGill has suggested that to create a 'Logical Clarinet' for 19-ET, it would be quite natural to keep the conventional keywork layout.[282] The key positions for the 19 chromatic divisions of the first mode on a 12-ET clarinet could be retained to give 19 chromatic steps in the octave. The player might find re-associating pitch and fingering disconcerting to begin with, but probably not for long. This possibility arises from a numerical coincidence, but nevertheless suggests that adapting keywork for logical woodwind might be done in a number of ways. It also suggests that by combining new and old philosophies we may uncover powerful techniques capable of dealing with large *n*-ET systems.

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[281] Equally, performers might rapidly develop techniques of performing, so to speak, 'with the correction, or 'against the correction'.
[282] In conversation, September 1997.

Players may, however, prefer completely new, 'logical' key systems. In Brindley and McGill's instruments, each finger operates usually one but no more than two switches. The power of the 'logical' approach would be hugely increased if (i) the number of switches available were equivalent to the number commonly used on conventional woodwinds, or (ii) the switches served more than one function, and/or were pressure sensitive (or have 'after-touch'). In the latter case one might imagine that, augmenting the tone-hole closing system, a crescent slide or an iris diaphragm could be actuated and varied by pressure, giving very fine pitch control. It is however difficult to know if this would be worth the effort (by which I mean, whether it would improve existing musical results and also give new and unimagined ones), given that in any case pitch varies with air pressure and embouchure. Other possibilities of logical woodwind include taking a more radical approach to the way combinations of switches operate - for example, giving the fingers of the left and right hands different combinative functions (continuing Norbeck's experiment); or, conceivably, assigning switches and switch combinations to intervals rather than notes (or a combination of the two). This, in turn, suggests an even more radical idea - a 'self-correcting' woodwind, which 'corrects' *consecutive intervals* rather than absolute pitches… although it is currently unclear to me exactly how this could be usefully controlled.

As mentioned above, it is arguable that creating a system of instruments in a single tuning system would result in an unwanted bias toward that system. Woodwind designers might therefore consider creating 'logical' woodwind in which a variety of different but precise tuning systems are available. One (conceivable) way of achieving this would be to make a number of inner sleeves for the instrument bore (not dissimilar to the 'transposing' Alberti flutes),[283] each with a different system of tone-holes, since closing pads on logical instruments need not be in fixed positions. A more practical way might be to design the configuration of tone-holes (more than that required for any one system) such that switching to an alternative (preset) fingering mode would sound an alternative tuning system (in each case using a combination of rational and forked holings). Conceivably, manually adjustable tone-hole sliding covers could be set to fixed positions for any system, being used to sharpen or flatten individual tone-holes: perhaps flautists and clarinettists could be heard to tune their instruments the way a harpist tunes the harp.

Combining these alternative approaches, we might imagine a networked-logical-self-correcting-multiple-bore-telescopic-slide clarinet, made of carbon fibre, and using custom-made solenoids, or piezo-electric plastic... Clearly, many collaborative research projects are suggested, including:

- the logic and ergonomics of electronic key operation and tone-hole closure;
- lightweight solenoids, 'iris' and 'sliding' tone-holes, piezo-electric applications;
- pitch-sensor applications and automatic tone-hole control;
- the relevance of contemporary playing techniques to new instruments;
- the logic of combinatory approaches.

[283] These flutes, built in 1914, had ingenious revolving inner sleeves which enabled them to transpose from C to anything up to D^b. Nancy Toff, *The Development of the Modern Flute*, Taplinger Publishing Company, New York, 1979, pp. 176-8.

Brass

A substantial recent historical survey of brass instruments and performance reports few if any major 20th Century innovations in brass instruments themselves.[284] However, building new brass specifically for ATS suggests some radical areas for research and innovation.

Brass intonation is achieved as a matter of course by lipping and with tuning slides, so it is sometimes argued that there is no sense in making special 'microtonal' instruments. But since great trouble is taken with conventional brass to maximise the facility of realising '12-ET' ('performable' not mathematical 12-ET), it seems logical that the same approach should be taken for other tuning systems. As is the case for voice and strings, the necessity of being able to imagine a note before playing it is essential - but an instrument which best encourages the right notes is preferable to one that does not. As Schönberg put it in a short essay entitled 'The future of orchestral instruments':

> The greater certainty afforded by the more easily playable instrument leaves the player time to concern himself with producing his sound.[285]

Brass players are especially dependent on being able to imagine a note before sounding it, and to what extent a specially conceived new brass instrument will help a player to realise an alternative tuning system accurately probably depends on the system as much as the instrument.

Martin Vogel, whose specially built enharmonic brass instruments are described below, has taken issue with the conventional view of lipping as an inevitable and effective way of achieving good intonation:

> Compensation using lip tension should always be a last resort, it should never become the rule. The trumpeter Ritterbach of Munich once wrote: 'Driving the tones with lip tension endangers the reliability of lipping and holding the tones, worsens the timbre, weakens the staying power of the lips and makes it difficult to produce a vibrato'.[286]

Vogel's brass instruments were therefore built with special valve tuning slides that provide arithmetically proportional adjustment to each valve length, giving just intonation. According to Vogel's philosophy of tone relations this minimises the need for lipping.[287] This system is described below, together with some questions arising from his discussion, particularly regarding brass performance for equal-tempered systems. This follows a brief overview of valve systems.

Valve systems

In principle, a brass instrument without any form of valve, slide or key system to alter the fundamental is limited to sounding only the pitches (or near approximations of the pitches) of the harmonic series of that fundamental - that is, leaving aside a player's ability to 'lip' up or down. Valved systems overcome this by providing the player with pistons (or levers) which extend (or sometimes diminish) the length of the cylindrical

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[284] *The Cambridge Companion to Brass Instruments*, edited T. Herbert & J. Wallace, Cambridge University Press, 1997.

[285] Arnold Schönberg**,** 'The future of orchestral instruments' (1924), in *Style and Idea*, edited by Leonard Stein, Faber & Faber, 1975, p. 322.

[286] Martin Vogel, *On the Relations of Tone*, translated from the German by Vincent J. Kisselbach, edited by Carl A. Poldy, Verlag für systematische Musikwissenschaft, Bonn, 1993, pp. 374-5.

[287] The more pragmatic view has been put by Doty: 'a valved brass instrument is really just several natural horns conveniently packaged for one player. By making proper adjustments of the tuning slides and by carefully selecting scales that make maximum use of natural harmonics, a great deal of Just Intonation can be gotten out of any brass instrument. Hornist Bruce Heim states that most horn players should be able to produce, at minimum, accurate deviations of 16 cents, and that the -31 cent shift necessary to produce harmonic sevenths is within the reach of many players. Heim has designed a series of exercises to help brass players achieve facility in such intonational shifts.' David Doty, op. cit., p. 67. Doty provides the following reference, which unfortunately I have been unable to obtain: David Bruce Heim, *Practical Tuning, Temperament and Conditioning for Hornists and Other Wind Instrumentalists: Understanding and attaining intonational flexibility in musical performance*, unpublished Masters thesis, University of Tulsa, 1990. However, it would appear that these figures underestimate the ability of many brass players to lip up or down while preserving quality of tone.

part of the bore. They may be pressed and released singly or in combination, thus providing a range of fundamentals and enabling the player to move rapidly and easily between the notes of different harmonic series.

For the smaller brass, the fundamental is not considered an ordinary part of the instrument's range, and the scale begins on the second harmonic. In order to obtain a fully chromatic range, the largest 'gap' of the harmonic series which must be 'filled in' is therefore between modes two and three - a fifth. For a '12-ET trumpet' in C, the valve system must therefore provide six extra fundamentals - B down to F#. The conventional instrument usually has three piston valves (sometimes four), the piston (almost) invariably having only two possible positions - 'in' and 'out'. A three-valve (two position) system therefore gives eight possible *piston position* combinations; a four-valve system gives sixteen; a five-valve system gives thirty-two. The lengths of extra tubing which are introduced into the bore by these combinations are carefully calculated at the manufacturing stage so that as accurate a chromatic scale as possible may be obtained from the resulting fundamentals and resonant modes.

In a three-valve 12-ET system, the 1st, 2nd and 3rd valves lower the fundamental by approximately 2, 1 and 3 semitones respectively. Ideally, the same lengths would, in combination, lower the fundamental by 4, 5 and 6 semitones, but some sort of adjustment is needed, since the extra lengths of tubing required to lower the pitch by (for example) a tone or a minor third are too short when combined to lower the pitch by a fourth.[288] This can more or less be achieved by adjusting slightly the extra lengths given by the first and second valves so that together they lower the fundamental by a minor third; the third valve can then be adjusted so that it flattens by more than three semitones - thus counterbalancing sharpness in other combinations.[289]

To avoid this compromise, in the mid-19th Century various compensating systems were built, in which length corrections are automatically introduced when pistons are pressed in combination.[290] Today, tuning slides on the first and third valves are normally provided, which are operated whilst playing. The intonational problem described above affects the larger brasses more severely, and the defect cannot be overcome by lipping or small slides - for this reason they are normally built with four, five or more independent valves.

Trumpets for Alternative Tuning Systems

To my knowledge, the majority of existing 'microtonal' trumpets are adaptations of the standard 12-ET instrument in which a single extra valve (or slide) has been added, lowering the fundamental by a quarter-tone, and (more or less) fitting the instrument for 24-ET;[291] alternatively, an extended telescopic U-slide is provided, allowing a wider range of inflection.[292] I am aware of only one instance of a trumpet built or adapted for a specific *n*-ET other than 24-ET - that built by Swift and Yunik, who took a conventional three-valve trumpet and adapted it for 19-ET.[293] They did this by shortening the lengths of valve tubing accordingly. Instead of lowering the fundamental by (approximately) 1/12, 2/12 and 3/12ths of an octave, the three valves lower the fundamental by (approximately) 1/19, 2/19 and 4/19ths. In combination the valves therefore provide chromatic flattening (theoretically) as far as 7/19ths (442.1 cents). This is unfortunately not as far as the 10/19ths (631.6 cents) which is needed to bridge the fifth in 19-ET. According to Swift and Yunik, the trumpet nevertheless provides an uninterrupted 19-ET chromatic range from D# (253 cents above middle C) upwards.

Vogel's approach is more sophisticated. Each of the nine enharmonic brass instruments he commissioned and had built were designed to realise just intonation. These were:

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288 Philip Bate, *The Trumpet and Trombone*, Ernest Benn, London, 1978, pp. 29-31; Anthony Baines, *Brass Instruments*, Faber & Faber, 1980, pp. 216-9. See also Vogel, op. cit., pp. 373-86.

289 Campbell and Greated point out that the introduction of additional tubing has an effect on the proportions of the cylindrical and flaring portions of tubing, which, in the case of the trumpet, somewhat mitigates the problem. Murray Campbell and Clive Greated, *The Musician's Guide to Acoustics*, Dent, 1987, pp. 356-7.

290 See for example the system designed by J. D. Blaikley. Bate, op. cit., p. 179.

291 The earliest known quarter-tone trumpet seems to have been built in 1893, and is now in the Odessa Conservatory. Other early examples include one built for Carrillo in the 1920s, another for Haba in 1931, by F.A.Haeckel. Hugh Davies, 'Microtonal instruments', *The New Grove Dictionary of Musical Instruments*, MacMillan, pp. 653-9.

292 In their current catalogue, the Holton *'Bb Maynard Ferguson Firebird'* speciality trumpet is of the latter type; in the 1960's Holton & Co. built quarter-tone trumpets for the whole trumpet section of a band led by the jazz trumpeter Don Ellis. Thanks to Kami Rousseau for pointing this out.

293 G.W Swift & M. Yunik, 'Modifying a trumpet to play 19 tone music', *Interval*, Spring-Summer 1980, p. 8.

two B^b Trumpets with rotary valves, a B^b Trumpet with piston valves, three B^b French Horns, a 3- valve F Tuba, a 4-valve F Tuba and a 3-valve B^b Tuba (Kaisertuba).[294]

In the 3-valve 'enharmonic concert trumpet', the extra tube lengths are in the proportion of 2, 1 and 4 units, relative to a basic bore length of 18 units. Thus, for example, depressing valves 1 and 3 will augment the tubing length by 6 units, making a total of 24 units, giving 24/18 or 4/3 (a perfect fourth) below the original fundamental. In addition, a thumb key or lever is provided, which enables the extra tube lengths to be extended simultaneously, such that their proportions remain constant relative to each other.[295] The slide may be moved to increase the draw lengths so that they equal (1+2+4)/16ths, (1+2+4)/15ths and (1+2+4)/14ths of the basic length. In this way, Vogel claims that the arrangement provides degrees of flattening of the available fundamentals - giving just intervals. Each of the above listed instruments is built on a similar principle.

Vogel in fact argues that

> it is… impossible to construct brass wind instruments in equal temperament… the only possible solution would be to equip the instrument with six valves… [but] what would be the point of having such instruments generally available? A trumpet in equal-temperament would sound just as out of tune in ensemble as an accordion or an electronic organ do. No one expects equal temperament, one expects pure intonation of a wind player.[296]

It is commonly argued that in chromatic music just intonation cannot be strict - neither for individual lines nor harmony - since, unless there are commatic shifts on individual notes, even quite simple harmonic progressions will imply a rise or fall in overall pitch.[297] But, as I understand it, Vogel has not produced these brass instruments so that players should play in pure intonation until such point as they are forced to make arbitrary leaps to recover concert pitch. Rather, Vogel proposes a more fluid conception which he describes as 'the unlimited use of pure tuning…[…] …A composer must at all times have unlimited access to the whole of all tone relations'.[298] Vogel makes a case for performing both existing and new music in this way. Exactly how and whether such a system is possible is beyond the scope of this essay - the most obvious obstacle is instruments of fixed intonation, but it is not the only one. However, Vogel's approach does pose an important question: if an equal-tempered scale were chosen for a new system of instruments, should they in fact be designed to produce that scale as exactly as possible?

In the following discussion, therefore, the limits of *n* for '*n*-ET trumpets' are considered with three provisos in mind: firstly, that although an instrument may be designed for an ATS, we will not expect its theoretical description here to model *exactly* that system; secondly, that, for *n*-ETs, the larger *n* is, the smaller the intonational adjustments (in general) become, therefore lipping becomes less of an issue; and thirdly, that the need for compensating mechanisms, simple or sophisticated, is taken for granted. In particular, when I describe what I call the '*nominal'* (integer) length proportions of a system (say 1, 2, 4 and 8), it should be clear that while the mathematical combination of these proportions gives the series (1,2,3,4,5,….15), this does not mean that this series will generate a *pitch* series corresponding exactly to those proportions. Thus, the available 'chromatic' combinations required to bridge the fifth may be marginally optimistic, but further detail is not necessary here. As stated above, as the number of divisions of the octave increases, the average compensation required becomes less.

The relation between the harmonic intervals and the use of the different modes is crucial with regard to new brass instruments. Firstly, to obtain the best possible tone (and to minimise effort) performers will, other

[294] Vogel, op. cit., p. 375, Note 1. Photographic illustrations of each of these instruments are included in the book.
[295] Vogel, op. cit., p. 376. Unfortunately the explanation of this mechanism (in English, at least), is not altogether clear. I have not had access to, but suppose the exact details may be found in: Martin Vogel, *Die Intonation der Blechbläser*, Neue Wege im Metallblas-Instrumentenbau, Düsseldorf, 1961. (In *On the Relations of Tone* Vogel does not compare his invention to other forms of brass compensating mechanisms).
[296] Vogel, op. cit., p. 374.
[297] Easley Blackwood, *The Structure of Recognisable Diatonic Tunings*, Princeton University Press, Princeton New Jersey, 1985.
[298] Martin Vogel, *On the Relations of Tone*, translated from the German by Vincent J. Kisselbach, edited by Carl A. Poldy, Verlag für systematische Musikwissenschaft, Bonn, 1993, pp. 413 & 418.

factors being equal, often prefer the lowest *mode* available (from amongst the different fundamentals) to produce a certain note. For example, on a standard C trumpet, the use of mode three of C3 rather than mode four of G2 would be preferred for sounding G4. (This is a convenient example - but in fact this preference is greater in the upper range). Secondly, assuming that the lowest available mode is preferred, then to obtain a chromatic scale in any system it is preferable to use modes three, five and six. In this case, unless the chosen tuning system contains good approximations of the fifth and the major third, the performer would inevitably be forced to lip (or slide) an unfortunately high proportion of notes. Thus, just as an ATS without a reasonable approximation of the fifth would limit the possibilities of the clarinet, new brass would be made more awkward unless a chosen ATS affords reasonable approximations of the fifth and the major third. Thus, as has been stated, unless the modes of overblowing of all the new woodwind and brass elements of an instrumental system can be adjusted to systematically agree with a chosen ATS, then good agreement between the harmonic series and the temperament would clearly be advantageous, or even essential.

Returning to Swift and Yunik's line of thought, four (2 position) valves would be needed to create a fully chromatic 19-ET trumpet. Depending on the compensating mechanism, the relative proportions of the 3rd and 4th valve lengths would benefit from experiment. A compensatory mechanism would have 'more to do' to adjust a system comprising 1/19, 2/19, 3/19 and 4/19ths (to provide accurate chromatic steps up to 10/19ths) than with some other series (from 1/19, 2/19, 3/19 and 5/19ths up to 1/19, 2/19, 4/19 and 8/19ths). The table below shows the 'redundancy' (the number of fingerings producing the same *nominal* interval) of these options: the greater the redundancy, the more combinations there are to choose from to produce a particular note; the less the redundancy, the greater the number of possible valve combination lengths.

Nominal pitch redundancy in 4 valve (2 position) brass systems

1	2	3	4		1	2	3	5		1	2	3	6		1	2	3	7		1	2	4	7		1	2	4	8	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1
0	1	0	0	2	0	1	0	0	2	0	1	0	0	2	0	1	0	0	2	0	1	0	0	2	0	1	0	0	2
0	0	1	0	3	0	0	1	0	3	0	0	1	0	3	0	0	1	0	3	1	1	0	0	3	1	1	0	0	3
1	1	0	0	3	1	1	0	0	3	1	1	0	0	3	1	1	0	0	3	0	0	1	0	4	0	0	1	0	4
0	0	0	1	4	1	0	1	0	4	1	0	1	0	4	1	0	1	0	4	1	0	1	0	5	1	0	1	0	5
1	0	1	0	4	0	0	0	1	5	0	1	1	0	5	0	1	1	0	5	0	1	1	0	6	0	1	1	0	6
1	0	0	1	5	0	1	1	0	5	0	0	0	1	6	1	1	1	0	6	1	1	1	0	7	1	1	1	0	7
0	1	1	0	5	1	1	1	0	6	1	1	1	0	6	0	0	0	1	7	0	0	0	1	7	0	0	0	1	8
1	1	1	0	6	1	0	0	1	6	1	0	0	1	7	1	0	0	1	8	1	0	0	1	8	1	0	0	1	9
0	1	0	1	6	0	1	0	1	7	0	1	0	1	8	0	1	0	1	9	0	1	0	1	9	0	1	0	1	10
1	1	0	1	7	1	1	0	1	8	1	1	0	1	9	1	1	0	1	10	1	1	0	1	10	1	1	0	1	11
0	0	1	1	7	0	0	1	1	8	0	0	1	1	9	0	0	1	1	10	0	0	1	1	11	0	0	1	1	12
1	0	1	1	8	1	0	1	1	9	1	0	1	1	10	1	0	1	1	11	1	0	1	1	12	1	0	1	1	13
0	1	1	1	9	0	1	1	1	10	0	1	1	1	11	0	1	1	1	12	0	1	1	1	13	0	1	1	1	14
1	1	1	1	10	1	1	1	1	11	1	1	1	1	12	1	1	1	1	13	1	1	1	1	14	1	1	1	1	15
Redundancy				**5**					**4**					**3**					**2**					**1**					**0**

Figure 8 : Nominal pitch redundancy in 4-valve 2-position brass systems

For the 'three-valve with added quarter-tone valve' 24-ET trumpet, the *nominal* (integer) length proportions are 1/24, 2/24, 4/24 and 6/24, providing (nominally if not acoustically) the necessary thirteen steps required to bridge the fifth (13 x 50 = 650 cents). The maximal proportional valve length distribution for a four-valve system is 1+2+4+8, giving fifteen discrete chromatic divisions to span the fifth. The upper theoretical *n*-division limit for a simple four-valve trumpet is 27-ET (15 x 44.44 = 667 cents). A fully chromatic trumpet for (say) 29, 31, 34, 36, or 41-ET would require 16, 17, 19, 20 or 23 divisions, respectively, to fill in the fifth. These *n*-ET systems would therefore require more than a four-valve (2 position) system

As mentioned above, five-valve systems are common among the larger brasses, but trumpets have also been built with five valves. Bate describes the *Merri Franqin* trumpet, for example, which

> could be played as a common three-valve trumpet, but two additional valves could also be brought into use. One, operated by the right thumb, *raised* the overall pitch by a tone, while the other, allotted to the left thumb, lowered it by a major or minor third according to how the slide

> was pre-set. Thus ascending and descending valves were combined, a thing uncommon on the trumpet.[299] (Bate's italics)

By all accounts this trumpet was indeed a highly refined and effective instrument. The reason why intricate instruments like this have not ultimately found favour is mainly because - at least to the popular ear, and for music composed employing '12-ET' - they have been proven unnecessary. But it is also because, in the long run, mechanical complications obstruct music making. As Bate says,

> In general… modern students prefer to concentrate on learning to play a simple instrument in tune rather than burden their minds with a complexity of alternative fingerings. The point is emphasised if we compare the tablatures of the ordinary three-valve instrument with that of the *Merri Franqin* trumpet…[300]

A five-valve (two-position) system gives 32 possible key position combinations - 32 renderings of the harmonic series. For the benefit of simplicity, therefore, consider two simpler alternative systems.

Firstly, consider a three-valve system, of which one valve is a *three-position rotary valve*, probably operated by the left hand.[301] Although this system would provide twelve possible piston/lever positions, it would not be possible to configure the length combinations to obtain twelve (nominal) discrete chromatic flattenings, because the three-position valve could only provide one length value at a time. There are two (integer) valve proportion options which (nominally) give the maximum of 11 chromatic steps:

	Rotary Valve	***Ordinary Valve***	***Ordinary Valve***	***Chromatic Steps***
Length Proportion	1 and 2	3	6	11
	2 and 4	1	6	11

Figure 9 : Nominal 11 chromatic step - one-3-position, two-2-position valve system

The upper *n*-division limit for this system is therefore 21-ET (11 x 57.1 = 629 cents), bridging the nearest (narrow) fifth (686 cents).

Secondly, consider a four-valve system, this time the left-hand operated three-position rotary valve supplementing three standard piston valves. This would provide 24 possible piston/lever position combinations; the maximum number of (nominal) chromatic steps is 23.

	Rotary Valve	***Ord. Valve***	***Ord. Valve***	***Ord. Valve***	***Chromatic Steps***
Length Proportion	1 and 2	3	6	12	23
	2 and 4	1	6	12	23

Figure 10 : Nominal 23 chromatic step - one-3-position, three-2-position valve system

The theoretical upper *n*-division limit would be 42-ET (23 x 28.6 = 657 cents) - again, bridging a somewhat narrow fifth (685.7 cents).[302]

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[299] Philip Bate, *The Trumpet and Trombone*, Ernest Benn, London, 1978, p. 181.
[300] Bate, op. cit., p. 183. A diagram comparing these fingering systems is given on p. 184, clearly illustrating the problem.
[301] Gustave Besson's two-way rotary valve of 1855 gives three positions. For such a mechanism to be effective in the context I am describing, it must be able to pass from any position A to any position B without having to pass through the remaining position C. It is unclear to me whether the Besson valve does this. The logic of this movement is of course possible, if we think of A, B and C as the three corners of a triangle. Bate, op. cit., p. 182.
[302] A five-valve (two-position) trumpet would provide 32 possible key position combinations, thus maximising the available valve length combination (1+2+4+8+16), giving a (nominal) upper *n*-division limit of 57-ET (32 x 21.05 = 674 cents). Hyper-hypothetically, then, a traditional valve design might be adapted for an instrument to encompass equal-temperaments as radical as 53-ET (the 'Mercator cycle' which gives exceptional accuracy to just intervals) or above. This is probably ludicrous, but at what point exactly does the possible become impossible?

This also suggests that for smaller *n*-ET a three-valve (12-position) system might be more practicable. For example, since only ten chromatic steps are required to cover the fifth in 19-ET, a three-valve (12 position) system would suffice. This might be preferable to the four-valve system described above. Another possibility might be to make a single instrument which is reliably adaptable for a number of different tuning systems, by providing replaceable valve lengths and an individual slide. The latter might be extremely finely 'notched' as an aid for a particular system, but whether this would be useful is unclear.

French Horn

The discussion so far has focused on the trumpet, but the general principle of providing an appropriate set of valve lengths and their combinations for an alternative tuning system should be applicable to any valve operated brass. George Secor has described how a 'multi-purpose microtonal' French horn might be built, using altered valve lengths, restricting his discussion to 19, 22, 24 and 31-ET.[303] Secor notes that the standard F/Bb instrument normally has four key-operated valves, the first three lowering the fundamental (F) in the manner of the trumpet, and the fourth (the thumb valve) raising the pitch by a fourth (to Bb). When the thumb valve is released, a second set of valve lengths comes into effect for the first three keys.

Taking 31-ET as a starting point, Secor notes that the fifth in this temperament must be spanned by 17 chromatic divisions for a fully chromatic instrument,[304] so a minimum of 5 single-position valves would be required. He then describes a problem specific to horn playing - how to compensate for the raise in pitch caused by hand stopping. On the F side the compensation is usually made by taking the fingering for a semitone lower than if the note were not hand stopped, or a special extra valve may be used; while on the Bb side the shorter valve draws may not be long enough to compensate for the change in pitch. If, therefore, a special 5-valve horn were built for 31-ET there would be no available finger to operate an extra valve. Moreover, Secor suggests that adjusting the pitch by approximately 2/5^ths^ or 3/5^ths^ of a tone (rather than a semitone) would be either too little or too much to adjust for hand stopping (although he does not seem to be altogether consistent in this).

The solution he suggests is to make 'one of the valves do double duty: to fill out tones in the range and serve as a hand-stopping valve', and he proposes the modification of a 'five valve F/Bb double horn with a separate hand-stopping valve and a separate Bb tuning slide'.[305] Having deduced the required tube lengths for each of the valves, which lower the fundamental pitch for both sides of the instrument by 2/5$^{\text{ths}}$, 1/5$^{\text{th}}$, 4/5$^{\text{ths}}$ and 3/5$^{\text{ths}}$ of a tone respectively, Secor shows that the (theoretical) deviation from 31-ET for any combination is never greater than 5 cents. The average deviation (in the intervals required to bridge the fifth) is less than 2 cents, and only four of these intervals stand substantially outside this. Secor claims this to be a smaller tuning error than that found for 12-ET brass instruments. Secor also reiterates a point made earlier:

> In determining the basic fingerings for the entire range of the 31-Tone horn, it is necessary to use the harmonics that agree most closely with the temperament. Therefore, basic fingerings will avoid the 9$^{\text{th}}$, 11$^{\text{th}}$, and 13$^{\text{th}}$ harmonics. All the others, through the 16$^{\text{th}}$, can be used for basic fingerings.[306]

Secor also shows that similar results may be obtained for instruments conceived for 19-ET and 22-ET. Unfortunately I have been unable to discover whether Secor's ideas for making these horns were ever realised.[307] It is not difficult to see how these ideas might also be extended to the tuba.

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303 George Secor, 'An approach to the construction of microtonal valved brass instruments - the French Horn', *Xenharmonikôn*, Vol. 3, No. 1, Spring 1976, pp. 1-3. Secor mentions that he hopes to realise a 'family of microtonal brass instruments', but I do not know if any of these were ever built. It is perhaps worth mentioning here that the microtonal composer Georgi M. Rimsky-Korsakov (the grandson of Nikolai) composed 'Pieces in quarter-tones' (1925-32) for an ensemble which included two 'modified French Horns' although what these modifications were I don't know. This composer was also the co-inventor of the 'Emeriton' (an electronic keyboard instrument), and a 'quarter-tone fisharmonium' - a precursor of the modern synthesiser'. See Larry Sitsky, *Music of the Repressed Russian Avant-garde 1900-1929*, Greenwood Press, 1994, pp. 326-7.

304 Secor actually says 18 divisions - but he is including 1/1. In the above discussion of the trumpet I omit 1/1 in the calculation.

305 George Secor, ibid., pp. 1-2.

306 George Secor, ibid., p. 3.

307 For those especially interested in the horn, Walter Hecht has described the radical horn designs of Mark Veneklasen, claiming that 'V1', the first prototype, was 'the best horn *anyone* had ever played'. 'V2', a radical second prototype, was completely modular -

Multiple Bore and Keyed Brass

Brass, like woodwinds, have also been made with more than one bore.[308] A 'duplex' instrument for ATS might be built, for example, in which a valve is used to move between two different basic bore lengths, but introducing the same (or different) extra tube lengths - to achieve the desired scale.

Prior to the invention of the valve there was a considerable history of tone-holes being operated by keys (or fingers) to alter the sounding length of brass instruments. In the hands of a brilliant performer, keyed brass encourage a special tone which is sometimes preferred by connoisseurs of Baroque brass. I have not dealt with keyed brass here because it would seem that valve technology has proved inherently more powerful in extending their capabilities and providing uniform tone. However, a keyed system might well prove useful for ATS, especially if driven by solenoids, as in the 'logical brass' discussed below.

'Logical' Brass

Following the discussion of 'logical woodwind', another potential method for creating brass instruments for ATS would be to control a combined system of valves and slide(s) electronically, programming them to give ideal length adjustments for each key combination (probably only one slide would be necessary). Conceivably, a 'logical' system could provide ideal pitch adjustment for a variety of ATS on a single instrument, by building in a number of alternative tuning 'presets' each defining a specific relationship between key combinations and valve/slide movement. (Alternatively, of course, the slide could remain manual). Light solenoids might be used to open and close the valves, and today, stepper motors could almost certainly manage the required movement with adequate speed.[309] Automatic slide adjustment might also be programmed to achieve very radical scales - that is, if a player were able to learn to pitch them.

We have seen, for example, that a two-position four-valve trumpet has 16 different possible valve combinations. Three electronic buttons each with a 'semi-sideways' mechanism (or 'aftertouch'), for example, could provide much more than this. This would retain, at least in physical terms, a relatively simple interface for the player. Some 27 combinations provided by a three-button three-position system would probably be excessive, but 'aftertouch' on one or another button might activate an alternative function. Perhaps it would be useful to control slide movement or some kind of mute. Alternatively, a trumpet (say) might have a much larger number of buttons (in effect a kind of 'keyboard') operated by both hands; this suggests a development of the trumpet which might only be practical with 'logical' control - that is, to include a system of draw lengths that bridges the octave rather than the fifth.

To my mind, the most promising approach would be to use solenoids to control something like Adolphe Sax's 'independent valve' system,[310] particularly because the need for both slide(s) and stepper motor could be done away with entirely. In Sax's 12-ET instrument, six separate independent pistons each introduce extra tubing lengths into a single 'circuit'. In this case each piston adds in a new length of tubing, and there is no longer the problem that single lengths of tubing must serve a double purpose (as with conventional systems). For traditional instruments this means that six fingers are required to operate the pistons, so it would normally be impossible that the arrangement could be applied to a tuning system with a larger number of divisions to the octave. This is however not so for a solenoid driven instrument, in which quite a large number of pistons could

different horns could be 'detached and reattached at will', and incorporated horizontal valves, light aluminium tubing, automatic water draining, and a novel mechanism in which depressing a valve-key *straightens* the air-stream instead of introducing bends and consequent deterioration of tone. See http://www.moffatt.demon.co.uk/horn/vhorn.html.

[308] Arnold Myers, 'Design, technology and manufacture since 1800', in *Cambridge Companion to Brass Instruments*, edited T. Herbert & J. Wallace, Cambridge University Press, 1997, p. 127; Philip Bate, *The Trumpet and the Trombone*, Ernest Benn, London, 1978, pp. 185-6.

[309] Although stepper motors can be compact they are also heavy; and they would need to be completely silent, which is not always the case.

[310] For an explanation of this system see: Arnold Myers, 'Design, technology and manufacture since 1800', *Cambridge Companion to Brass Instruments*, edited T. Herbert & J. Wallace, Cambridge University Press, 1997, pp. 129-30. A diagrammatic illustration of Sax's trumpet using a six-valve independent system can be seen in Philip Bate, *The Trumpet and the Trombone*, Ernest Benn, London, 1978, p. 185.

be controlled by relatively few electronic keys working in combinative logic. No compensating system would be required, and the basic simplicity of the system suggests considerable potential.

A self-correcting pitch ‘feedback’ mechanism was described for woodwind (pp. 73 - 74), and it is conceivable that an analogous approach might be used for brass.[311] The pitch monitor would send information to a self-adjusting slide. A good degree of co-operation between the ear, the physical acoustics of the instrument, and the software controlling the slide, would perhaps make an extremely accurate ATS brass instrument possible. Again, if this was successful, it might also be applied to create an instrument capable of multiple ATS; it would be particularly appropriate for achieving strict temperament, or strict JI, and especially so where the number of divisions of the octave is large because the mean corrections are smaller. The provisos mentioned in reference to woodwind are of course also appropriate to brass. Likewise the suggestion of creating ‘networked’ brass sections, to which various parameters of global adaptive or fixed pitch control might also be applied.

Lastly, as an instrument of variable intonation, the trombone would not appear to need ‘microtonal’ redesign. It is difficult to assess whether it would be practical or useful to create an electronic slide trombone, in which the positioning of the slide is controlled by electronic keys, giving extremely accurate length definition on the slide, and again, programmable for any tuning system.

[311] The recent Yamaha ‘*Silent Brass*’ system for trumpet, trombone, French horn and flugelhorn, comprises a ‘pickup mute’, digital processor and earphones. The mute is inserted into the bell and damps the acoustic sound while transforming the intended acoustic signal into digital audio - the player can therefore practice without disturbing the neighbours, or play along with an electronic source such as CD or MIDI, mixing the sounds on the headphones (or via loudspeakers). Unerringly accurate multiple ATS acoustic-electronic brass instruments might therefore be made by combining technology along these lines with an adapted version of the *Antares Auto-Tune* (see note 279).

Bowed Strings

Virtual Frets - visual and tactile

Aside from special aural training, special fingering techniques, and providing various forms of pitch reference, there are a variety of ways in which adaptations to the instrument itself will help the bowed string instrument player to project an ATS reliably. Some of these are familiar temporary adaptations, such as marking '*visual* (virtual) frets' on the fingerboard, which help to locate, not necessarily exactly, where the string should be stopped. Partch did this by embedding 'brads' in the (cello) fingerboard of his 'adapted viola';[312] more commonly, performers have used sticky tape, sometimes of different colours, to wrap around the neck or fingerboard; chalk (and tippex) are also used.

For radical ATS visual frets can be helpful, but in the long run they are perhaps dispensable. At best they help the performer to feel confident they are playing what is intended; at worst, they can distract a player from good intonation - players do not want to look at the fingerboard when performing. Visual frets are of most help during the process of learning a new work.

It would therefore be useful to research the possibilities of '*tactile* (virtual) frets'. A number of approaches seem feasible, and each of the following examples might be implemented either on existing instruments, using a non-permanent 'sleeve' attached over the fingerboard, or for new instruments, using either a permanent or an interchangeable fingerboard.[313] Sleeves, made of very thin but suitably hard and comfortable material, and interchangeable fingerboards, would both have the advantage of affording interchangeable tuning systems, but would in any case need to be tailored for a given tuning system for individual instruments.

'Tactile frets' might take various forms: (i) very slight protrusions *between* adjacent strings might be made along the length of the surface of the sleeve, so that the player would easily be able to feel where the pitches of any given system lie, but without the protrusions adversely affecting sound production in any way; (ii) very fine cuts could be made directly (not very deep) into the fingerboard, which would be wide enough to feel but not so wide as to have any effect on the sound of *glissandi* or *portamenti*, nor so wide as to have the effect of damping were the string to be stopped directly on a cut. The placement of cuts would not be in the expected acoustic position: the point at which the finger stops the string, and the point at which the cut in the fingerboard is sensed by the finger will not match exactly - experiments would be needed to 'fine tune' this relationship.[314]

In these cases, intonation would not normally be strictly determined by the tactile frets, unless composer or performer intend it. Yet if something along these lines were to prove successful, they would help a player feel confident with new tunings and intonations. In addition, tactile frets would be less likely to distract the player than visual ones. As was suggested regarding to the design of new woodwind, this kind of adaptation would not merely facilitate accurate pitching, but importantly, would help to *confirm* an alternative intervallic system in the player's ear.

The 'Undulating Fingerboard'

Another means of aiding the performer, and perhaps of revolutionising the instrument, is that suggested by Lewis Jones, who has experimented with the idea of an 'undulating fingerboard'. In this case the fingerboard is sculpted in a series of asymmetrical waves running from the nut towards the bridge, and the series of distances from the peak of each wave to the bridge effects the desired tuning system.[315] The waves are asymmetrical in the sense that the peaks would normally become progressively closer from nut to bridge, and that the slope of each wave would be gentle on the nut-side, but slightly steeper on the bridge-side. This is shown (very roughly) in the diagram below.

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[312] See the illustration in Partch, op. cit.. p. 199.
[313] Interchangeable fretboards for guitars are made by Mark Rankin. Reference XXX.
[314] Thanks to Lewis Jones for comments on this.
[315] For an equal-tempered system, the lateral direction of the waves would run at 90 degrees to the edge of the fingerboard. Although the violin is an instrument of variable intonation, it might conceivably be advantageous for the wave crests to be staggered across the fingerboard, as on guitar fretted for JI.

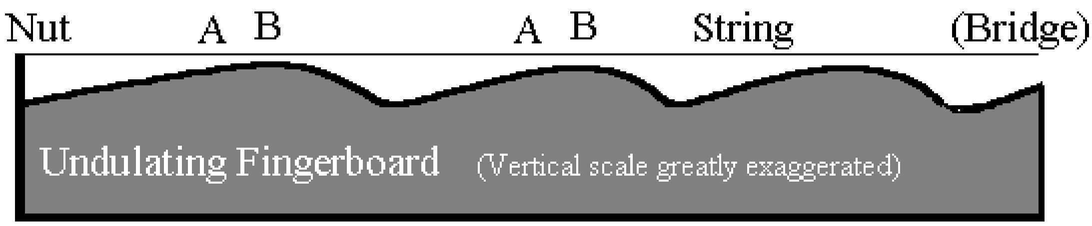

Figure 11 : Undulating fingerboard for string instruments (after a drawing by Lewis Jones).

The wave in the fingerboard would of course be much more subtle than as shown, but could be felt clearly under the player's fingers. Stopping the string at the peak of a wave (B) will provide the required pitch so long as the descending angle of the wave towards the bridge is great enough not to interfere with the vibration of the string. Alternatively, when the string is stopped at (A) on the side of the wave towards the nut, then the string will *also* sound the same pitch (or one slightly higher), because in this situation the peak of the wave acts as a 'fret'. As Jones says:

> Mode (A) playing, in which the 'fret' [wave-peak] defines the pitch, would have considerable didactic value in demonstrating the required intervals, and would suit music requiring a clear, non-vibrato tone. It would also act as a pitch safety-net in rapid passages in which vibrato might not be important (or possible), and in passages where the player has the dexterity to play the notes but with less than perfect intonation. Mode (B) playing would sound very much like normal violin playing. The wave-peaks would not be so high as to make the experience of stopping *on* them uncomfortable. Gross misplacement of the fingertip would lead to the production of one of the primary tones of the scale, according to mode (A). [316]

Where necessary, therefore, a player will be able to secure a degree of intonational accuracy by tending toward mode (A) rather than mode (B) playing. As Jones has pointed out, the drawback to this scheme is the loss of glissandi between intervals greater than a scale step, 'which would be replaced by an effect mid-way between violin and guitar glissandi, in which sliding alternates with stepped pitch-change'. It remains to be seen whether players and composers could accept this loss, and (possibly) a reduction in string players' intonational freedom. But as Jones points out, there may be considerable benefit to be gained from the undulating fingerboard in terms of learning a new ATS. It might also be thought that with an undulating fingerboard it would be necessary to ensure that the strings do not lose their tuning - otherwise every 'fret' will mis-tune - but this is equally true regarding the open strings of a conventional instrument. It is likely for an undulating fingerboard the pegs may need to fit slighlty more securely.

The 'undulating fingerboard' is important because it may overcome one of the basic stumbling blocks of acoustic 'microtonality'. In this case, the smallest discrete unit interval for string writing would be determined by the smallest physical undulation that is practical, and its registral position. Further research could establish the practicality and limits of this. However, performances of Ezra Sims' music (by the cellist Ted Mook, for example), and that of the jazz violinist Matt Maneri,[317] for example, both ostensibly employing 72 octave divisions, suggest that very fine division systems are possible on conventional instruments[318] - at least for the highest calibre players - yet a fingerboard with '72 undulations to the octave' would certainly be unworkable. However, in addition to its didactic value, the undulating fingerboard could perhaps enable the realisation of the more 'awkward' smaller-division tempered systems, and bring some radical ATS within reach of players of sub-orchestral level.

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[316] The undulating fingerboard was explained to me in a private communication from Lewis Jones, March 1998.

[317] Maneri plays a custom-built six-string electric violin - whether this instrument has any special feature which aids the performance of very fine octave divisions, I have been unable to find out.

[318] As mentioned previously, there is a movement in Holland, following the work of Fokker particularly, of performing music for 31-octvae divisions. Brian Mclaren mentions the violin duo Bouw Lemkes and Jeanne Vos from Utrecht who specialise in playing '31-tone' music (and other 31-tone ensembles elsewhere). Clearly, these pockets of activity indicate the feasibility of performing ATS with many octave divisions on conventional versions of what are (normally) the smallest of the bowed string family. B. Mclaren, 'A brief history of microtonality in the twentieth century', *Xenharmonikôn* 17, Spring 1998, pp. 87, 89, etc.

Logical Strings?

It seems unclear whether the self-correcting pitch-feedback idea described for woodwind (pp. 73 - 74) and brass (pp. 82 - 83) could be applied to strings. Suppose four pitch sensors were placed within the body of the violin, such that each sensor 'listens' only to one string; and suppose that the strings were not held by pegs, but wrapped around the nut into a cavity in the neck. The pitch sensors transmit to an extremely acute corrective mechanism operating independently on each string - varying tension directly, or via a lateral pressure screw. It is, I think, more difficult to imagine exactly how this kind of mechanism could be made beneficial for instruments of entirely variable intonation, unless we assume (again) that corrections are instantaneous, and effect very small corrections which are at the same time significant. Success would seem to depend on whether a player's intonation was very accurate in the first place: clearly, a new learning process would be implied by such instruments. However, if such an idea could be made to work, it might provide an aid in achieving 'multiple' strict temperaments or strict JI systems for large *n*-division systems.[319] Similarly, as for wind and brass, an ensemble of 'logical feedback' strings could be 'networked' so that pitch correction operated on a complete string section. This might be set up in many different ways - for example, as previously suggested, the ensemble could take its pitch from the basses, or from a fixed reference; equally, composers could explore setting different reference pitches for different elements of an ensemble.

The open strings of a 'logical' string instrument would tune automatically - this would be useful for compositions employing ATS, which implicitly require scordatura if the fifth is significantly altered.

Scordatura and Resonance

Research has shown (broadly speaking) that the finest Italian violins (*Stradivari, Guaneri* etc.) are those which provide optimal resonance throughout the widest range, and do not especially privilege the open strings or other particular notes.[320] It would seem, therefore, that little would be gained by attempting to maximise body resonances at the specific pitches of a given alternative scale. As a related issue, scordatura for ATS are not uncommon in the avant-garde[321] and in contemporary pieces they are normally used to secure specific tunings on the open strings, and as a simple way of obtaining a range of alternatively tuned chords. However, if a radical tuning system were adopted in which the 'fifth' differed significantly from just (or if a 'standard' string tuning were based on another interval altogether), then compensation in the setup of body resonances would almost certainly be necessary. Since the change in string tension and the change of pitch of the open string will have an audible effect on the resonance and timbre of the instrument, the question would therefore arise as to exactly what and how compensation might be achieved.

Amongst the string family the violin is often thought to have the most perfect tone. This is usually explained in terms of the relative proportions of the size of the strings to the violin body (as opposed to the same proportions in the viola, cello or double bass, each of which differ from the other). Carleen Hutchins, in conjunction with *Catgut Acoustical Society*, has designed and built a new family of eight bowed string instruments, in which each member is modelled on the scaling and resonances of the violin. About 100 'octet instruments' have been completed so far.[322] The eight instruments range from a 'contrabass violin' (whose lowest note is the same as a normal double bass but whose body is much larger) to a treble violin (tuned an octave above the normal violin).[323] Perhaps another maker will in future attempt a similar project, building a family of strings modelled

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319 Thanks to Joseph Sanger for comments on this.

320 See Fletcher and Rossing, op. cit., pp. 272-7. On the other hand Beament has argued that 'there appear to be no characterising differences between the perceived sound from well made orthodox [bowed string] instruments of any age when played by a skilled player.' James Beament, *The Violin Explained - Components, Mechanism, Sound*, Clarendon Press, Oxford, 1997, p. 90. However, this controversy does not affect our present purpose.

321 Nono and Scelsi, for example, both called extensively for microtonal scordatura. The use of scordatura was familiar in the Baroque period, and perhaps used most famously in the '*Mystery Sonatas*' by Heinrich Biber, in which the intention was to give each Sonata its own timbral character. Thanks to Mieko Kanno for comments on this.

322 Of these, there are six full sets in prestigious institutions - The Metropolitan Museum of Art, New York City; Music Museet, Stockholm, Sweden; University of Edinburgh, Scotland; Shrine to Music Museum, South Dakota; St. Petersburg Conservatory, Russia, and Catgut Acoustical Society, New Jersey. See: Catgut Acoustical Society Homepage at http://www.marymt.edu/~cas/ .

323 C.M Hutchins, 'Founding a family of fiddles', *Physics Today*, 20, 1967, pp. 23-7; C.M Hutchins, 'The new violin family', in *Sound Generation in Winds, Strings, Computers*, pp. 182-203, Royal Swedish Academy of Music, Stockholm. See also: Fletcher and Rossing,

upon the scaling and resonances of the viola or cello, or some other sound ideal. Possibly, the kind of research which lies behind creating a homogeneous timbre for a family of instruments will be of importance for the 'coevolution' of strings and alternative tuning systems. The reason is not so much to correlate tuning and timbre, but because new refinements in pitch complement new refinements of tone (by which I mean the shading of timbre).[324]

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op. cit., pp. 279-83; James Beament, *The Violin Explained - Components, Mechanism, Sound*, Clarendon Press, Oxford, 1997, pp. 87-8; Bart Hopkin, *Gravichords, Whirlies and Pyrophones*, Ellipsis Arts, 1996, pp. 52-3.

[324] In the UK various works have recently be written in a contemporary idiom for an ensemble of viols - particularly for the ensemble *Fretwork*. It is therefore worth mentioning that Paul Waters, a student at London Guildhall University, has successfully fretted a tenor viol for 31-ET. 'The viol as is normally fretted has 7 frets tied to the neck, the last of which marks out the interval of a fifth. To allow the same range of notes in 31-ET requires that 18 frets be tied to the neck with each fret marking out an interval of 38.71 cents. The frets that approximate the just intervals are marked using a bright coloured dye to facilitate the playing of the more common intervals.' Some multitrack recordings of the 31-ET music of Nicola Vincentino were made using the instrument. Paul Waters, *A method of fretting the viol in 31 note equal temperament. Including performance of music composed by Nicola Vincentino.* Project completed as part of B.Sc., LGU, May 1997. Would composers also be interested in composing for fretted *violins*? There are, for example, attractions to a slightly stricter use of temperament in string playing, although, as Bart Hopkin has pointed out, frets change the character of the instrument by significantly reducing damping. But it is controversial whether applying movable gut frets to the violin, viola etc., could prove useful as an aid to learning an ATS.

The Piano

The piano has a special place in relation to *alternatives* to 12-ET because, despite its stretched octaves, it was the ‘soul of 12-ET’, and the most influential instrument of fixed intonation in the 19th and 20th Centuries. It is unlikely that a *new* piano could play a similar role in future - particularly because of the emphasis on an ‘electroacoustic’ conception of sound in much contemporary (instrumental) music, and because electroacoustic technology itself will continue to become increasingly influential. In any case, the cost of the adaptations described below would be prohibitive, especially in comparison with electronic keyboards. Nevertheless, as I argue in what follows, a new piano could have an important role amongst 21st Century acoustic instruments.

For obvious reasons we seldom hear any of the pianos which have been specially built for ATS in concert,[325] although works for re-tuned piano(s) are less rare. In *Conclusions about New Instruments* (p. 105) it is suggested that new instruments commissioned using public funds should initially be made available to soloists and chamber groups on a hire or lending basis. For a new piano, cost and size would place restrictions on this. In addition, therefore, MIDI mother-keyboards with an identical keyboard layout to a new acoustic instrument would be needed for performers to practice on. Experimental composers have been calling for alternative MIDI keyboards for some years, and it’s a shame how conservative companies such as *Yamaha* or *Roland* are in this area, especially since alternative keyboard layouts would be ‘cool’ amongst pop musicians.[326] Perhaps a new Centre and the existence of other new acoustic instruments might have some impact in persuading manufacturers to produce alternative keyboards.[327] There is, however, a MIDI controller keyboard available (or soon to be available) in the USA called the *Microzone,* based on a design by Ervin Wilson, which is described as:

> a 1990’s rendition of the Bosanquet generalised keyboard…[featuring]…768 keys in an 8 x 96 [hexagonal] array.[328]

Given limitations of funding, it might also make sense to consider the possibility of making a ‘dual-’ or ‘multi-tuning’ acoustic piano. But the cost of research in this direction, and the likelihood of unacceptable compromises in sound quality perhaps stand against this. A very considerable number of factors - from how heavy the instrument will be, to how the keyboard action adjoins the strings, and the time required to tune the

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[325] Hugh Davies lists nine specially built pianos (two unfinished) and two pianinos built between 1892 and 1931, including those built for Hans Barth, Hába, Arthur Lourié, and Wyschnegradsky. Barth’s 3-manual piano was built by George L. Weitz of Baldwin (New York, 1928). Hába commissioned a 3-manual piano built by Grotrian-Steinweg (1924), a 2-manual built by August Förster (1924), and a 3-manual also built by Förster (1925). Förster had patented a quarter-tone piano in 1924 (see note 261) - typically, in the 3-manual instruments the length of the keys becomes smaller the further they are from the player - the third and furthest manual is included to allow alternative fingerings. Lourié’s quarter-tone piano was begun by Maison Diederichs (in St. Petersburg?, c. 1913-14) but remained unfinished. Wyschnegradsky commissioned a 2-manual ‘pneumatic action’ quarter-tone piano which (according to Franck Jedrzejewski) was completed by Pleyel in 1921; dissatisfied, Wyschnegradsky returned this instrument to Pleyel in 1922, and later commissioned a 3-manual quarter-tone piano from Grotrian-Steinweg (1924), and a 2-manual from Förster (1928), both of which were completed. (I am unclear about the meaning of one of Davies’ entries, where both Hába and Wyschnegradsky are listed as the inventors - or commissioners, or owners - of a single piano: or was this a *model* of piano?). Davies also mentions Carrillo’s *pianos metamorfoseadores* - a series of ‘microtonal uprights with conventional keyboards, each in a different tuning from 1/3 to 1/16 tones’ (that is, 18- to 96-ET). The latter instrument apparently had 97 rather than 88 keys, and spanned one octave. Davies mentions that these pianos were planned as early as 1927; Brian Mclaren reports the plans were patented in 1940. Davies says they were built between 1957-8 (by Carl Sauter Pianofortefabrik); Mclaren states these pianos were built over a period of 10 years from 1947 onwards (referencing Carrillo’s 1962 history of ‘Sonido 13’). However, both writers report that an 18-ET *grand* piano was built for Carrillo in 1947. Davies also mentions the *Novares* - ‘pianos that sound less percussive than normal and are tuned in such divisions of the octave as 14, 15, 19, 22, 31 and 53…’, built by Carrillo’s student Augusto Novaro (from the 1930s onwards). See: Hugh Davies, ‘Microtonal instruments’, *The New Grove Dictionary of Musical Instruments,* MacMillan, and B. Mclaren, ‘A brief history of microtonality in the twentieth century’, *Xenharmonikôn* 17, Spring 1998, pp. 61, 64, 71 etc. In addition, Anthony Baines mentions a piano built by V. Odoyevsky in 1864, having 31 keys to the octave in a manner resembling Trasuntino’s 1606 keyboard (see below), which is now held in the State Central Museum of Musical Culture in Moscow. Anthony Baines, ‘Keyboard’, *The Oxford Companion to Musical Instruments,* Oxford University Press, 1992. Thanks also to Franck Jedrzejewski for information on Wyschnegradsky ‘s pianos - see for example http://music.dartmouth.edu/~franck/ - which also contains some relevant photographs.

[326] Alternative layouts keyboards are potentially very useful for pop music, although for different reasons to those advocated here.

[327] For discussion of relevant electronic applications in this area, see Section 6.

[328] From the Starr Labs website catalogue at http://www.catalog.com/starrlab/uzone.htm. Erv Wilson has confirmed that the latest version of the *Microzone* will accommodate 108 keys per octave - personal communication, 1998. The keyboard is briefly described below - pp. 110- 111. For more on the Bosanquet generalised keyboard, see pp. 95 - 96.

instrument - would influence the design of a new instrument for ATS. These brief remarks deal only with the following issues:

- the interrelation between tuning system, the stretching of the scale, and piano timbre;
- the feasibility of large-number divisions of the octave;
- the keyboard layout - 'acoustic presentation' and 'transpositional invariance';
- weight, size, stringing - and the relation between keyboard and hammer action;
- the 'Logical Piano';
- new technologies for piano tuning.

Piano Timbre and a New Piano

Aside from the organ, there is no other acoustic instrument which equals the conventional piano in terms of harmonic power. This aspect should, I believe, be emphasised in a new instrument, particularly in the relationship between tuning system and timbre (and in the keyboard layout). The piano may be an example of the 'coevolution' of tuning and timbre - for it seems possible that in the evolution from fortepiano to pianoforte, timbre and tuning may have 'coevolved'. Tuning has evolved through various unequal temperaments to equal-temperament, and from not-much-stretched to more-stretched; timbre has evolved from the soft but somewhat rough-edged, short sustain but slightly jangly sound of early pianos, to the brilliance, purity, power and sustain of the modern Steinway. Much work would be needed to confirm whether this hypothesis is true, and which physical, acoustic and musical factors have had an important influence on the process. But if the piano remains a vital element of the acoustic instrumentarium, then the overall agreement between tuning system (including 'stretched-ness') and piano timbre should probably remain an important factor regarding preferred systems of tuning.[329]

Due to stiffness, piano strings vibrate in a manner which combines the behaviours of an ideal string and a bar (or rod). Consequently, the frequencies of the partials of the tone of each string are 'stretched' relative to the harmonic series, and each partial is in turn further from harmonic than the one immediately below it. The following example is given by Benade, showing the discrepancies between the perfect harmonic series of an organ tone, and the increasingly stretched series of partials of a piano tone (in this case 'middle C'):

Component	***1***	***2***	***3***	***4***	***5***	***6***
Piano String (Hz)	261.63	523.51	785.91	1049.23	1313.23	1578.68
Pipe Organ (Hz)	261.63	523.26	784.89	1046.52	1308.15	1569.78
Discrepancy (Hz)	0.00	0.25	1.02	2.71	5.08	8.90
Discrepancy (cents)	0.00	0.8	2.3	4.5	6.8	9.8

Figure 12 : Comparison of inharmonic frequency components of a piano string with harmonic components of a pipe organ tone of the fundamental frequency. (After A. H. Benade, *Fundamentals of musical acoustics*, Dover, 1990, p. 315).

As is well known, if a conventional piano is to be tuned for maximum consonance it cannot be tuned to 'just intonation' in terms of 'just ratios'. Discussing this same example, Doty shows that, with this piano spectrum and corresponding spectra for other strings, there are at least three conflicting settings to which one might tune a perfect fifth, but that it is 'impossible to tune this or other simple intervals by 'zero-beating' its defining pair of partials'.[330]

[329] Ivor Darreg has argued , for example, that 'The timbre of the piano evolved during the 19th Century in such a direction as to make it progressively worse for the intervals involving quarter-tones… trial and error… brought the hammer-striking point to about 1/7th the vibrating string length, which almost eliminates the seventh harmonic. The intervals having the number 7 in their ratios… are thus deprived of their definition…[…]… The piano is just about the worst possible instrument for quarter-tone music… The idiosyncrasies of the piano have operated to the quite undeserved detriment of the quarter-tone system…'. Ivor Darreg, 'The quarter-tone question revisited', *Xenharmonikôn*, No. 2, 1974, pp. 6-7. The detail of Darreg's argument is flawed because septimal ratios do not equate to the intervals of 24-ET (see note 33 and APPENDIX II Table 24), and, as Darreg himself points out, hammer striking points vary. But the gist of what he says - that the effectiveness of a tuning system depends upon the harmonic structure of instrumental sound, that contemporary piano sound is not ideally suited to 24-ET, and that this lack of suitability has regrettably had an adverse effect on the development of microtonality in the 20th Century - seems correct, and remarkably prescient.

[330] David Doty, op. cit., pp. 61-63.

The stretching of the scale on a new instrument is one of a number aspects of the tuning/timbre relationship to consider, and to adapt the timbre of a new piano for a particular scale many design elements might be taken into account. It is unlikely the relationship could be exact because spectra are variable across the instrument's range, are relative to the stretching of each single string, and vary with dynamics.[331] The largest grand pianos require noticeably less octave-stretching than smaller instruments because longer strings are less inharmonic. But piano strings offer a number of parameters for timbral adaptation, including scaling (relative string lengths) and string gauges, tension, material and the number of strings per unison, and the exact tuning of the strings comprising a unison. As Sethares has pointed out,

> if the contour of the string is changed, or if the density of the string is not uniform, or if the string is weighted at strategic points, then the partials can deviate significantly from harmonicity. Devising a method for readily specifying the kinds of physical manipulations that correspond to useful spectral deviations is an important first step.[332]

Similarly, if very regular and specific fluctuation of the diameter or density were introduced into the windings of bass (or higher?) strings, perhaps the resulting distribution of weight in the string might be made to affect string harmonics in a predictable (and musically interesting) way. Equally, many aspects of the piano action influence timbre, including the hardness, material and voicing of the hammers, the strike point on the string (the proportions into which the strike point divides the string), the relative mass of hammer and string, shank thickness (more generally their flexibility), and the relationship between the keyboard design and hammer velocity. The soundboard, case and frame each affect timbre too, but it should be emphasised that most of these factors affect the relative *amplitude* of different partials, rather than their *frequencies*.[333]

With regard to the number of strings per unison, and the exact tuning of each string, David Doty has pointed out that:

> It is… impossible to keep three strings tuned in a perfect unison for any appreciable period. In practice, however, piano tuners do not normally even attempt such a unison. The strings for a given note on a piano are typically detuned over a range from two to eight cents. This detuning is part of the characteristic sound of the piano. It not only adds a chorus effect to the timbre of the instrument, but also has a significant effect on the amplitude envelope of the sound. Because the two or three strings of a piano tone are not perfectly in tune, their phase relationship is in a state of constant variation. Hence, when one string is pushing down on the bridge, another may be pulling up. Energy is cyclically transferred from the strings to the bridge and back to the strings. As a result, the transfer of energy from the strings to the bridge and soundboard is considerably slower than for an instrument with only one string per note, and the decay time of the tone is lengthened accordingly.[334]

Is it the case, then, that the typical combined tuning setting of the two or three strings in a unison has itself evolved so that it is 'adapted' to 12-ET? Would it be possible to control the *exact* 'per string tuning' to advantage in some ATS? As discussed below, in respect of the piano tuning technologies available today (p. 97), achieving an extremely exact tuning is less of an obstacle than it used to be, although the question remains as to whether an instrument could be made to hold a tuning long enough for this to be worthwhile.

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[331] For a technical discussion of attack transients and the general spectral make-up of piano timbre see R. D. Weyer, 'Time-varying Amplitude-Frequency-Structures in the Attack Transients of Piano and Harpsichord Sounds - II', *Acoustica*, Vol. 36., No. 4, 1976-7, pp. 241-258; see also, Clarence Barlow, *Bus Journey to Parametron - all about Çogluotobüsisletmesi*, Feedback Papers 21-23, Feedback-Studio-Verlag, Cologne, 1980, pp. 64-70.

[332] William A. Sethares, *Tuning Timbre, Spectrum, Scale*, Springer Verlag, 1977, p. 277.

[333] For a lucid and up to date account of piano acoustics, see: *Five Lectures on the Acoustics of the Piano*, ed. Anders Askenfelt, Royal Swedish Academy of Music, Stockholm, 1990; see also Stephen Birkett, *Technical Foundations for Analysis of Pianos*, at www.ptg.org.

[334] David Doty, op. cit., p. 61. Doty also comments that: 'Most Just Intonation pianists avoid the tuning problems produced by multiple strings by using only a single string per note. They either remove the additional strings, damp then using tuner's mutes…, or modify the action so that the *una corda* pedal causes the hammers to strike only a single string per course. A single-strung piano has a sound that is quite distinct from that of a normal instrument… the timbre is more string-like in character than that of a normal piano, with a more incisive attack, somewhat akin to the tone of a hammered dulcimer'.

The next time you hear a Schubert song with a slow moving chordal piano accompaniment, reflect for a moment on the sound of the piano. Suppose this is a first class instrument, and has been very well tuned. What is sometimes astonishing is the fusion of piano harmony, the illusion of hearing a chord as if it were *one tone*. This (somewhat intangible) experience is not uncommon, but it is remarkable that it occurs when listening to an instrument tuned in '12-ET'. This is precisely because the tuning of the piano has been 'adapted' to the spectra of its tones. This would not occur (in a comparable way) on a modern instrument without reasonably accurate octave stretching relative to the characteristics of the individual piano. [335]

The point here is that it seems important that a new piano (to continue to have an important role in the instrumentarium) is able to provide both a high degree of harmonic fusion, and of melodic 'directness'. This is because a new, combined system of instruments and tuning should give composers the means to say *one* thing at a time. When composers want to say two or more things simultaneously, they can find ways of doing so, but, from this perspective, singular building blocks are important.

The Keyboard Layout and Large-Number Divisions of the Octave

It is reasonable to assume that no pianist or composer will accept easily, on a future instrument, a reduction in the interval stretch which can be made with one hand. To be able to stretch an octave seems a minimum requirement, but a greater stretch would certainly be preferable.[336] At the same time, keys cannot be so close together that the fingers are cramped when two adjacent keys are played together; nor can the keyboard be altered so radically that the usefulness of the thumbs is reduced (as in a typewriter-style keyboard, for example). It could be reasonably simple for electronic keyboards to be adapted for smaller hands, less so for acoustic instruments.[337] If a new piano were built to realise a unique tuning system, then the keyboard layout should be optimised for ease of performance and to correspond to the acoustic (harmonic/melodic) properties of that particular system. As Douglas Keislar concludes in his survey of microtonal keyboards:

> For tuning systems with many notes per octave, or for those in which extensive transposition is possible, an equidistant key layout such as the Bosanquet generalised keyboard [see below] is definitely preferable. In cases where the user plans to explore only one specific tuning system, a special-purpose keyboard highlighting properties of that system may be more desirable…[338]

The traditional keyboard layout evolved from the diatonic keyboard which was the norm in the Middle Ages. Thus seven diatonic keys, in one layer, were interspersed with five further keys, in a higher layer. This layout first appeared in 1361 on the 'Halberstadt' organ, and it was not long before it was adopted as a standard, although, of course, tuning itself was not standardised.[339] Later, to achieve greater purity of intonation, split keys were sometimes introduced, in which one or more keys in the upper layer was divided, either length-wise, or split into front and back portions. Some early experiments were astonishingly sophisticated:

> Enharmonic keyboard designs were carried further by sixteenth- and seventeenth-century theorists and composers including Gioseffo Zarlino (1558), Nicola Vincentino (1555), Franciso Salinas (1577), and Marin Mersenne (1637). One of the most important instruments was Vincentino's "*Archicembalo*", whose tuning was apparently very close to 31-tone equal temperament… In addition to splitting the black keys, these designs often called for split white keys, additional black keys, or extra manuals. Note that these techniques of splitting keys and

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[335] These are of course qualitative judgements, since harmonic fusion can occur even when a piano is out of tune. I have noticed the effect especially in Schubert, which may have to do with voice leading and chordal spacing, but it can occur in any piano music.

[336] The octave span of the conventional modern piano keyboard is 16.5 cm, that of electronic piano keyboards is normally 16.3 cm. Keys cannot be narrower than fingers, unless they are buttons, and are arranged in a two-dimensional array. Wilson has suggested a useful guide: if one is designing a keyboard for one's own hand, make the distance between adjacent whole tones (the idea might be adapted in different ways) the same as the average distance between the knuckles. Ervin Wilson, 'The Bosanquetian 7-rank keyboard after Poole and Brown', *Xenharmonikôn,* No. 1, Spring 1974, p. 3.

[337] It seems hard to believe, for example, that economic reasons alone have determined that pianos with narrower keyboard octaves are not available for women: surely there are as many women pianists as men?

[338] Douglas Keislar, 'History and principles of microtonal keyboards', *Computer Music Journal*, Vol. 11, No. 1, Spring 1987, p. 27.

[339] Keislar, ibid., p. 19.

> inserting new ones, without disrupting the established layout, represent a continuation of the process of accretion that led to the standard keyboard.[340]

Nicolas Meeus describes Vincentino's Archicembalo as having 35 keys to the octave, and comments that Vincentino

> appears to have been one of the very few Renaissance or Baroque theorists to realise that the best purpose of an enharmonic keyboard would be the playing of microtones, and some of his compositions use the quarter-tone as a melodic interval.[341]

Another keyboard built by Vito Trasuntino (1606), which is today preserved in Bologna, has 31 divisions to the octave. '[E]ach regular accidental key is divided into four parts, and additional keys divided into two are inserted between E and F and between B and C.'[342] A sketch of this layout is given in APPENDIX VI (b).

Meeus reports that in 1573 'Zarlino mentioned a harpsichord made by Domenico Pesarese with raised keys inserted between E and F and between C and D, as well as the five regular raised keys split into two'.[343] This scheme, in which seven extra 'black' keys are added to the standard keyboard layout (with the extra key between B and C, not C and D as Meeus states), has been commonly advocated for 19 division systems, an example of which can be found in Joseph Yasser's book *A Theory of Evolving Tonality*.[344] A diagram of the simplest layout given by Yasser is shown in APPENDIX VI (a). The scale reads:

C, C#, Db, D, D#, Eb, E, E# (or Fb), F, F#, Gb, G, G#, Ab, A, A#, Bb, B, B# (or Cb).

As can be seen from APPENDIX II Table 19, the white notes in this arrangement form an acceptable major scale, and in general much of the physical and aural geography of the traditional keyboard is retained. In Yasser's (and other) versions of this layout the adjacent black keys (C# and Db, D# and Eb, etc.) are shown as being of equal height and length. This poses problems to the player, however, and a more sensible arrangement is to have (say) the C# key longer than Db, but the Db raised higher than C#; D# longer than Eb, but Eb raised higher, and so on. Each black key is therefore placed so that it does not cause obstruction (nor are two black keys in danger of being struck at once), and each is relatively accessible from either direction on the keyboard. In addition, movement of the thumb is not constricted, and the arrangement provides a reasonable compromise between familiarity and ergonomics.

In its simpler aspects, the acoustic presentation of the 'accretional' keyboard is more or less self evident and keyboard players can orient themselves to the placement and patterns of familiar intervals without great difficulty. However, with an accretional layout, the more unfamiliar the harmonic territory, the more the asymmetry (of the physical patterns of scales and chords) of the conventional layout become exaggerated. Ezra Sims, for example, has drawn up some keyboard designs which show the feasibility, but also the difficulties of such systems. In one 36 division scheme the 7-white/5-black layout remains unchanged at the front of the keyboard, but the black keys are much shortened, and behind each of them are placed two similar short 'black' keys; further behind these are two more rows of raised keys, completing six divisions to the whole-tone. A drawing of this layout is shown in APPENDIX VI (a).[345] There are thus six tiers of keys, each key sitting slightly to the right of the previous key in terms of ascending pitch. However, as the scale ascends, the order of pitch (in terms of *tiers*) is 1, 6, 3, 2, 4, 5, where '1' refers to the 'white keys' at the front of the keyboard, and '6' refers to the tier furthest to the back - this is clearly not obvious to the performer. It would not be impossible for such a layout to become familiar and useful, but it is far from ideal.

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[340] Keislar, ibid., p. 19. Zarlino's 19 division harpsichord was, according to Groves, built in 1548 (by Dominicus Pisaurensis). Edwin M. Ripin et al., *Early Keyboard Instruments* - (*The New Grove musical instrument series*), Macmillan,1989, p. 23.
[341] Nicolas Meeus, 'Enharmonic Keyboard', *The New Grove Dictionary of Musical Instruments,* MacMillan. Whether in 31 or 35-ET, the term 'quarter-tone' is being used in the loose sense adopted by Barbour (see note 31).
[342] Meeus, ibid. A clear photograph of this keyboard can be found in Edwin M. Ripin et al., *Early Keyboard Instruments* - (*The New Grove musical instrument series*), Macmillan,1989, p. 24.
[343] Meeus, ibid. Should this read 'B and C' rather than 'C and D' Is this the same keyboard as mentioned above, or a different one?
[344] Joseph Yasser, *A Theory of Evolving Tonality*, Da Capo Press, New York, 1975, p. XXX??. A similar diagram can be found in Bart Hopkin, *Musical Instrument Design - Practical Information for Instrument Design*, See Sharp Press, Tucson, Arizona, 1996, p. 27.
[345] This illustration is copied from an unpublished design drawn by Sims, and used with the composer's permission.

As Keislar suggests, a keyboard for a dual- or multi-tuning instrument must be laid out to accommodate the performer's needs for multiple ATS. The layout of Sims' 36 division keyboard could easily be used for some smaller divisions (eg., for 19 divisions the front three rows could be used with the addition of keys 15 and 36 which might be slightly lengthened to resemble Yasser's keyboard); but other systems are less easily accommodated (eg., for 22 divisions the major third (above 'C') will fall on the fourth rank black note above 'D', rather than on 'E' itself). If a keyboard layout were intended for a unique system, the best layout is the one which affords the most effective application in the long-term. It may be argued that this is another reason for adopting an ATS standard.

Jankó Layout

An alternative keyboard layout patented in 1882 by Paul von Jankó (1856-1919) had moderate but short-lived success before the turn of the century.[346] A number of piano manufacturers (Blüthner, Kaps, Ibach, Broadwood, Decker Brothers) built pianos using the system,[347] and the keyboard layout could be applied to a piano with any normal action.[348] The keyboard provides six tiers of 'touch-pieces' which actually comprise two sets of keys, each of which connect to three (alternate) tiers - that is, for example, middle C can be played by depressing the middle C touch-piece on each of the three tiers, as can C#, and so on.[349] Tiers one, three and five sound the whole-tone scale from C natural; tiers two, four and six sound the whole-tone scale from C#. The keyboard covers the standard seven and a quarter octaves, acting on a single bank of strings.

> The advantage of this system is that all major scales are fingered alike, as are all minor, and an octave span is only 13 cm instead of the standard 16.5 cm. Huge arpeggios can be negotiated with barely any arm movement, by moving the hand up or down the tiers. The system is unique in that it compensates for the unequal length of the fingers.[350]

By partitioning 12-ET into two symmetrical sets, and by duplicating the tiers above and below, every interval on the keyboard can be taken with the same physical shape of the hand. The reduction in octave span means that a hand which will stretch an octave on a conventional piano would be able to stretch a tenth on the Jankó keyboard; stretching a ninth (conventionally) would be equivalent to stretching a major tenth or a sharp eleventh; and stretching a major tenth (conventionally) would be equivalent to stretching a sharp eleventh or sharp twelfth.[351] It is possible that the keys might be made slightly narrower, but almost certainly not enough to make much improvement on this.

Translating these stretches (respectively) to numbers of scale-steps shows that the Jankó layout would provide an equivalent keyboard of approximately 14-16, 16-18, or 18-20 scale degrees to the octave. A modified version of this layout might therefore be used to realise the even-numbered *n*-ET systems between 14-ET and 20-ET (depending on a 'hand size standard'). For odd-numbered *n*-ET systems in the same range, spatial/intervallic consistency would be compromised, but the latter could perhaps be compensated for by reverse colour-coding in each octave.

Bosanquet Generalised Keyboard

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[346] A description of this system may be found in Paul von Jankó, *Eine neue Klaviatur*, Wien, 1886 - but as yet I have been unable to obtain this volume.

[347] David Crombie, *Piano : Evolution Design and Performance*, Balafon, 1995, p. 68. Many other attempts to improve the piano layout might be relevant here, for example, the Adam (1901), Durand (1904), Kuba (1907) and Nordbo (1915) keyboards. The 'Clavier Hans' (1917) was also an adaptation of the Jankó design which employed only two tiers of keys.

[348] A working example of a Jankó piano is held in the National Museum in Washington D.C. Jeremy Siepman, *The Piano,* Everyman, 1996.

[349] A photograph of a Jankó piano can be seen under that entry in *The New Grove Dictionary of Musical Instruments,* MacMillan. Because each touch-piece was connected to two others, in its day the action was heavy. As Edwin M. Good comments: 'A very light but tough plastic key material might now serve better.' Encyclopaedia of the Piano, ed. Robert Palmieri, Garland, 1996, p. 186.

[350] Margaret Cranmer, 'Jankó (Paul von)', *The New Grove Dictionary of Musical Instruments,* MacMillan. This passage actually reads '18.5 cm' for the octave span. I assume this is a mistake.

[351] Conventionally, 7 whites = 16.5 cm; therefore 8 whites = 18.86 cm; 9 whites = 21.21 cm. On the Jankó keyboard 6 keys = 13 cm, therefore each key is 2.166 cms; the available stretch is therefore approximately 7.62, 8.7 or 9.8 Jankó keys on one tier.

In the appendices to Helmholtz's *On the Sensations of Tone,* Alexander J. Ellis describes a number of radical 19th Century keyboard designs, the most celebrated of which are Colin Brown's *Voice Harmonium*,[352] a organ keyboard design by Henry Ward Poole,[353] and R.H.M. Bosanquet's *Generalised Keyboard* (1876) and *Enharmonic Harmonium*.[354] While differing in various respects, the purpose of each of these keyboards was to obtain an instrument of fixed intonation which provided both extensive modulation and increased intervallic purity. In Bosanquet's keyboard the keys are placed rising diagonally from left to right and set in multiple tiers (3, 4 or 5 in the examples given by Helmholtz and Ellis). The keys are placed immediately behind one another, interspersed diagonally by the keys of an 'in-between' row. The layout is shown in APPENDIX VI (b). The design combines acoustic presentation with 'transpositional invariance' - that is, the ability

> to move chords or musical passages to any pitch level while maintaining exactly the same fingering and the same spatial relationship between the keys involved.[355]

Some remarkable features of Bosanquet's design are that the conventional keyboard pattern (7-white/5-black) remains embedded in the new layout (albeit 'diagonally'); all major and minor scales (for example) can be fingered in the same way as the key of *A major* or *F# minor* on a conventional keyboard;[356] and it is particularly successful (in terms of acoustic presentation) for mapping systems of equal-temperament which comprise a unique cycle of fifths (the design was created with this in mind). Extra tiers may be added depending on the number of octave divisions, or, for example, for a tempered system requiring more than one cycle of fifths. This layout, and adaptations of it, have also been used for extended JI systems, most particularly in the designs of Ervin Wilson.[357]

Many Bosanquet type keyboard layouts have been realised on working instruments, although (to my knowledge) none of these have been applied to the piano - they have been for organ, harmonium or percussion. Perhaps the most famous of these is the 31-ET *Fokker Organ* (1948-50) built for the Dutch composer and scientist Adriaan D. Fokker. In this case the keyboard layout is 'horizontal' rather than diagonal:

> The Fokker organ has two 31-tone manual keyboards and one 31-tone pedal keyboard… [Each] keyboard consists of eleven 'horizontal' rows of keys… Two horizontally adjacent keys (although always separated by keys of the rows immediately behind and in front) always have the interval of a whole tone. Two vertically adjacent keys in a column (two rows apart) always have the interval of a diesis (the higher key having the higher pitch)… The natural keys are white; the keys called sharps or flats are black… The keys that represent truly microtonal

352 Helmholtz, op. cit., Dover, pp. 470-3.
353 Helmholtz, ibid., pp. 474-9.
354 Helmholtz, ibid., pp. 479-81.
355 Keislar, 'History and principles of microtonal keyboards', *Computer Music Journal*, Vol. 11, No. 1, Spring 1987, p. 20. Bosanquet wrote of his own invention: 'The most important practical point about the keyboard arises from its symmetry; that is to say, from the very fact that every key is surrounded by the same definite arrangement of keys, and that a pair of keys in a given relative position corresponds always to the same interval… any passage, chord, or combination of any kind, has exactly the same form under the fingers in whatever key it is played… Some simplification of this kind is a necessity if these complex phenomena are to be brought within the reach of persons of average ability; and with this particular simplification, the child or the beginner finds the work reduced to the acquirement of one thing, where twelve have to be learnt ton the ordinary keyboard.' R.H.M. Bosanquet, *Temperament - An elementary treatise on musical intervals and temperament*, Macmillan and Co., 1876, pp. 20-21.
356 John S. Allen has pointed out that the fingering of scales is in fact dependent both on the tuning system and on whether the keyboard is what he calls '*left-rising*', '*right-rising*' or '*horizontal-row*' - that is, whether the keys are laid out rising diagonally from right-to-left or from left-to-right, or straight, as in a conventional layout. In tunings such as 12-, 19- or 31-ET, a 'left-rising' generalised keyboard requires fingering as for the keys of Db major and Bb minor; a 'right-rising' keyboard requires fingering as for the keys of B major and G# minor. On the other hand, for systems such as 29-, 41-, or 53-ET a left-rising keyboard requires fingering as for A major and F# minor, and a right-rising keyboard fingers as for Ab major and F minor. See John S. Allen's website at: http://web0.tiac.net/users/jsallen/music/bosanqet.htm. Allen has also pointed out that 'the horizontal-row generalised keyboard has the advantage of being equally adaptable to either type of fingering' although it may in the long run be less intuitive than the left and right rising layouts (email to the author from John S. Allen.). The 'Fokker keyboard' considered below is an example of the horizontal-row layout.
357 Ervin Wilson, 'The Bosanquetian 7-Rank Keyboard after Poole and Brown', *Xenharmonikôn,* No. 1, Spring 1974; Ervin Wilson, 'On linear notations and the Bosanquet keyboard', *Xenharmonikôn* 3, Vol. 2, No. 1, 1975.

> pitches [relative to C? PO-L], such as the semisharps and semiflats, are light blue. The range of the keyboard is five octaves, from C to c'''' (143 pitches, 319 keys). [358]

In the 1960's Fokker also commissioned a portable electronic 31-ET organ known as the *Archiphone,* in which typewriter keys are used for the keyboard (which is set out in a somewhat similar manner to the organ). 'Since 1970 four Archiphones have been built...'[359] Other realisations of the generalised keyboard have included the *Motorola Scalatron* (in a special model developed commercially by George Secor in the 1970's);[360] a highly sophisticated adaptation of the layout developed by Martin Vogel for a 48-division organ;[361] and quite a few others.[362] As mentioned above, the *Starr Labs* Bosanquetian *Microzone* is, to my knowledge, the only alternative keyboard (electronic or acoustic) which is available (or soon to be available) commercially; it is also worth noting that amongst John S. Allen's current projects is a generalised microtonal MIDI keyboard (see p. 109).

The problem of an 'ideal layout' for a microtonal keyboard is yet to be resolved. It would seem that progress on this front is more likely if a unique ATS was adopted as a standard, although the discussion of the 'logical piano' given below suggests a more radical solution. These are certainly areas where collaborative research is essential.

Weight, Size, Stringing - and the Relation between Keyboard and Hammer Action.

The sheer size and weight of a many-octave-divisions acoustic piano has often been cited as the main stumbling block to construction. The modern grand supports string tension of about 210,000 Newtons (47,000 lbf),[363] and to support a greatly increased number of strings, the frame of a new piano would have to be of increased size and strength. The weight of the action, strings and structure would also be greater.

On the traditional piano the lateral distance separating each key corresponds roughly to the distance between each string (or unison group of strings). Broadly speaking this is to enable the action - from key to hammer - to have as direct and simple a relation to the strings as possible. But if an alternative keyboard layout is used, in which many more keys are crowded into the same width, a corresponding increase in the number of strings and string groups cannot be made (in a single plane), unless the width of the piano frame is increased very substantially, and the relation between key and hammer is made oblique.

A number of approaches might be taken to solve these problems:

- accept the consequences of size and weight;
- restrict the pitch range;
- use fewer strings per unison;
- rethink the stringing of the piano;
- use an electronic interface between key action and hammer action.

The *Klavins Model 370* upright piano was built in 1988, and is

> believed to be the largest stringed instrument in the world…[it] was initially built as an experimental instrument designed to obviate the compromises of the modern piano. The

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358 Adriaan Daniël Fokker, *Selected Musical Compositions (1948-72)*, ed. Rudolf Rasch, The Diapason Press, 1987, p. 33. The organ is installed in Teyler's Museum in Haarlem, Holland. John S. Allen has reported from his correspondence with Dr. Leigh Jardine (who translated Fokker's book 'New Music with 31 Tones' into English) as follows: 'Dr. Gerdine has had experience playing a Fokker keyboard (a 'horizontal-row' keyboard) which was installed at Webster University in St. Louis. This keyboard also had rather short keys. Gerdine reports that he found the keyboard difficult to play for that reason. However, the difficulty might result from the expectation of being able to use standard keyboard technique.' Email to the author from John S. Allen.

359 Adriaan Daniël Fokker, ibid., pp. 34-5.

360 Keislar, 'History and principles of microtonal keyboards', *Computer Music Journal*, Vol. 11, No. 1, Spring 1987, p. 23.

361 Vogel, op. cit., pp. 358-70.

362 See Keislar, 'History and principles of microtonal keyboards', *Computer Music Journal*, Vol. 11, No. 1, Spring 1987, p. 23.

363 Harold A Conklin Jr., 'Piano design factors - their influence on tone and acoustical performance', *Five Lectures on the Acoustics of the Piano*, ed. Anders Askenfelt, Royal Swedish Academy of Music, Stockholm, 1990, p. 32.

> instrument stands 12 ft 1½ inches tall, enabling it to be straight strung and houses a bottom bass string of about 10ft in length. This reduces inharmonicity and gives a more natural sound. The soundboard has a surface area of more than four square yards, and is mounted vertically, thus projecting the sound towards the audience rather than the ceiling. The strings exert a force of more than 27 tons.[364]

An instrument of this size should probably be considered a permanent feature of a concert hall, in the same way that an organ is. Given this precedent, as well as the various quarter-tone and other pianos mentioned above, it would not seem outlandish to build an especially large and radical piano to be housed in a prestigious concert hall - specifically for (one or more) ATS. Equally, in recent years in the UK the pianistic duo David Nettle and Richard Markham have toured with a '*Duoclave*' or double grand piano. This is one of a number of such instruments built by *Pleyel*, in which two overstrung grand pianos are united (yin-and-yang-like) in a single rectangular case, with keyboards at either end. The soundboard is shared, but the actions and strings are independent.[365] So a larger piano is not impossible…

Equally, the range of the instrument may be restricted, and/or strung with fewer strings in the middle and upper registers, minimising weight and size, although the consequences for tonal power are not ideal for a concert instrument (note 334). However, APPENDIX VII (a) and (b) show the number of strings required for pianos having between 12 and 41 divisions per octave - firstly for a 7¼ octave, and then for a 6¼ octave range. The typical Steinway grand has a total of 243 strings, that is, 8 wrapped single bass strings (A0 - E1), 5 sets of wrapped strings double-strung (F1 - A1), 7 sets of wrapped strings triple-strung (Bb1 - E2), and 68 sets of unwrapped strings triple-strung (F2 - C8). In the APPENDIX this distribution is shown as three rather than 4 groups, (8/88, 5/88 and 75/88) corresponding to single strings, bichords and trichords.

If we suppose, for example, that the quarter-tone pianos of Hába and Wyschnegradsky (note 325) were made with (roughly) double the number of strings of a conventional design, we may provisionally assume 486 strings is an approximate upper limit of feasibility.[366] Thus APPENDIX VII would suggest that by varying the proportions of single strings, bichords and trichords (or reducing the range by one octave) a piano with a large number of octave divisions is possible. For example, a 31 division 7¼ octave instrument with a string distribution corresponding proportionally to 27/88, 24/88 and 37/88 - which might still be reasonably effective - would require 481 strings. A 41 division 7¼ octave instrument with a string distribution of single strings and bichords only (distributed in the proportions 27/88, 61/88) would require 509 strings. Equally, a 31 division 6¼ octave instrument with a string distribution not dissimilar to that of a typical upright (15/76, 11/76 and 50/76) would require 483 strings; a 41 division 6¼ octave instrument with a string distribution corresponding to 27/76, 24/76 and 25/76 would require 513 strings.

In the case of a piano with many divisions and which had a reduced number of strings in unisons, would it, for example, be possible to counterbalance the change in tone by adding a further bank of purely *resonant* strings which are coupled to the soundboard (somewhat as on a sitar)? Here, the extra strings would require to be automatically damped and/or immediately released by opposite motions of the sustain and damper pedals, or perhaps a 4th pedal. The idea of completely rethinking the stringing of the piano, so that, for example, the strings are no longer in a single plane, or their relation to the soundboard is radically changed, is outside the limits of this paper. I am not aware of any such instrument. Is it at all conceivable, for example, that a grand piano might be built in which there is one extended keyboard, of which the action ends of the keys are narrow and activate two actions - the first acting from above on a bank of strings (above the soundboard), and a second acting from below a second bank of strings (below the soundboard)?

The Logical Piano

[364] David Crombie, *Piano: Evolution Design and Performance*, Balafon 1995, p. 79.
[365] David Crombie, ibid., p. 69.
[366] This depends of course on the number of strings used per unison in the different registers of these instruments - something I have unfortunately been unable to find out.

Another approach would be to control electronically the relation between the keyboard and the action. In this way a keyboard layout with many more keys could be made to activate a (nominally) remote action having an equally large number of hammers and string groups. The pianist would play as normal, but rather than the action being activated directly by the key, information from the keys would be passed to the action electronically. From a technological point of view, this mechanism would in principle be similar to the well known digital player piano systems manufactured (and patented) by *Yamaha* (the *Disklavier* series) or *Bösendorfer* (the *290 SE* 'computer supported concert grand').[367] The major difference would be that 'playback' is eliminated from the process: play and 'playback' must occur simultaneously, and it would be critical that the pianist did not experience any loss of 'touch' or connection with the sound.

Creating a virtual (electronic) rather than fixed (mechanical) relationship between the interface and the pitches which result, suggests some interesting possibilities, as it does for the other 'logical' adaptations put forward. If the keyboard were 'remote' or detachable from the action, strings and soundboard, then a number of different keyboard layouts might be connected to one action/string/soundboard unit; similarly, different action/string/soundboard units could be controlled (at different times, or conceivably at the same time) by one keyboard controller. It is probably true to say that, like 'touch', the physical feedback from being in some kind of contact with the actual vibrating source of sound in the instrument is important to the pianist, so it might be wrong to make the player completely remote from the strings and soundboard. Similarly, just as the use of combinations of keys (such as 'shift' or 'control') add versatility to the computer keyboard, key combinations might be configured for 'microtonal' and other purposes on a 'remote' acoustic piano keyboard.

The point about this for ATS is that the sound-producing unit might be built to be tuned in many different ways, or have so many hammers and strings available that only a proportion of them would be used in one work, or section of a work, while the assignment of key to pitch is controlled in software. If the keyboard and sound production unit are separable, this would help to make an 'ATS piano' more portable, especially if it were in some way standardised. But if a '21st Century Concert Grand' were installed in the concert hall, somewhat as if it were an organ, there is little to preclude it having a very large number of strings and hammers, controlled from a keyboard of manageable size which has preset switches to alter the hammer and string combinations that are available at any given time. This might serve as a working compromise (!), for an acoustic instrument of fixed intonation, considering Vogel's idea that '[a] composer must at all times have unlimited access to the whole of all tone relations'. More practically, the 'logical piano mechanism' might be applied to an instrument of more sensible size to create an 'adaptive' acoustic piano, in which intonational correction is automatically controlled in software (see note 96).

New Technologies for Piano Tuning

Modern tuning equipment such as the *Sanderson Accu-tuner,* the *Reyburn Cyber Tuner* and others[368] are accurate to a thousandth of a semitone (0.1 cents), and are specially designed for piano tuners and technicians. Whether a tuning is established by ear or with the aid of the machine, the result may be automatically recorded and saved on a database, either as a template for general use, or as a tuning for a specific piano. (Equally, the tuning can be made or adjusted purely by ear, and subsequently recorded for the same purpose. Because every piano is different, the exact amount of stretching or the precise setting of the initial scale may differ subtly. Thus, each time the tuner returns to a particular piano the specific tuning for that instrument may be recalled (or, of course, any other tuning), and the piano can be re-tuned using that record or template as a starting point. The device provides the exact tone required for each note, and the piano tuner may tune by ear to this, or in

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[367] For a brief overview of computer controlled player pianos, see Bart Hopkin et al., 'Computer control for acoustic instruments', *Experimental Musical Instruments*, Volume VII, #1, June 1991, pp. 1 and 8-11. Thomas Levenson, for example, describes a Bösendorfer owned by Micheal Hawley (a researcher at MIT) as having a 'Kimball digital playback and recording mechanism - an automated piano control mechanism. The Kimball device uses lights to measure the timing and speed with which the Bösendorfer's hammers strike its strings. The Kimball then transforms the flicker of the optical sensors into the digital data that computers can read. Finally it plays back its digital musical information by activating a stack of electromechanical devices that can drive the instrument's hammers and keys'. Thomas Levenson, op. cit., p. 276. Also worth mentioning here is Trimpin's unique MIDI controlled *Instant Prepared Piano* IPP 71512 - see David Crombie, ibid., p. 79.

[368] I am grateful to Ed Foote on the Mills Tuning List for advice on this topic. The *Sanderson Accu-tuner,* for example, is available from Inventronics Inc., 9 Acton Road, Chelmsford, Massachusetts 01824-341, USA; details of the *Reyburn Cyber Tuner* can be found at: http://www.reyburn.com/index.html.

conjunction with a strobe. If the tuner returns regularly to a particular piano, then the tuning may be refined on each occasion, until it can hardly be improved upon. Manufacturers claim that the tuner's job is in no way diminished by these devices, but rather that they make the process of tuning somewhat quicker, more reliable and more accurate.

The relevance of this equipment here is that it might be used to help tuners tune for ATS. If, for example, a piano with 31 divisions to the octave needed to be tuned regularly, this would be time-consuming and expensive. These devices can provide an ideal initial tuning for the tuner to aim for, and the ability to record the actual tuning on a database when the tuner is satisfied. If a future piano were to have a multi-purpose keyboard on which a variety of tunings were from time to time required, then computerised tuning devices would also make the job easier. To my knowledge, existing tuning devices are currently only manufactured for 12 division systems. A workaround is to use more than one setting to obtain all the required intervals, but then a number of other features are lost. In the event of creating a new piano for ATS, creating an equivalent tuning device for the chosen *n*-division system chosen would be an essential co-project.

One of the more sophisticated features of these devices is that a tuner or technician can record a few notes of a particular piano, and then instruct the device (click with the mouse) to estimate the optimal degree of stretching for the particular instrument based on the spectrum of the recorded tone.[369] The device will then generate exactly that tuning, and the tuner can base a first approximation on the computer's calculation. Typically, these systems store a range of historical (or user defined) temperaments which can be automatically adjusted for an individual instrument. It is very likely these devices will become increasingly powerful - for example, measuring higher partials, and enabling cross-comparisons of all possible notes.

A further use for these machines is that composers (and performers) might come to prefer a particular degree of stretched-ness or a particular initial setting of the scale for a particular work or concert programme. The devices enable an artist to request a specific realisation of a particular tuning system. It is not expected that this will have exact results because, as has been said, every piano is different. Nevertheless, such devices would allow composers and performers to have more say in the way an instrument is tuned for a particular concert or composition.[370]

This brings us to one last idea regarding the piano: the 'self-tuning piano'. If a '21st Century Concert Grand' was to be created as a permanent installation in a concert hall, and had very many strings, would it make sense to build in a robotic motor which co-ordinates with a pitch sensor to tune the piano automatically? Presets for many alternative tuning systems might be made available (although too frequent re-tuning may cause wear on the pin-block). However, since a 'piano' such as this would seldom be moved, its pitch ought to be more than usually stable. Technicians would have the pleasure of fine tuning.

[369] For example, the *Sanderson Accu-tuner* measures the relative inharmonicity of the sampled notes; similarly, the *Reyburn Cyber Tuner* measures and matches up to 21 partials.
[370] For example, a tuning established in one part of the world can be sent over the internet like any other digital file.

The Harp

Both the pedal and the chromatic harp may be re-tuned quite easily to give some alternative tuning configurations, as has been done in many contemporary pieces.

On the pedal harp, however, pedal changes give 12-ET semitone shifts. One adaptation might therefore be to re-design the tuning mechanism to give *n*-division chromatic changes rather than semitones. Instead of three possible tuning positions there might be four or five, and the resulting shifts need not be as large as a semitone, nor necessarily equal-tempered.[371] Given seven strings (the number on a conventional pedal harp) to the octave, the instrument might thus achieve up to (say) 35 discrete divisions to the octave. In this way a radical ATS harp could be made without necessitating a special body (given that, for instance, a 5-position tuning arrangement could fit into the conventional housing). Perhaps it would also be possible to re-design the tuning mechanism so that the degrees of shift are not permanently fixed, but variable by some controlled amount, so that a single harp could realise a wider variety of tuning systems.

The above idea presupposes that octave linkages be retained - that is, each pedal shifts all the strings of one pitch class. This means that even for a '35-ET harp' there would remain only seven pitch classes available at any one time. This need not be the case since linkages need not be restricted to the octave. For octave based ATS the cent value of the tuning shifts will be an aliquot part of 1200, and linkages might connect the nearest interval greater or smaller than the octave.[372] Such an arrangement might unduly restrict the variety of musical styles appropriate to the instrument. By moving the linkages, say, to the interval of a fifth, such problems might be avoided.

A more radical solution would be to make a pedal harp with more than seven strings to the octave, adapting the tuning shift mechanism and linkages accordingly, although this would reduce the instrument's range. Alternatively, a 'double-harp', with two parallel banks of strings, giving 14 strings to the octave might be made. The two banks might break the range into two separate registers, or the scale could be interleaved. In either case each of the normal seven pedals might control a set of strings from each of the two banks. None of these systems need be unplayably complicated.

For a chromatic harp, re-tuning (and perhaps re-stringing) could suffice for systems which do not have very many octave divisions. Restrictions of range (due to arm length) might be overcome by creating two or more separate instruments, for example an SATB harp quartet.

It is worth mentioning here Joel Garnier's Camac 'memory' harp (1984). An on-board computer controls pneumatic 'pedal' movements, which are programmed in advance for a particular work. Each 'pedal change' is thus activated at the appropriate moment in performance by a single pedal or button press.[373] This harp is mentioned because it suggests that the complexities of performing a '35-ET pedal harp' might be reduced by an automated pedal-tuning control of some kind. In addition to a single button which recalls a sequence of pre-programmed tuning positions, one might imagine a system in which all possible pedal positions are stored in memory, each of which can be uniquely recalled by 'typing' in a number with the feet. This number could be written on the player's part instead of traditional pedal indications.

[371] Note, however, that in an equal-tempered system composers may be able to take advantage of what harpists call 'homophones' - in 12-ET this is the practice of tuning (say) the 'A' string to Ab and the 'G 'string to G#, thus enabling *glissandi* to be made with relatively simple harmonic material. This depends on the intervals of the neutral positions of the strings, and the system adopted.

[372] The combination of *n*-division chromatic shifting and non-octave linkages could provide a way of achieving non-octave (or 'morenoctave') scales. The cent value of the tuning shifts must not be a sub-multiple of 1200, and linkages might connect the nearest interval greater or smaller than the octave.

[373] Roslyn Rensch, *Harps and Harpists*, Duckworth, 1989, pp. 258-60.

Voice, Guitar, Organ, & Tuned Percussion (omitted)

These sections are temporarily omitted.

Amongst the acoustic mainstream, tuned percussion, the guitar, and (to lesser extent) the organ, are probably the instruments which have been most commonly and easily adapted or developed for ATS.

The voice also has a special status with regard to ATS.

Discussion and bibliography for these topics will be available at a later date.

An inharmonic instrumentarium?

Following the discussion of sensory consonance and relating scale to spectrum, the physical possibility of creating acoustic instruments having timbres 'related' to an ATS seems a crucial area for future investigation. Collaborative research in this field could be particularly important for the future of instrumental microtonality.

The sounds of impulsively driven instruments, such as the piano, percussion, or plucked strings, are intrinsically inharmonic to a greater or lesser degree. On the other hand, sustained instrument tones resulting from the normal playing mode of woodwind, brass and bowed strings, are intrinsically harmonic. Instrument makers take pains to refine the timbres of impulsively driven instruments, often trying to make them as harmonic as possible - but it would seem that the opposite has seldom been attempted. Since the mutual integration of both kinds of instrument depends on spectral correspondences between the two, what possibilities are there for developing inharmonic sustained instruments - for a deliberately inharmonic instrumentarium?

Joseph Yasser argued, for example, that for conventional instruments with harmonic spectra, there is an audible dissonance between the 3rd (and 6th) harmonics and the 'fifth' in 19-ET, there being approximately a 7 cent difference between the two intervals (in octave reduced form).[374] This difference is present for melodic as well as harmonic combinations, since the 3rd and 6th harmonics may be present in resonance, and in the fusion of harmonic complex tones. Following this line of thought, Tillman and Piehl built a metallophone in 19-ET in which it is thought they 'avoided adjusting any of the overtones to octave displaced fifths over the fundamental'.[375]

This kind of treatment is possible for percussion instruments (further discussion of this will be added at a later date);[376] and timbral adaptations of the piano have also been mentioned. Similarly, the resonating body of string instruments can be made to boost or suppress certain partials - but adaptations of plucked strings would normally have awkward consequences if they are also to be bowed - unless separate instruments were envisaged for each playing mode. In particular, unconventional weighting of strings where pitch changes are made by stopping, for example on the violin or guitar, will produce interesting but unpredictable results, because the centre of mass of the string is changed as the sounding length varies.

In *Tuning, Timbre, Spectrum, Scale* Sethares suggested that:

> With stringed instruments, the trick is to find a variable thickness and/or variable stiffness string that will vibrate with partials at the desired frequencies. The partials of a drum head can be tuned by weighting or layering sections of the drumhead. The partials of reed instruments can be manipulated by the contour of the bore as well as the shape and size of the tone holes. Bells can be tuned by changing the shape and thickness of the walls.[377]

This is correct for impulsively driven instruments. But, as discussed previously, in the case of sustaining instruments the fact that the *resonances* of these systems can be adjusted to be inharmonic does not mean that their *spectral structure* will be correspondingly inharmonic, a point about which Sethares has since agreed he was mistaken.[378] There is a body of acoustic theory which suggests that deliberately inharmonic sustained

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[374] Joseph Yasser, *A Theory of Evolving Tonality*, Da Capo Press, New York, 1975, [exact page ref? XXX]. See also, APPENDIX II, Table 19. (Temporary note: Paul Erlich has remarked that this description of Yasser's argument is not accurate. Unfortunately I no longer have access to Yasser's text, and need to check both the content and the references. Possibly this explanation derives from Tillman and Piehl's interpretation of Yasser - see next footnote).

[375] Personal communication from Bart Hopkin, to whom thanks for the correction. See Tillman and Piehl, 'Musical instruments in nineteen-tone equal temperament', *Journal of the Acoustical Society of America*, 1947, 19(4), p.730. Mandelbaum has noted that makers building instruments for 12-ET sometimes try to suppress the 7th harmonic for similar reasons. M. Joel Mandelbaum, 'Multiple divisions of the octave and the tonal resources of 19-tone temperament: Chapter 11 - Joseph Yasser', reprinted in *Xenharmonikôn* 4, 1975, p. 4.

[376] Doty provides a useful discussion of the controversial issue of using idiophones (percussion instruments using bars, rods etc.), which normally have inharmonic timbres, in the context of Just Intonation. Doty, op. cit., pp. 65-66.

[377] Bill Sethares, 'Relating Timbre and Tuning', *Experimental Musical Instruments*, Volume IX, #2, December 1993, p. 68.

[378] Personal communications between Bill Sethares and Neville Fletcher (1998), to both of whom I am grateful for allowing me access.

acoustic instruments are not possible because mode-locking (or 'entrainment')[379] is essential to the physical production of sustained tones. But it must also be pointed out that *Tuning, Timbre, Spectrum, Scale* - Sethares' recent book - is the first to have so thoroughly demonstrated the validity of relating spectrum to scale. In the past, instrument makers have focused on finding perfectly harmonic timbres - and none would have had convincing reasons for creating inharmonic sustaining instruments, or thought it important had they stumbled across ways of doing so. As a result of Sethares and others' work, this uncharted territory might soon become an area of intensive research. In the following paragraphs it is assumed that the investigation of impulsively driven instruments is likely to be a valuable area of research: the question is - is it possible that results in that area could be matched by results with sustaining instruments?

Three approaches suggest themselves: firstly, investigation of the extent to which the amplitudes of various harmonic partials might be *boosted* or *suppressed* - maximising those that correspond to, and minimising those which conflict with, the 'related' degrees of a given alternative scale (this would not, of course, be a truly 'inharmonic' adaptation). For an *n*-ET which has, for example, a good fifth, major and minor third and harmonic seventh, the adjustments required need only affect the higher partials, and the overall timbral quality and fusion of familiar instruments need not be radically disturbed; nor, ideally, would an instrument's stability and responsiveness. Further, it is unlikely that there is a need to contrive a very strict relationship between spectrum and scale on a sustaining instrument. This is because (i) the timbre produced on a wind, brass or string instrument varies from one player (or instrument) to another; (ii) spectra differ in terms of amplitude from one register or note to another; (iii) conventional acoustic spectra are imperfectly 'related' to 12-ET but function very adequately, partly because (iv) most sustaining instruments are in any case of variable or fixed-but-variable intonation.

Secondly, to investigate radical approaches to adapting (or creating new) sustaining instruments for which the frequencies of certain partials are actually 'related' to a scale. In correspondence, Sethares and Fletcher have defined the problem as how to find a way of driving an acoustic system (a physical instrument) which has inherently 'related' inharmonic properties in such a way as to avoid 'mode-locking'. For example, if the driving energy is very small then no periodic motion has enough energy to 'entrain' another; alternatively, partials may be so far out of harmonic alignment that mode-locking cannot occur. Familiar examples of this include blowing down a recorder very softly, or using a very odd fingering. The result is typically a multiphonic, and (in the present context) this is not what is wanted. As Bart Hopkin has commented:

> Whether for physical reasons, some in-born psycho-acoustic reason, or simply due to acculturation, our ears are amazingly adept at hearing composite tones possessing harmonic overtones. We hear an array of tones appearing as a single sound… [But] we bring no comparable facility to inharmonic overtone mixes… timbres rich in inharmonic partials, while they may be appealing or intriguing, can be musically confusing. The ear may fail to settle on a single partial as the main pitch, or we may find our sense of the pitch shifting from one partial to another…[380]

And Sethares has made clear that (for the purposes we are exploring):

> when dealing with nonharmonic partials… it is important for the partials to fuse, to be perceived holistically as a single entity rather than as a collection of oddly placed sine waves.[381]

In the electronic examples which accompany *Tuning, Timbre, Spectrum, Scale*, spectra are typically constructed to imitate the harmonic series as closely as the 'related' scale permits, and the resultant tones are generally heard as 'fused'.[382] Neville Fletcher has suggested an exceptional instance of an instrument and playing technique which might be used as a model for constructing a woodwind with a 'related' inharmonic timbre:

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[379] 'Entrainment' is the influence of one periodic motion over an other - in this case, for example, pressure waves inside the bore of a woodwind or in the vibrating motion of a violin string.

[380] Bart Hopkin, *Experimental Musical Instruments*, Volume III, #6, April 1988, pp. 116-17.

[381] William A. Sethares, 'Local consonance and the relationship between timbre and scale', *Journal of the Acoustical Society of America*, 94 (3), Sept. 1993, p. 1218.

[382] As an example of the basic principle: for the 11-ET pieces, harmonic partials at f, $2f$, $3f$, $4f$, $5f$, $6f$ etc., are mapped to f, $r^{11}f$, $r^{17}f$, $r^{22}f$, $r^{26}f$, $r^{28}f$, $r^{31}f$, $r^{33}f$, $r^{35}f$, $r^{37}f$, $r^{38}f$, where $r = \sqrt[11]{2}$. William A. Sethares, *Tuning, Timbre, Spectrum, Scale*, Springer Verlag, 1997,

> [A] rather different example is the method of playing the pan-flute that is used by South-American musicians. Instead of adjusting the jet from the player's lips to excite the pipe in the normal way… they use a wide turbulent jet and excite the pipe by turbulence noise with very little feedback. This gives the breathy sort of sound familiar on recordings of their music. If you look at the spectrum of this sound, then it mimics quite well the slightly inharmonic admittance curve of the (closed) pipe. There is nothing much in the way of sum and difference terms because there is little feedback to the jet and nothing locks in to the generator mechanism. If you fiddled with the geometry of the pipe to make the resonances very inharmonic (blowing roughly across the top of a drink bottle is an example!), then this too would be echoed in the spectrum of the sound, since there is very little feedback. This might give you the sort of sound you are looking for (although the spectral peaks are rather broad, as you might expect from the turbulent excitation), so that the tone is not very "tonal".[383]

More experimentally, Sethares has postulated a system in which the motion of a modified (inharmonic) metal string is 'monitored' by some kind of electronic or optical sensor, and fed back into the system in such a way that the string sustains an inharmonic tone.[384] If this were achievable, there also remains the question whether a conventional acoustic instrument resonator will amplify an inharmonic spectrum adequately, or whether this too could be modified accordingly. We might speculate that, like the electronic pitch-feedback correction mechanism for 'logical' woodwind suggested above (pp. 73 - 74), it is perhaps conceivable that a kind of electronic feedback mechanism could 'replace' mode-locking, stabilising inharmonic tones in woodwind and brass. It would be especially interesting to hear of research into applications along similar lines of thought, or adaptations of woodwind bore contours, brass horn perturbations, violin strings, etc., which aims at creating inharmonic timbres in sustaining instruments - whether or not the intention is to control consonance/dissonance for an ATS.

Lastly, in the case of brass instruments and to a lesser extent the winds, there is the possibility of adjusting resonances so as to relate the overblown ratios to the intervals of the temperament or scale. For example, Fletcher and Rossing have stated that 'the instrument designer can… adjust the horn shape slightly in order to properly align or displace horn resonances',[385] and that '[i]f the flaring part of the horn extends over a reasonable fraction of the total length, for example around one third, then there is still enough geometrical flexibility to allow the frequencies of all the modes to be adjusted to essentially any value desired.'[386] Clearly, this is not the same as modifying a brass timbre to 'relate' to a given scale, but such modifications will have an effect on the timbre of the instrument (boosting or suppressing partials), and might be combined with the above approaches.

Relating scale to spectrum or spectrum to scale is clearly less difficult to apply to electronic than acoustic instruments. However, if specific and consistent manipulation of the *frequency* components of real acoustic instruments seem far fetched, the timbres of acoustic instruments *are* dependent on instrumental design (and the performer). As Sethares has shown, digital samples of instrumental sounds can be manipulated to conform closely to a 'related' spectral ideal for a given tuning. The latter could therefore provide a good *aural* model of timbre for the instrument maker to aim for, and against which to judge further physical/timbral modifications. Equally, the timbral results of physical modifications could be sampled and tried out in musical situations by using simulations. More sophisticated still, mathematical modelling is used to simulate instruments, not just with 'fixed' samples, but so that fine parameters (such as bow speed, bow pressure or breath pressure etc.) can be built into the simulation. In the case of trying to create an 'inharmonic' acoustic instrument, where the aim may be to drive an inharmonic system in a linear way, mathematical modelling could therefore be extremely useful.[387] All this suggests a kind of dialectical process of approximation, using physical and electronic media.

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p. 236. It is unclear whether there is any 'absolute' limit to fusion relative to inharmonicity, or how far this is affected by familiarity. For discussion of the limits within which an inharmonic complex tone is heard as fused, see Elizabeth Cohen, 'Some effects of inharmonic partials on interval perception', *Music Perception*, Spring 1984, Vol. 1, No. 3, pp. 323-349.

[383] Email from Neville Fletcher to Bill Sethares (1998).

[384] Email from Bill Sethares to Neville Fletcher (1998).

[385] Fletcher & Rossing, op. cit., p. 198.

[386] Fletcher & Rossing, op. cit., p. 367.

[387] Thanks to Neville Fletcher for comments on this.

Note also that when inharmonicities (of string or resonance) are pronounced in the sound of a piano, a tuner can have difficulties achieving a satisfactory tuning. This is because the tuner is caught between trying to tune to a given system, and listening for consonance or a particular frequency of beats. It is difficult to predict, therefore, whether playing 'in tune' in an intended alternative tuning system, on designedly inharmonic instruments of non-fixed intonation, is likely to be successful (or possible). This may also depend on the extent to which different listeners each perceive tones with a varying degrees of inharmonicity with varying degrees of fusion; and whether inharmonic or quasi-inharmonic sustaining instruments suffer problems of instability or responsiveness. However, as we know from the music of other cultures, such qualities may contribute to their attractiveness. Similarly, as Sethares has pointed out, singers and rebab players can and do tune together successfully with the inharmonic sounds of the Javanese gamelan. The only way to really discover if an 'inharmonic acoustic instrumentarium' is feasible is to actually try to build and play radically inharmonic instruments, utilising electronic simulations for reference and research.

Sethares has given substance to an important area of exploration. At very least, this discussion, together with our experience of much microtonal music, should underline the extremely important relationship between harmonic and timbral systems. Music for piano(s) tuned in quarter-tones, for example, can so easily sound as if we are hearing an out of tune bar-piano, rather than a radical and (ostensibly) excitingly new harmonic idiom.[388] There is little doubt that this effect results from the specific relationship between the spectral structure of piano sound and the quarter-tone system. If a new, inharmonic, acoustic instrumentarium were a serious future possibility, two further 'criteria' concerning a provisional ATS standard should be added to our list: (i) if more than one spectral structure is 'induced' by a single tuning system, how do we choose between them, in terms of musical effectiveness and flexibility?; and (ii) if an alternative tuning systems 'induce' preferred (or at least maximally 'consonant') spectral structures, how do we evaluate the tuning/spectral structure *combinations*, if not in terms of 'absolute consonance', or whether we like them?

[388] I am not suggesting that Charles Ives' rather beautiful '*Three Pieces for Two Quarter-Tone Pianos*', for example, sound like bar-piano music - but rather that the success of these pieces on a particular occasion is more dependent on the exact tuning which the pianos are given, than is conventional two-piano music dependent on exact tuning. Possibly, 'cross-tuning' between the two pianos, especially in terms of the stretched scale, is significant.

Conclusions about New Acoustic Instruments

This survey of instrumental possibilities has had various functions:

- to gather together information on acoustic instruments for ATS;
- to suggest new instrumental possibilities;
- to suggest areas of research, especially with regard to establishing criteria for establishing a provisional alternative tuning standard.

In conclusion, there seem to be a number of major routes which the development of acoustic instruments might take:

- to adapt instruments by changing the scale but retaining existing timbres;
- to follow a 'coevolutionary' approach, in which tuning and timbre develop in tandem;
- to make instruments which are optimised for a single scale;
- to make instruments which may be optimised for a number of scales;
- to make instruments optimised for a restricted (traditional) continuum of timbre;
- to make instruments in which timbre is variable and controllable - that is, to a greater extent than in existing instruments.[389]

Clearly, serious investigation of creating new acoustic instruments in conjunction with establishing a provisional alternative tuning 'standard' is a complex task which would undoubtedly benefit from a wide-ranging collaborative project, co-ordinated by one or more research Centres.

If a Centre and new instruments were created from public funding then it would seem right that the instruments themselves should become, in some sense, a 'public' resource. They would perhaps need to be retained and maintained by the Centre, but loaned to performers for study and performance. In addition, new instruments might be privately commissioned by composers, performers or institutions, and specific new projects undertaken collaboratively with professional instrument manufacturers.

[389] Ideally, this might be modelled on the voice - where the articulation of both discrete and continuous spectral changes are possible - but such that these could be specified reliably by composers, and the harmonic implications of timbral variation integrated in the harmonic/melodic flow. Currently, I have no suggestions how this might be achieved - other than electronically.

6. Digital Technology and the realisation of ATS using Acoustic Instruments [390]

As suggested previously, interactive CD-ROM and sequencer type programs may be invaluable for aural training - the success of alternative tuning systems depends on aural abilities above all else - and digital technology will be important in other areas too. There are five obvious areas of application:

- computerised ear-training programs;
- the tuning process itself (for example, for tuning a new piano);
- acoustic and instrumental research;
- digital simulation as a compositional tool;
- digital simulation of scores and parts, to help performers and conductors get to grips with new works.

For most avant-garde musicians working with electronics, absolute fidelity in the simulation of acoustic instruments is not a key issue - it is seldom this which really arouses their interest in digital music technology. For a broader spectrum of musicians, however, the quality of simulation is important. Furthermore, to learn a new work in a radical ATS on an acoustic instrument of non-fixed intonation, three stages or approaches may be necessary or helpful: (a) the tuning system is studied and internalised from first principles; (b) aural references are provided, from within the instrument itself or externally, or with a 'pitch track' which makes no pretence to 'simulation' of the work in question; (c) the performer is provided with a full-blown simulation, in which an attempt is made to make interrelationships of pitch, timbre, dynamics, expression, phrasing etc., self-evident to the performer, in conjunction with a score. In the latter case fidelity is highly advantageous.

For the performer, a simulation (of a work using an ATS) helps the ear and eye associate pitch and notation quickly; likewise, 'music-minus-one' tapes or MIDI files enable a player to rehearse with a 'virtual' ensemble (although the last mentioned technique is controversial). The current drawbacks are the huge amount of time it takes to create a simulation which is in any sense realistic, and the fact that, unfortunately, performers may actually be alienated by the inevitably limited musical qualities of a simulation - even if it is made with the highest quality digital equipment. This of course varies from instrument to instrument: for example, solo string instruments are notoriously difficult to simulate; sampled piano sounds may be lifelike individually, but do not form satisfactory chords, especially if the chords are complex; woodwinds and brass fare slightly better; percussion simulations are often excellent. There is of course no doubt that fidelity is also useful for composers working directly with ATS simulations.

The simulation of acoustic instruments using samplers and synthesisers is commonplace in popular and film music, but normally too crude for use in contemporary classical music (unless used for special effect). On the whole, synthesisers sound too 'artificial' for convincing simulation, although some recent demonstrations of woodwind simulations using physical modelling systems are impressively realistic.[391] Compared to most synthesisers, however, high quality samples normally provide better realism for individual sounds, so long as they are not looped, since they are simply recordings of acoustically generated sounds.[392] Unfortunately, each time a sample is triggered its sound is more or less identical (excepting dynamics), and even if a ludicrously huge amount of time is spent sampling every possible instrumental attack or diminuendo, and layering or combining them just so, the results are lifeless in comparison to 'real' performance on acoustic instruments.[393]

Moreover, using a sampler specifically for ATS is extremely laborious. Typically, there are three logical layers in the sampler architecture which are relevant (in effect, three relational data sets): the samples, the keyboard

[390] This section deals specifically with the relevance of digital technology to making music on new acoustic instruments for ATS, not as a more general tool for acoustic and electronic music. Obviously, electronic instruments can be and are be used in their own right, in any kind of music a composer wishes - this is not at issue here.

[391] For example, a demonstration track of a simulated saxophone solo created using a wind controller in conjunction with one of the Yamaha VL modules shows outstanding realism.

[392] Sampling technology would be particularly useful for sampling new, inharmonic acoustic instruments, and using the results for research and composition with those new timbres.

[393] Physical modelling technology or something similar may soon make more realistic simulations less arduous. Another approach, which is even more time consuming, is to use digital recordings of performances on acoustic instruments and to edit, re-tune and re-assemble them as required.

map, and an intermediate layer which is used (among other things) to link samples and keyboard notes (or zones).[394] To create a simulation of a single instrument tuned in some ATS, the user must painstakingly create, map, and re-tune one intermediate field for every one (or perhaps two) [395] notes of the 'instrument', telling the sampler: which sample to use, how it should be tuned, and which keyboard (or wind controller) note should trigger it. This is an extremely long-winded and error prone way of obtaining a simulation - but there is no adequate alternative on most current samplers.[396]

The problem could fairly easily be remedied by built-in 'macro' and/or 'template' features, which allow the user to generate and organise the tuning and assignment of samples to keyboard zones semi-automatically. The user might be presented with 'check boxes' or 'picking lists', or a menu, somewhat like the following:

- choose equal-tempered (default) or non-equal-tempered;
- choose octave dividend (default) or non-octave dividend;
- choose tonic (1/1) keynote;
- if equal-tempered:
  - set number of divisions to octave (or other dividend);
- if non-equal-tempered
  - set details of scale structure;
  (a keyboard map appears, and user types in scale definition);
- select range of scale (up to full midi range - 127 notes);
- select ordered set of samples to be mapped onto range;

When the command is executed the sampler would then automatically create an 'instrument' with the selected set of samples mapped according to the required scale across the specified range. The samples would be mapped according to their absolute pitch, so that traditional acoustic instruments or sounds retain the intended quality of sound relative to the desired pitch. The automatic assignment could be treated as a first pass, which could then be 'tweaked' by the user.

Even more restrictive is the limited number of 'intermediate fields' usually available. Of recent units, for example, the Roland sampler range (S770, 750, 760) gave only 255; the Akai S-series only 200.[397] Consider, for example, the problem of attempting to simulate a string quartet in 19-ET, in which a bowed sample of each 19th division of the octave (for the full range of violin, viola and cello) has been created.[398] Assuming that each 'instrument' has a (restricted) range of three octaves and a fifth, then each one will require 68 tuning fields (one per note of the scale). Assuming that the two 'violins' will use the same tuning fields, the 'four' instruments will require (3 x 68) = 204 fields. Obviously, even this very limited task is impossible with 200 fields, and with 255 fields there are therefore not enough to add a 19-ET 'pizzicato violin' - let alone create a simulation of a larger chamber ensemble. In ATS with more divisions to the octave the problem is greatly increased. The way around this is to use more than one sampler, or to multi-track: but the expense and inconvenience are prohibitive.

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[394] Different manufacturers call the 'fields' of this intermediate layer by different names: for example, Akai call them 'keygroups', Roland 'partials', and EMU 'zones'.

[395] When samples are transposed up or down in pitch (normally beyond about a minor third - depending on the sampler technology), the character of the tone is distorted and normally becomes unusable for the purposes of simulation.

[396] Exceptions to this rule include the sampler modules manufactured by Ensoniq, and apparently the *Ensoniq ASR-X* supports a variety of ATS; the Kurzweil K2000 and K2500 also implement some advanced tuning options which are may be used in conjunction with the sample-based element of the unit. Unfortunately I do not have direct experience of using either of these systems. On some Ensoniq models, for example, it is possible to specify a fragment of a non-equal non-octave division scale and automatically extrapolate this structure across the complete MIDI range. I am unclear to what extent the assignment of samples is automatically dealt with so that the mapping does not result in 'mickey mouse' effects. My thanks to members of the Tuning list for information on this.

[397] The EMU range (E4, E6400 etc.) does give an unlimited number of these fields (at least, it is only limited by memory capacity). It has been claimed that for use with ATS the tuning resolution of 1/64th of a semitone (764 steps to the octave) is a drawback on the EMU systems, and is therefore less attractive than the Akai and Roland systems (according to the manufacturers these units actually tune accurate to1 cent). At the time of writing, both Akai and Roland are in the process of making new and powerful samplers, and it must be hoped that these machines will remedy these problems to some degree. The new Akai S6000 for example is expandable to 256 Mbs RAM, and it is likely that the *proportion* of 'fields' to RAM will be similar to that in previous machines. If the number of 'fields' is greatly increased then it will be even more necessary to be able to assign samples to MIDI notes automatically.

[398] More often than not samples originally recorded in 12-ET would re-tuned and mapped to the fields accordingly. This employs less samples, but the number of fields required is the same.

Synthesisers are more popularly used than samplers for simulating ATS, partly for the above reasons, and often because they are less expensive. If 'micro-tuning' is implemented, synthesisers typically have one of two tuning architectures: the most common is the 'octave-table', in which each of the twelve conventional pitch classes may be tuned up or down by (about) 50 cents, which is automatically mapped across the keyboard range. This is convenient if a twelve division scale of some kind is required, but there is seldom (if ever) an option to 'stretch' the scale for different timbres, and of course it is impossible to configure the scale for greater than twelve divisions to the octave, or for non-octave scales. The alternative architecture is a 'keyboard table' in which each note of the standard keyboard range (88 notes), or the full MIDI range (127 notes), can be tuned independently - clearly this is more flexible. The ideal architecture is one which incorporates both approaches in a single function, somewhat as described above. In a 'tuning table template page' an option would be available to select the interval and the scale-degree at which the tuning system repeats - the default would naturally be 12, but could also be saved as something else - automatically reconfiguring the sounds as appropriate; scales could be equal-tempered or otherwise. A number of tables should be available in memory at any time, as a separate (global) parameter which can be applied to a selection of instruments, to a specific instrument, to a 'part', or track, or MIDI channel etc.. Similarly, it should be possible to switch instantly from one table to another via a standard MIDI protocol (SysEx). Equally, it should also be possible save and retrieve the tuning templates independently, and 'apply' them to an 'instrument', or group of instruments, automatically. Ideally, a standardised tuning interface could be implemented for various units -including samplers and synthesisers.

Various other new features in MIDI equipment would be advantageous. Samplers, especially, could have an on-board absolute pitch meter which would ideally be accurate to 0.1 of a cent or less; moreover, the pitch resolution of the machine ought to display its *actual* tuning resolution, not an approximation.[399] There is the vexed question of the ideal tuning resolution: just intonationists advocate higher tuning resolution (to avoid beating in just intervals) than those working with equal-temperaments; in this respect, MIDI instrument manufacturers could do worse than take a lead from the makers of the piano-tuning devices discussed earlier, which normally have a tuning resolution of 0.1 cents. Some composers, however, have argued for a better resolution than this.[400]

To simulate instruments of variable or fixed-but-variable intonation, each musical line must be recorded individually, with an appropriate use of pitch bend, otherwise normal intonational subtleties are lost.[401] To overcome the limitation of instruments of fixed intonation, samplers or synthesisers might have user-configurable 'adaptive' algorithms built-in to the module.[402] Alternatively, controller interfaces may be made more user-friendly. For example, the *Notebender* electronic keyboard was patented in 1978 by John S. Allen and *Key Concepts Inc.*, and a prototype was completed in 1983. The seven-black five-white layout of the keyboard is conventional, but the *Notebender* is significant for having introduced

> the use of undercut black keys to allow all keys to move either forward or back by fully three-quarters of an inch (19.5mm) without interfering with one another.[403]

Rubber inserts in the keytops allow the fingers to control longitudinal movement which may be assigned to control pitch (lateral grooves had been used in an earlier prototype). Allen claims that

> with the longitudinal key notion controlling pitch over a range of ± 2 semitones, the *Notebender* is somewhat more difficult to play than a conventional keyboard, though much less difficult

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399 As a consequence of these approximations (where for example the scale on the machine reads in cents, but the actual machine resolution is 1/64ths of a semitone) it must often the case that users unwittingly tune a pitch less accurately than necessary. I am grateful to the Mills Tuning List for discussion of these issues, and particularly to John Loffink, whose synthesiser website and discussion of a 'wish-list' for ATS have been particularly helpful. See http://freeweb.pdq.net/jloffink/

400 The MIDI Tuning Dump Standard supports a potential resolution of 0.0061 cents.

401 Another minor drawback for this kind of work is that the speed of vibrato of two tones double-stopped on an acoustic violin or cello are synchronised by the movement of the hand; this effect is lost in simulation.

402 See note 96.

403 See John S. Allen's website at http://web0.tiac.net/users/jsallen/music/introduc.htm. *Key Concepts* and *Notebender* are registered trademarks of *Key Concepts, Inc*. The *Notebender* keyboard is described in U.S patents: 4,068,552, 4,498,365, and 4,665,788; and in related foreign patents.

> than a clavichord. When the longitudinal key motion controls a variable which does not need as precise control as pitch, the *Notebender* keyboard is no more difficult to play than a conventional keyboard. The ability to adjust pitch either up or down gives the *Notebender* keyboard great expressive ability in the hands of a talented performer.[404]

The clavichord, of course, allows slight upward adjustment of pitch when pressure is sustained on a key to press the tangent against a string. It would seem that for composers wishing to make electronic simulations of instruments of non-fixed intonation, especially strings and woodwind, the ability to adjust minutely the pitch of each note individually at the keyboard, in real time, would be invaluable. For this to be possible the sound module must respond to the pitch changes for each note independently. Although this is 'possible' using 'aftertouch', it is not effective.

Allen is currently working on a 'generalised keyboard' for MIDI, based on a 'left-rising' Bosanquetian keyboard layout:

> My keyboard currently under construction has 48 keys per octave (with the design option to increase this to as many as 72 in future implementations), but the keys will be about 100 mm from front to rear and 1.375 cm across, the same size as the black keys of a conventional keyboard, allowing a conventional finger-over-thumb playing technique. The keyboard with 48 keys per octave will be about 60 cm deep and will be concave, allowing an easy reach to the rear keys. I intend to compensate for the smaller number of keys by using foot buttons to advance or retreat the note pattern by one row at a time [by this is meant the assignment of keys to musical notes]. This will only be necessary with scales that have very large numbers of notes per octave - not with the 31-tone scale on which I will base the most usual tuning. The hand, after all, can play on no more than three adjacent keys (from front to rear) at a time on this keyboard - a range sufficient to take in all the notes of the 31-tone scale.[405]

As we have seen, if this or another keyboard proved successful as a MIDI controller, then it might be adapted to an acoustic instrument. Allen's webpages contain a number of inventive alternatives for electronic keyboard design which are worth close examination.

As mentioned above, the Starrlabs *Microzone* keyboard features a 12 by 96 hexagonal array of black and white keys, which are arranged in a plane so that different patterns within the array will correspond to preferred scalic patterns and divisions of the octave - the expected variation being between 12 and 108 divisions. The keys themselves are spaced 1 inch (2.54 cm) apart, centre to centre.

> The purpose of the keyboard with its large array of keys is to implement any variety of tonal systems and divisions of the octave. The intent is to allow the user to define any arrangement of notes and fingerings which seem to be musically applicable to the situation at hand… In [certain] cases an octave [may be] laid out in any horizontal line by one full hand-span which includes a group of twelve black and white keys, as they might appear on a traditional piano keyboard.[406]

The unit supports all the expected functions of a MIDI mother-keyboard, such as velocity sensitivity, polyphonic aftertouch, layering and hotkeys etc., and is configured using an external computer (PC or Macintosh). The use of very small keys brings an extraordinary variety of tuning systems within the compass of the ordinary hand. It is to be hoped that the simplicity of design and components will enable the keyboard to be relatively affordable; it may also be available in different versions - for example with a full or reduced keyboard layout, depending on the user's requirements.

[404] As previous reference.
[405] Email correspondence with the author.
[406] From the description of the *Microzone* on the Starrlabs websiste: http://www.catalog.com/starrlab/uzone.htm.

Wilson has pointed out, naturally enough, that 'the Microzone is not a piano', and that it should not be judged on the extent to which conventional keyboard skills may be transferable to it. The 'keys' are rather like buttons, and quite a different technique will be required. However, as John S. Allen has commented -

> It should be remembered that accordionists use a button board successfully with the left hand, and on the Russian baran accordion, with both hands. The right hand buttons, used for melody playing as well as chords, are larger than the left-hand buttons - about 2 cm in diameter. Both sets of buttons are set in hexagonal arrays.[407]

Equally, the buttons of the *Microzone* may not provide the degree of dynamic subtlety that is expected from a piano (or from a high quality MIDI mother-keyboard), because the finger action is different to that of a conventional keyboard, and because the buttons do not 'depress'. This may mean that

> the minimum pressure to sound a note must be [slightly] increased to prevent false triggerings, and on a music keyboard, it reduces the dynamic range which can be controlled through the keys.[408]

However, as Wilson has pointed out, to replace these buttons with a more sophisticated technology would increase the cost of the Microzone to an unacceptable level.

An important aspect of the proposed Centre and overall project could be to channel and lend weight to new ideas as to how electronic and acoustic instruments might complement each other, and how electronic technology can support the development of new acoustic instruments. Leaving aside the Bosanquetian *Microzone*, the fact that there is today no commercially available electronic mother-keyboard which differs from the conventional 7-white 5-black layout, indicates how powerless the microtonal lobby is. Aside from commercial reasons, there is no overriding technical obstacle to creating a keyboard which acts as a kind of 'variable template' (comprising, say, two banks of 127 slots - there being 127 MIDI note numbers), and into which any selection of differently shaped or coloured keys could be slotted at will by the composer or performer. One would have to decide the layout of the slots into which the keys would fit, and the size of the keys themselves, and these might be variable.[409] Moreover, user configurable electronic keyboards could be used to good effect by pop musicians, many of whom would relish an 'alternative' keyboard. As suggested above, if a new acoustic piano were built with a non-standard keyboard layout, it would be necessary to make MIDI keyboards for performers and composers with that layout.

As part of the envisaged Centre, electronic facilities would serve a number of important functions:

- to provide instrument makers with the tools needed to simulate and 'speculate' about the results of acoustic instrument modifications;
- to provide for the production of interactive CD-ROMs for ear training for ATS;[410]
- to provide for the development of an ATS sample library for composers wishing to work with simulations of instrumental works. This could be developed in accordance with existing commercial products, and with special reference to the development of new instruments;
- as the basis of a performance and recording console;
- as a studio for composition.

There are many features for which composers working with ATS would be interested in pressing electronics manufacturers, including dedicated hardware and software. Again, the adoption of one or a few ATS standard(s) is probably an essential step toward encouraging the development specialist hardware and software

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[407] Email correspondence between J. S Allen and the author.
[408] Email correspondence between J. S Allen and the author.
[409] The order of black and white keys on most MIDI keyboards can be swapped, providing alternative keyboard layouts, but this does not diminish the problem of how far the hand can stretch.
[410] Special computerised ear-training programs would be especially useful for musicians interested in ATS. A manual that has been recommended to me is: Joseph Maneri and Scott A. Van Duyne: *Preliminary Studies in the Virtual Pitch Continuum*, Accentuate Music, Plainview, NY, 1986. (This is referenced in Douglas Leedy, 'Giving number a voice', *Xenharmonikôn* 15, 1993, p. 58). As yet I have been unable to obtain this work.

- especially alternative MIDI keyboards, and corresponding sequencing and notational software. A Centre should effect a formal channel through which a consensus of opinion could be presented, and play a catalytic role in new developments.

7. Conclusions

In this paper I hope to have represented:

- the need for new acoustic instruments;
- the value of an independent Centre and 'project network';
- the range of relevant current activities in the field;
- the wide range of criteria relevant to evaluating alternative tuning systems;
- some benefits which would result from establishing a provisional ATS standard;
- the possibility of a coevolutionary approach to instrumental design, timbre and tuning system;
- the viability of new design technologies;
- the range of relevant research areas;
- the potential for international collaboration in the field.

A Giant Jigsaw Puzzle

The case for new acoustic instruments is overwhelming. Although electronic instruments have immense potential, a very considerable proportion of contemporary composers write, and wish to continue to write, for acoustic instruments. There is no doubt that, for the vast majority of these composers, electronic instruments do not yet 'replace' acoustic instruments; nor can we be certain that electronic instruments will 'supersede' them. Meanwhile, whilst mainstream acoustic instruments remain essentially unchanging, they are in danger of acquiring an increasingly historical aura.[411] Electronic instruments may give birth to the 'orchestra' of the future - but mainstream acoustic instruments too must be brought into the 21st Century.

In 1987 the editor of *Computer Music Journal* prefaced a special issue on microtonality with the following words:

> the proof of a microtonal theory is not a set of mathematical expressions but rather a body of successful musical compositions.[412]

This is right. Composers working with ATS are not just interested in proving mathematical theorems - they want to create new music - not only with electronic but also acoustic instruments. New acoustic instruments - a new palette of tones and timbres - are essential if composers are to be inspired to create, and performers to realise, those 'successful compositions'. The contemporary acoustic instruments of mainstream Western music should both reflect and stimulate new directions.[413] The main purpose of this text has been to show that a wide ranging collaborative project is necessary if new instruments are to be created. Instrumental technologies for ATS should be researched; investigation of new acoustic timbres and of the 'coevolutionary' relation of timbre and tuning carried out; a provisional ATS standard (or standards) for a new system of instruments should be agreed; new instruments built; and new works composed and performed.

It is commonplace to hear that 'acoustic instruments cannot be built for microtonality', and that, 'even if they could, no manufacturer will make them because there is no market'. Similar complaints are heard regarding instrumental adaptations for other musical approaches. It seems that composers, performers and instrument makers are resigned to, or accept as inevitable, the want of radical developments of 'mainstream' acoustic instruments during the last 150 years. Some are pleased to move on to electronics, to what is feasible now, and to what may be feasible in future. But the moratorium on acoustic innovations is premature, and seems to have

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[411] Of course, claims of anachronism can be exaggerated - the violin, for example, has had an equally valid role in the 20th Century as it had in the 18th Century. Yet as electronic instruments become more sophisticated, the timbres they are capable of producing will become increasingly complex and dynamic. Paradoxically, it is precisely this aspect of acoustic instrumental sounds which as yet has proved so elusive to electronic simulation - while the dynamic aspect of electroacoustic production threatens the 'simplicity' of conventional instrumental timbre.

[412] Curtis Roads, *Computer Music Journal*, Vol. 11, No. 1, Spring 1987, p. 3.

[413] It is taken for granted here that the development new instruments for ATS is just one new acoustic direction amongst others; but, as I have tried to show, since 'composition with tones' is not 'over', there are very many factors, such as the relations of tuning and timbre, manufacturing standards, new technologies etc., which suggest the centrality of this project.

been the result, not of having reached the limits of acoustic possibility, but of the absence of new standards, and the commercial and technical monopoly achieved by corporate and well-established instrument manufacturers. It is also due, of course, to an accustomed and understandable conservatism amongst players, who have worked long and hard to master existing instruments, and '12-ET'. Nevertheless, so long as no change in this general situation occurs, radical innovations of electronic instruments will totally outpace acoustic developments. Unfortunately, there is a lack of consensus as to what can be done about these problems.

It has been argued above that it is neither 'impossible', nor 'unrealistic', nor 'too expensive', to build mainstream orchestral instruments for (one or a number of) ATS. Nevertheless, the problem *is* a giant jigsaw puzzle. If we are to create a new, provisional, 'quasi-universal' system of instruments and tuning, then instrumental feasibility, the relations between timbre and tuning, the mathematical and logical analysis of ATS, performance and notation, - must all fit together. The view has also been put forward here, perhaps controversially amongst some advocates of ATS, that there may be considerable value in 'modelling' (in a deep sense) new developments on the extraordinarily successful '12-ET system' itself, and on the instrumental and tuning systems of other cultures. In Western music, the division of the octave into 12 is deeply 'coincidental': the receptive structure of the ear appears to be attuned at an early age to harmonic sounds (Terhardt); the just intonation (major) scale is the 'related' scale to the 'dissonance curves' for harmonic spectra (Sethares); 12-ET is the most nearly 'related' equal-tempered scale (Sethares); the spectra of vibrating strings and pipes are harmonic; and 12-ET has proved manageable and versatile - vocally, instrumentally and compositionally.

The practice and success of 12-ET (or at least '12' in some form) relies on the coincidence of aural intuition and performance practicality, together with compositional and instrumental (manufacturing) standards. Piecemeal attempts to alter one parameter of this coincident system, without simultaneously considering others, are liable to fail - as in some respects has been the case for a proportion of Western music exploring ATS. However - 12-ET is not alone in having 'coincidental' properties. To devise a new acoustic instrument/tuning system, the coevolutionary process itself may have to be 'modelled', in all its parameters, to arrive at another deeply coincident system. Such a project can hardly be undertaken by individuals alone.

A Project Network

Suppose that a network of composers, instrument makers, performers etc., establish a long-term, collaborative, national or international project. Imagine that a number of research institutes or universities,[414] some commercial instrument manufacturers, and considerably more individual makers and researchers each agree to undertake a limited period of research into a single new instrument, or specific project area. These research projects could include: the upper practical limit of discrete octave divisions for a given instrument (ET or otherwise); the possibility of achieving unique *and* multiple ATS; relevant individual technologies such as solenoids, pitch sensors, piezo-electrics, 3-position brass valves, 'silent' stepper motors, and combinative logic for electro-mechanical keying of woodwind and brass etc.; the relations of tuning and timbre; mathematical tuning theory; and many others. Suppose that interested composers are invited to compose new works to demonstrate the musicality and musical resourcefulness of an ATS system of their choice. Suppose that all of these activities are networked via an internet forum.

Suppose that after a sensible period these groups are brought together in local conferences, and an international conference; results are reported, prototypes presented, and a selection of new compositions are performed - where possible using existing acoustic instruments, but at least as electronic simulations. Before it is automatically assumed that such conferences are not affordable - perhaps an organisation such as the *International Society for Musical Acoustics*, or similar, might host the project as a theme for one of its annual conferences? Existing concert series might adopt the project in the early years of the new millennium, as the first prototype new instruments become playable. Suppose that a provisional conclusion may be reached regarding whether a single technology (such as solenoids, piezo-electrics, pitch sensors etc.,) could be applied

[414] This is only feasible if the project is incorporated into the official and remunerated academic agenda of the institutions and researchers involved. Local funding might however be found to create new posts; in Europe a strong case could be made for EC funding to support an international collaboration; likewise, there are surely many opportunities for postgraduate research fellowships to be associated with the project.

to a wide range of instrumental forms, substantially reducing the costs of new instruments; suppose that a single, provisional ATS standard, or a 'group of standards', can be agreed.

We may suppose too, that research could turn up new instrumental forms, new tuning and timbral possibilities, and new kinds of acoustic-electronic hybrids. As things stand today, individual instrument makers seldom hope to benefit from small-scale improvements or inventions, because patenting is too expensive, and an individual cannot hope to market an idea against corporate competition. Secondly, as has been mentioned, the absence of an ATS standard (or standards) cripples innovation. Thirdly, major commercial instrument manufacturers guard the details of their products, naturally enough for commercial interest.

The project outlined therefore has the following practical advantage. Outside the commercial field, universities, research institutes and music colleges sponsor important work in acoustics, instrument design and manufacture, psychoacoustics, relevant technologies, as well as composition, performance and other relevant topics. These institutions retain a wealth of expertise. At the same time, throughout Europe and the United States (at least) there are many instrument makers (and inventors) who make a living from their considerable knowledge and skills of instruments and instrument making, without affiliation to either commercial or academic institutions. What better way, then, to create new momentum for acoustic instruments, than to create a project in which many instrument makers (and technologists, composers, etc,) - who own a computer and have a passion for new acoustic instruments - can link up, contribute to and partake of an exciting new movement in instrument design, co-ordinated by a dedicated Centre, and overseen by the finest academics in the field? [415]

Harmony and Revolution

'Alternative tuning systems' have been presented here neither as a messianic nor as an independent 'way forward' in contemporary and other musics. It has been taken for granted that the exploration of ATS is just one amongst many interrelated aspects of new music, and is inextricable, both in practice and theory, from issues of timbre, rhythm, tempo, expression and form. The relationship between 'style' and ATS, for example, is complex, and has hardly been touched on here. The main issue has been the formidable implications of ATS for acoustic instrumental innovation.

Tuning and temperament have been essential to Western musical experience, at least up to the electroacoustic era. Correspondingly, Western harmony has evolved slowly - but in the 20th Century it has been shaken by an earthquake. The enjoyment of atonality amongst a large majority of informed musicians is well established, but the more fundamental change over the last 50 years has been the 'emancipation of sound', not only in avant-garde instrumental music, but especially in electroacoustic and popular music. In the past, the historical development of Western tuning systems has been inseparable from the evolution of harmony, understood in its deepest sense. Today, that relationship is thrown into question in a way that is extremely complex. For example, the experience of 12-ET (or '12 in some form') is more or less harmonic, yet the Western system of tuning/instruments has enabled new musical languages to emerge which, on the surface, are not at all harmonic.

Moreover, tuning and temperament, allied to issues of timbre, will continue to have an important role in the current and foreseeable development of classical and other musics. After over 500 years of 'harmonic' music, in which diatonic and chromatic systems have become second nature to all Western musicians and listeners, it is clear that the adoption of a parallel, extended, 'enharmonic' system of tuning poses a series of complex interrelated questions. The most difficult of these to confront is that the adoption of an alternative system might seem to threaten conventional musical practice and knowledge. But this is not really the case, since, just as the native tongue is essential in learning a new language, so is existing musical knowledge essential in learning an alternative tuning system. The reward for embracing a new tuning and instrumental system would be a radically new universe of vocal and instrumental music, which is at the same time deeply connected to a tradition.

[415] An issue here is whether a principle of complete openness about new ideas or inventions is practical or acceptable. Certainly, if new ideas are shared on an internet forum, and are regularly published as part of an on-going project, to some extent they are protected from patent by a corporate manufacturer. On the other hand, publicising an idea without patent makes it difficult or impossible, other than through sheer economic power, for anyone to obtain the kind of monopoly that can make an idea profitable.

Musical history, in the sense of ‘compositional progress’, is made by composers with special gifts, who have something new and profound to say. Ultimately, it is *what* a composer expresses that is most important, not the resources, style or technique through which that expression is made, even if those aspects are inseparable. Nevertheless, there is a fundamental interdependence between available musical resources and the direction which an individual composer or a tradition may take. Over the last 50 years especially, Western acoustic instruments themselves have failed to adapt, and are perhaps in danger of becoming fossilised. At the same time, in terms of the instrumental realisation of works in alternative tuning systems, Western music has as yet merely been star-gazing: the truly revolutionary cosmography of ‘other’ harmonic universes still awaits us. The acoustic instrumentarium must be revolutionised.

APPENDIX I (A)

‘Interval zones’ and ‘rational intervals’ against which the tuning systems in APPENDIX II are compared.

Interval Zone		Just Ratios (dominant intervallic area(s) within a zone)					
Interval	**Abbr**	**3-limit &‘Pythagorean’**	**5-limit**	**7-limit**	**11-limit**	**13-limit and above**	**Cents**
Unison	U	1/1					0
‘1/8th or 1/6th Tone’	U^{+}	(UNRESOLVED	IN	TERMS	OF	RATIOS)	25.0 - 33.3
33rd Harmonic or ‘1/4-Tone’	m2^{11}				(33/32)		53.27
Small Semitone	m2-		25/24				70.67
Subminor 2nd or	m2^{7}			21/20			84.47
Pythagorean Semitone	m2^{py}	(256/243)					90.23
17th Harmonic	m2^{17}					17/16	104.96
Just Semitone	m2		16/15				111.73
Large Semitone or	m2+		27/25				133.24
Large Semitone (13)	m2^{13}					13/12	138.57
Neutral 2nd or	N2				12/11		150.64
Ptolemy’s 2nd	M2^{pt}				11/10		165.00
Small Just 2nd	M2-		10/9				182.40
Just 2nd	M2	9/8					203.91
Septimal 2nd	M2^{7}			8/7			231.17
Septimal Minor 3rd	m3^{7}			7/6			266.87
Pythagorean Minor 3rd or	m3^{py}	32/27					294.14
19th Harmonic	m3^{19}					19/16	297.51
Quasi ‘Tempered’ Minor 3rd	m3^{qt}			25/21			301.85
Just Minor 3rd	m3		**6/5**				315.64
Neutral 3rd (11) or	N3^{11}				11/9		347.41
Neutral 3rd (13)	N3^{13}					16/13	359.47
Just Major 3rd	M3		**5/4**				386.31
Pythagorean Major 3rd or	M3^{py}	(81/64)					407.82
Large Major 3rd (11)	M3^{11}				14/11		417.51
Just Large 3rd or	M3+		32/25				427.37
“Supermajor” 3rd	M3^{7}			9/7			435.08
49th Harmonic Undertone	4^{49}			(64/49)			462.35
or 21st Harmonic	4^{21}			21/16			470.78
Just Perfect 4th	4	**4/3**					498.045
11th Harmonic	4^{11}				11/8		551.32
Septimal Augmented 4th or	4+^{7}			7/5			582.51
Just Augmented 4th	4+		(45/32)				590.22
Just Diminished 5th or	5-		(64/45)				609.78
Septimal Diminished 5th	5-^{7}			10/7			617.49
11th Harmonic Undertone	5^{11}				16/11		648.68
Just Perfect 5th	5	**3/2**					701.955
21st Harmonic Undertone	5^{21}			32/21			729.22
49th Harmonic	5^{49}			(49/32)			737.65
Septimal Minor 6th or	m6^{7}			14/9			764.92
Small Just Minor 6th	m6-		25/16				772.63
Small Minor 6th (11) or	m6^{11}				11/7		782.49
Pythagorean Minor 6th	m6^{py}	(128/81)					792.18
Just Minor 6th	m6		**8/5**				813.69
Neutral 6th (13) or	N6^{13}					13/8	840.53
Neutral 6th (11)	N6^{11}				18/11		852.59
Just Major 6th	M6		**5/3**				884.36
Quasi ‘Tempered’ Major 6th	M6^{qt}			42/25			898.15
or Major 6th (19)	M6^{19}					32/19	902.49
or Pythagorean Major 6th	M6^{py}	27/16					905.87
Septimal Major 6th	M6^{7}			12/7			933.13
Harmonic minor 7th	m7^{7}			7/4			968.83
Minor 7th	m7	16/9					996.09
Large Minor 7th	m7+		9/5				1017.60
Large Minor 7th (11)	m7^{11}				20/11		1035.00
Neutral 7th (11)	N7				11/6		1049.36
Small Major Seventh (13) or	M7^{13}					24/13	1061.43
Small Major 7th	M7-		50/27				1066.76
Just Major 7th	M7		15/8				1088.27
Major 7th (17)	M7^{17}					32/17	1095.05
Pythagorean Major 7th	M7^{py}	(243/128)					1109.78
Septimal Major 7th	M7^{7}			40/21			1115.53
Diminished Octave	M7+		48/25				1129.33
33rd Harmonic Undertone	M7^{11}				(64/33)		1146.73
Small Octave	U^{-}	(UNRESOLVED	IN	TERMS	OF	RATIOS)	1166.6 - 75
Unison (Octave)	U	2/1					1200

APPENDIX I (B)

This provisional table shows dyadic intonational tolerance for sustained violin tones (with and without vibrato). The 'Upper' and 'Lower Zone Limits' describe subjective limits within which a dyad, considered in isolation, retains an aural identity or character. The 'nominal zone centre' would define the 'zone score' in APPENDIX II, but in itself is not acoustically significant.

Interval Zone		Intonational Tolerance						
Interval	**Ratio**	**Lower Zone Limit**	**'Rational' Interval**	**Upper Zone Limit**	**Grey Area - undefined?**	**Zone Width (?)**	**'Nominal' Zone Centre (?)**
'1/8th or 1/6th Tone'	-	18??	**N/A**	48??	all grey!	?	?
33rd Harmonic or '1/4-Tone'	33/32	41??	**53.27**	63??	all grey!	?	?
Small Semitone	25/24	64?	**70.67**	78?	all grey!	14?	71
Subminor 2nd or Pythagorean Semitone	21/20 256/243	79?	**84.47** **90.23**	95?	all grey!	16?	87.0
17th Harmonic Just Semitone	17/16 16/15	95?	**104.96** **111.73**	118?	all grey!	23?	106.5
Large Semitone or Large Semitone (13)	27/25 13/12	128?	**133.24** **138.57**	143?	all grey!	15?	135.5
Neutral 2nd or Ptolemy's 2nd	12/11 11/10	146?	**150.64** **165.00**	170?	all grey!	24?	158
Small Just 2nd	10/9	174?	**182.40**	190?	all grey!	12?	182
Just 2nd	9/8	194?	**203.91**	210	211-224?	16?	202
Septimal 2nd	8/7	225?	**231.17**	237	238-259?	12?	231
Septimal Minor 3rd	7/6	260?	**266.87**	286	287-290?	26?	273
Pythagorean Minor 3rd or 19th Harmonic or Quasi 'Tempered' Minor 3rd	32/27 19/16 25/21	290(?)	**294.14** **297.51** **301.85**	308?	All grey!	18?	299
Just Minor 3rd	**6/5**	308?	**315.64**	320 (h) 322 (m)	321-342	12?	314
Neutral 3rd (11) or Neutral 3rd (13)	11/9 16/13	342?	**347.41** **359.47**	363?	364-381?	21?	352.5
Just Major 3rd	**5/4**	382 (h) 380 (m)	**386.31**	402 (h) 408 (m)	all grey!	20? (h) 28? (m)	392
Pythagorean Major 3rd or Large Major 3rd (11)	81/64 14/11	406?	**407.82** **417.51**	420?	421-424?	14?	413
Just Large 3rd or "Supermajor" 3rd	32/25 9/7	425?	**427.37** **435.08**	445?	446-456?	20?	435
49th Harmonic Undertone or 21st Harmonic	64/49 21/16	457?	**462.35** **470.78**	475?	476-491?	18?	466
Just Perfect 4th	**4/3**	492 (h) 488 (m)	**498.045**	504 (h) 508 (m)	509	12?(h) 20? (m)	498
11th Harmonic	11/8	541	**551.32**	558	559-575?	17?	549.5
Septimal Augmented 4th or Just Augmented 4th	7/5 45/32	576?	**582.51** **590.22**	600??	all grey!	24?	588
Just Diminished 5th or Septimal Diminished 5th	64/45 10/7	600??	**609.78** **617.49**	627?	628-634?	27?	613.5
11th Harmonic Undertone	16/11	635?	**648.68**	670?	671-?	35?	652.5
Just Perfect 5th	**3/2**	694 (h) 690 (m)	**701.955**	710 (h) 712 (m)	713-726?	16? (h) 22? (m)	702
21st Harmonic Undertone 49th Harmonic	32/21 49/32	727?	**729.22** **737.65**	744?	745-756?	17?	735.5
Septimal Minor 6th or Small Just Minor 6th	14/9 25/16	757?	**764.92** **772.63**	774?	775-778?	17?	765.5
Small Minor 6th (11) or Pythagorean Minor 6th	11/7 128/81	779?	**782.49** **792.18**	795?	N/A all grey!	16?	787
Just Minor 6th	**8/5**	796?	**813.69**	822?	823-834?	26?	809
Neutral 6th (13) or Neutral 6th (11)	13/8 18/11	835?	**840.53** **852.59**	855?	856-876?	20?	845
Just Major 6th	**5/3**	877?	**884.36**	892?	N/A	15?	884.5
Quasi 'Tempered' Major 6th or Major 6th (19) or Pythagorean Major 6th	42/25 32/19 27/16	893?	**898.15** **902.49** **905.87**	915?	916-926?	22?	904
Septimal Major 6th	12/7	927?	**933.13**	939?	940-961?	12?	933
Harmonic minor 7th	7/4	962?	**968.83**	978?	979-985??	16?	970
Minor 7th	16/9	986??	**996.09**	1006??	N/A	20?	996
Large Minor 7th	9/5	1007??	**1017.60**	1022??	1023-1029??	15?	1014.5
Large Minor 7th (11) Neutral 7th (11)	20/11 11/6	1030??	**1035.00** **1049.36**	1054??	1055-1057??	24?	1042
Small Major Seventh (13) or Small Major 7th	24/13 50/27	1058??	**1061.43** **1066.76**	1072??	all grey!	14?	1065
Just Major 7th Major 7th (17)	15/8 32/17	1082??	**1088.27** **1095.05**	1106??	all grey!	24?	1094
Pythagorean Major 7th Septimal Major 7th	243/128 40/21	1107??	**1109.78** **1115.53**	1120??	all grey!	?	1113.5
Diminished Octave	48/25	1126??	**1129.33**	1136?	all grey!	?	1131
33rd Harmonic Undertone	64/33	??	**1146.73**	??	all grey!	?	?

APPENDIX I (C)

APPENDIX I (C) resembles APPENDIX I (B), in this case showing 22 intervallic zones corresponding the following rational intervals: 3/2 (4/3), 5/4 (8/5), 7/4 (8/7), 9/8 (16/9), 6/5 (5/3), 7/6 (12/7), 11/8 (16/11), 7/5 (10/7), 10/9 (9/5), 9/7 (14/9) and 16/15 (15/8). These are the 'primary zones/intervals' for which APPENDIX IV details some comparisons of equal-temperaments.

Interval Zone		**Intonational Tolerance**					
Interval	**Ratio**	**Lower Zone Limit**	**'Strongest' Rational Intervals**	**Upper Zone Limit**	**Grey Area**	**Zone Width (?)**	**'Nominal' Zone Centre (?)**
Just Semitone	16/15	95?	**111.73**	118?	119-128?	??	106.5
Small Just 2nd	10/9	176	**182.40**	188?	189-193?	??	182
Just 2nd	9/8	194	**203.91**	210	211-224?	??	202
Septimal 2nd	8/7	225	**231.17**	237	238-259?	??	231
Septimal Minor 3rd	7/6	260	**266.87**	286	287-290?	??	273
Just Minor 3rd	**6/5**	308?	**315.64**	320 (h)	321-342	??	314
Just Major 3rd	**5/4**	382 (h)	**386.31**	402 (h)	N/A	??	392
"Supermajor" 3rd	9/7	425?	**435.08**	472?	473-491?	??	435
Just Perfect 4th	**4/3**	492 (h)	**498.045**	504 (h)	509	??	498
11th Harmonic	11/8	541	**551.32**	558	559-575?	??	549.5
Septimal Augmented 4th	7/5	576?	**582.51**	600??	all grey	??	588
Septimal Diminished 5th	10/7	600??	**617.49**	627	628-634?	??	613.5
11th Harmonic Undertone	16/11	635	**648.68**	670?	671-?	??	652.5
Just Perfect 5th	**3/2**	694 (h)	**701.955**	710 (h)	713-731?	??	702
Septimal Minor 6th	14/9	?	**764.92**	774?	775-778?	??	765.5
Just Minor 6th	**8/5**	796?	**813.69**	822?	823-834?	??	809
Just Major 6th	**5/3**	877?	**884.36**	892?	N/A	??	884.5
Septimal Major 6th	12/7	927?	**933.13**	939?	940-961?	??	933
Harmonic minor 7th	7/4	962?	**968.83**	978?	979-985??	??	970
Minor 7th	16/9	986?	**996.09**	1006?	N/A	??	996
Large Minor 7th	9/5	1007?	**1017.60**	1022?	1023-1029??	??	1014.5
Just Major 7th	15/8	1082??	**1088.27**	1106??	N/A??	??	1094

APPENDIX II

44 TUNING SYSTEMS COMPARED TO THE RATIOS AND INTERVAL ZONES GIVEN IN APPENDIX I

These tables show the 'goodness of fit' to small integer ratios from 1/1 for a selection of scales and temperaments, comparing each interval to its nearest equivalent in APPENDIX I (a). The 'Just Major' and 'Just Minor' scales, the 'Pythagorean' scale, and 'Werkmeister III' are listed in Tables 1 to 4; equal-temperaments from 5- to 41-ET are listed in Tables 5 to 41; Table 43 shows Harry Partch's 43 division Just Intonation scale. Note the broad similarity between Partch's scale and the intervals of APPENDIX I (a). Lastly, Tables 42 (a) and (b) show Erv Wilson's (2/4) 1 3 5 7 Hexany, and the (3/6) 1 3 5 7 9 11 Eikosany (these are not in Wilson's original format) and the analysis of these scales is left to the reader.

However, the purpose of these tables has not been entirely completed. Considering the controversial issues surrounding the 'primariness' or 'naturalness' of low integer ratio intervals, it was intended that these tables *also* show to what degree the intervals of the ET systems fall within the interval *zones* of APPENDIX I (b) (and alternatively relative to the less 'microtonal' APPENDIX I (c)). Unfortunately, it proved impossible to establish a set of definite results in this respect, due to the provisional nature of APPENDICES I (b) and (c). The intention, however, was to provide a rough quantification of the 'xenharmonic' usefulness of each temperament, either independently or in conjunction with the comparison to just ratios.

For example, for each interval of each temperament, a 'zone score' was variously calculated in terms of functions of its closeness to the 'nominal zone centre'; the 'zone-harmonicity' of each step of the temperament was then calculated - by combining the 'zone score' with a function (eg. 'flattened' versions of the Euler GS value or Barlow's 'harmonicity' value) of the nearest ratio in APPENDIX I (a); or, by choosing the 'lowest harmonicity value' for each interval which arose from that function, considering each of the nearest ratios within a 'zone' (using either APPENDIX I (b) or (c)). For each temperament, these results were considered independently, or combined in a further function with the mean deviation of the temperament from the rational intervals. Three ET systems consistently fared better than the others across each of these tests: 12-, 31- and 41-ET.

However, due to the highly provisional status of the 'zone' analysis of APPENDIX I (b), and the somewhat arbitrary status of the GS and 'harmonicity' functions,[1] it was decided to withhold these results until a more sophisticated analysis was completed. For example, it was thought that replacing the prime factor functions with values from a dissonance curve (taking into consideration a relatively high number of harmonic partials) would provide more objective results. The analysis would combine three significant features: sensory consonance, approximation of low integer ratios, and accuracy relative to a 'zone'.

The comparison of octave division equal-tempered systems to 'rational intervals' is always symmetrical about the tritone. A 'zone' component in this analysis is therefore also relevant since the relative consonance of symmetrical elements cannot claimed to be equal - that is, both halves of the scale are shown because the 'zones' are not symmetrical about the tritone. Note that Table 12(a) compares 12-ET to the Just Intonation (major) scale; Table 12(b), like all the others, compares 12-ET to the intervals of APPENDIX I.

This APPENDIX was prepared jointly by Joseph Sanger and Patrick Ozzard-Low.

[1] Chalmers has commented that 'GS [*gradus suavitatis*] would predict slightly mistuned consonances to be extremely dissonant, a prediction not consistent with observation. This failure, however, is a feature shared by the other simple theories of consonance based upon the prime factorisation of intervals. Helmholtz's beat theory… and the semi-empirical 'critical band' theories of Plomp and Levelt and Kameoka and Kuriyagawa avoid predicting infinite dissonance for mistuned consonances, but are more complex and difficult to use. The prime factor theories are adequate for theoretical work and for choosing between ideally tuned structures.' John H. Chalmers, *Divisions of the Tetrachord,* Frog Peak Music, 1993, pp. 66-7. Yet, for example, the Euler value for the interval of 19/16 (297.51 cents) is 23; the Euler value of 256/243 (207.75 cents) is 19; that is, the function predicts that 256/243 is more consonant (or more primary) than 19/16. Barlow's function makes similarly unconvincing predictions; moreover, the two functions predict a different *order* of harmonicity for the same values. This leads one to seek explanations of the functions in empirical terms: otherwise it is tempting to think that a function exists to confirm a pre-decided ordering of the smaller ratio intervals, which may then be extrapolated on trust.

Table 1 Just Intonation (Major)

Just (Major)		**Interval Zone**		**Closest Ratio from APPENDIX 1 (A)**							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	111.73	Just Semitone	m2		16/15				111.73		0
2	203.91	Just 2nd	M2	9/8					203.91		0
3	315.64	Just Minor 3rd	m3		6/5				315.64		0
4	386.31	Just Major 3rd	M3		5/4				386.31		0
5	498.04	Just Perfect 4th	4	4/3					498.04		0
6	590.22	Just Augmented 4th	4+		45/32				590.22		0
7	701.96	Just Perfect 5th	5	3/2					701.96		0
8	813.69	Just Minor 6th	m6		8/5				813.79		0
9	884.36	Just Major 6th	M6		5/3				884.36		0
10	996.09	Minor 7th	m7	16/9					996.09		0
11	1088.27	Just Major 7th	M7		15/8				1088.27		0
12	1200	Unison (Octave)	U	2/1					1200		0

Table 2 Just Intonation (Minor)

Just (minor)		**Interval Zone**		**Closest Ratio from APPENDIX 1 (A)**							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	70.67	Small Semitone	m2-		25/24				70.67		0
2	182.40	Small Just 2nd	M2-		10/9				182.40		0
3	315.64	Just Minor 3rd	m3		6/5				315.64		0
4	386.31	Just Major 3rd	M3		5/4				386.31		0
5	498.04	Just Perfect 4th	4	4/3					498.04		0
6	590.22	Just Augmented 4th	4+		45/32				590.22		0
7	701.96	Just Perfect 5th	5	3/2					702.96		0
8	813.69	Just Minor 6th	m6		8/5				813.69		0
9	884.36	Just Major 6th	M6		5/3				884.36		0
10	996.09	Minor 7th	m7	16/9					996.09		0
11	1088.27	Just Major 7th	M7		15/8				1088.27		0
12	1200	Unison (Octave)	U	2/1					1200		0

Table 3 Pythagorean Scale - Wolf at F#/C#

Pythagorean		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3- limit	5- limit	7- limit	11- limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	90.23	Pythagorean Semitone	$m2^{py}$	256/243					90.23		0
2	203.91	Just 2nd	M2	9/8					203.91		0
3	294.14	Pythagorean Minor 3rd	$m3^{py}$	32/27					294.14		0
4	407.82	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		0
5	498.04	Just Perfect 4th	4	4/3					498.04		0
6	609.78	Just Diminished 5th	5-		64/45				609.78		+2
7	701.96	Just Perfect 5th	5	3/2					701.96		0
8	792.18	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		0
9	905.87	Pythagorean Major 6th	$M6^{py}$	27/16					905.87		0
10	996.09	Minor 7th	m7	16/9					996.09		0
11	1109.78	Pythagorean Major 7th	$M7^{py}$	243/128					1109.78		0
12	1200	Unison (Octave)	U	2/1					1200		0

Table 4 Werckmeister III[2]

W. III		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3- limit	5- limit	7- limit	11- limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	90	Pythagorean Semitone	$m2^{py}$	256/243					90.23		0
2	192	Small Just 2nd	M2-		10/9				182.40		+10
3	294	Pythagorean Minor 3rd	$m3^{py}$	32/27					294.14		0
4	390	Just Major 3rd	M3		5/4				386.31		+4
5	498	Just Perfect 4th	4	4/3					498.04		0
6	588	Just Augmented 4th	4+		45/32				590.22		-2
7	696	Just Perfect 5th	5	3/2					701.96		-6
8	792	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		0
9	888	Just Major 6th	M6		5/3				884.36		+4
10	996	Minor 7th	m7	16/9					996.09		0
11	1092	Just Major 7th	M7		15/8				1088.27		+4
12	1200	Unison (Octave)	U	2/1					1200		0

[2] This table is the sole example of an irregular temperament in this APPENDIX. This circulating temperament has also been called 'Werckmeister's correct temperament, No.1 (1/4 comma)', see: Murray Barbour, *Tuning and Temperament*, East Lansing Michigan State College Press, 1951, p. 162, Table 140. The temperament is given correct to the nearest cent, and is compared to just ratios accordingly.

Table 5 5 Division Equal Temperament

5-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.		Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	240.00	Septimal 2nd	$M2^{7}$			8/7			231.17		8.83
2	480.00	21st Harmonic	4^{21}			21/16			470.78		9.22
3	720.00	21st Harmonic Undertone	5^{21}			32/21			729.22		-9.22
4	960.00	Harmonic 7th	$m7^{7}$			7/4			968.83		-8.83
5	1200.00	Unison (Octave)	U	2/1					1200		0

Table 6 6 Division Equal Temperament (Whole Tone Scale)

6-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	200.00	Just 2nd	M2	9/8					203.91		-3.91
2	400.00	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		-7.82
3	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
4	800.00	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		+7.82
5	1000.00	Minor 7th	m7	16/9					996.09		+3.91
6	1200.00	Unison (Octave)	U	2/1					1200		0

Table 7 7 Division Equal Temperament

7-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	171.43	Ptolemy's 2nd	$M2^{pt}$				11/10		165.00		+6.43
2	342.86	Neutral 3rd (11)	$N3^{11}$				11/9		347.41		-4.55
3	514.29	Just Perfect 4th	4	4/3					498.05		+16.24
4	685.71	Just Perfect 5th	5	3/2					701.95		-16.25
5	857.14	Neutral 6th (11)	$N6^{11}$				18/11		852.59		+4.55
6	1028.57	Large Minor 7th (11)	$m7^{11}$				20/11		1035.00		-6.43
7	1200.00	Unison (Octave)	U	2/1					1200		0

Table 8 8 Division Equal Temperament

8-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	150.00	Neutral 2nd	N2				12/11		150.64		-0.64
2	300.00	Quasi Tempered Minor 3rd	$m3^{qt}$			25/21			301.85		-1.85
3	450.00	49th Harmonic Undertone	4^{49}			64/49			462.35		-12.35
4	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
5	750.00	49th Harmonic	5^{49}			49/32			737.65		+12.35
6	900.00	Quasi-Tempered Major 6th	$M6^{qt}$			42/25			898.15		+1.85
7	1050.00	Neutral 7th	N7				11/6		1049.36		+0.64
8	1200.00	Unison (Octave)	U	2/1					1200		0

Table 9 9 Division Equal Temperament

9-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	133.33	Large Semitone	m2+		27/25				133.24		0.09
2	266.67	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		-0.2
3	400.00	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		-7.82
4	533.33	11th Harmonic	4^{11}				11/8		551.32		-17.99
5	666.67	11th Harmonic Undertone	5^{11}				16/11		648.68		+17.99
6	800.00	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		+7.82
7	933.33	Septimal Major 6th	$M6^{7}$			12/7			933.13		+0.20
8	1066.67	Small Major 7th	M7-		50/27				1066.76		-0.09
9	1200.00	Unison (Octave)	U	2/1					1200		0

Table 10 10 Division Equal Temperament

10-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	120.00	Just Semitone	m2		16/15				111.73		+8.27
2	240.00	Septimal 2nd	$M2^{7}$			8/7			231.17		+8.83
3	360.00	Neutral 3rd (13)	$N3^{13}$					16/13	359.47		+0.53
4	480.00	21st Harmonic	4^{21}			21/16			470.78		+9.22
5	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.220 609.78		+9.78 -9.78
6	720.00	21st Harmonic Undertone	5^{21}			32/21			729.22		-9.22
7	840.00	Neutral 6th (11)	$N6^{11}$					13/8	840.53		-0.53
8	960.00	Harmonic 7th	$m7^{7}$			7/4			968.83		-8.83
9	1080.00	Just Major 7th	M7		15/8				1088.27		-8.27
10	1200.00	Unison (Octave)	U	2/1					1200		0

Table 11 11 Division Equal Temperament

11-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	109.09	Just Semitone	m2		16/15				111.73		-2.64
2	218.18	Septimal 2nd	$M2^{7}$			8/7			231.17		-12.99
3	327.27	Just Minor 3rd	m3		6/5				315.64		+11.63
4	436.36	“Supermajor” 3rd	$M3^{7}$			9/7			435.08		+1.28
5	545.45	11th Harmonic	4^{11}				11/8		551.32		-5.87
6	654.55	11th Harmonic Undertone	5^{11}				16/11		648.68		+5.87
7	763.64	Septimal Minor 6th	$m6^{7}$			14/9			764.92		-1.28
8	872.73	Just Major 6th	M6		5/3				884.36		-11.63
9	981.82	Harmonic 7th	$m7^{7}$			7/4			968.83		+12.99
10	1090.91	Just Major 7th	M7		15/8				1088.27		+2.64
11	1200.00	Unison (Octave)	U	2/1					1200		0

Table 12(a) 12 Division Equal Temperament

(Compared to Table 1 - Just Intonation (Major))

12-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	100.00	Just Semitone	m2		16/15				112		-12
2	200.00	Just 2nd	M2	9/8					204		-4
3	300.00	Just Minor 3rd	m3	6/5					316		-16
4	400.00	Just Major 3rd	M3	5/4					386		+14
5	500.00	Just Perfect 4th	4	4/3					498		+2
6	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590 610		+10 -10
7	700.00	Just Perfect 5th	5	3/2					702		+2
8	800.00	Just Minor 6th	m6	8/5					814		-14
9	900.00	Just Major 6th	M6	5/3					8814		+16
10	1000.00	Minor 7th	m7	16/9					996		+4
11	1100.00	Just Major 7th	M7		15/8				1088		+12
12	1200.00	Unison (Octave)	U	2/1					1200		0

Table 12(b) 12 Division Equal Temperament

(Compared to nearest ratios in APPENDIX XXX)

12-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	100.00	17th Harmonic	$m2^{17}$					17/16	104.96		-4.96
2	200.00	Just 2nd	M2	9/8					203.91		-3.91
3	300.00	Quasi-Tempered Minor 3rd	$m3^{qt}$			25/21			301.85		-1.85
4	400.00	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		-7.82
5	500.00	Just Perfect 4th	4	4/3					498.04		+1.96
6	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
7	700.00	Just Perfect 5th	5	3/2					701.96		-1.96
8	800.00	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		+7.82
9	900.00	Quasi-Tempered Major 6th	M6			42/25			889.15		+1.85
10	1000.00	Minor 7th	m7	16/9					996.09		+3.91
11	1100.00	Just Major 7th	M7					32/17	1095.05		+4.95
12	1200.00	Unison (Octave)	U	2/1					1200		0

Table 13 13 Division Equal Temperament

13-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3- limit	5- limit	7- limit	11- limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	92.31	Pythagorean Semitone	$m2^{py}$	256/243					90.23		+2.08
2	184.62	Small Just 2nd	M2-		10/9				182.40		+2.22
3	276.92	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		+10.05
4	369.23	Neutral 3rd (13)	$M3^{13}$					16/13	359.47		9.76
5	461.54	49th Harmonic Undertone	4^{49}			64/49			462.35		-0.81
6	553.85	11th Harmonic	4^{11}				11/8		551.32		+2.53
7	646.15	11th Harmonic Undertone	5^{11}				16/11		648.68		-2.53
8	738.46	49th Harmonic	5^{49}			49/32			737.65		+0.81
9	830.77	Neutral 6th (13)	$N6^{13}$					13/8	840.53		-9.76
10	923.08	Septimal Major 6th	$M6^{7}$			12/7			933.13		-10.05
11	1015.38	Large Minor 7th	m7+		9/5				1017.60		-2.22
12	1107.69	Pythagorean Major 7th	$M7^{py}$	243/128					1109.78		-2.09
13	1200.00	Unison (Octave)	U	2/1					1200		0

Table 14 14 Division Equal Temperament

14-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3- limit	5- limit	7- limit	11- limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	85.71	Subminor 2nd	$m2^{7}$			21/20			84.47		+1.24
2	171.43	Ptolemy's 2nd	$M2^{pt}$				11/10		165.00		+6.43
3	257.14	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		-9.73
4	342.86	Neutral 3rd (11)	$N3^{11}$				11/9		347.41		-4.55
5	428.57	Just Large 3rd	M3+		32/25				427.37		+1.20
6	514.29	Just Perfect 4th	4	4/3					498.04		+16.24
7	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9/78
8	685.71	Just Perfect 5th	5	3/2					701.96		-16.25
9	771.43	Small Just Minor 6th	m6-		25/16				772.63		-1.20
10	857.14	Neutral 6th (11)	$N6^{11}$				18/11		852.59		+4.55
11	942.86	Septimal Major 6th	$M6^{7}$			12/7			933.13		+9.73
12	1028.57	Large Minor 7th (11)	$m7^{11}$				20/11		1035.00		-6.43
13	1114.29	Septimal Major 7th	$M7^{7}$			40/21			1115.53		-1.24
14	1200.00	Unison (Octave)	U	2/1					1200		0

Table 15 15 Division Equal Temperament

15-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	80.00	Subminor 2nd	$m2^{7}$			21/20			84.47		-4.47
2	160.00	Ptolemy's 2nd	$M2^{pt}$				11/10		165.00		-5.00
3	240.00	Septimal 2nd	$M2^{7}$			8/7			231.17		+8.83
4	320.00	Just Minor 3rd	m3		6/5				315.64		+4.36
5	400.00	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		-7.82
6	480.00	21st Harmonic	4^{21}			21/16			470.78		9.22
7	560.00	11th Harmonic	4^{11}				11/8		551.32		+8.68
8	640.00	11th Harmonic Undertone	5^{11}				16/11		648.68		-8.68
9	720.00	21st Harmonic Undertone	5^{21}			32/21			729.22		-9.22
10	800.00	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		+7.82
11	880.00	Just Major 6th	M6		5/3				884.36		-4.36
12	960.00	Harmonic 7th	$m7^{7}$			7/4			968.83		-8.83
13	1040.00	Large Minor 7th	$m7^{11}$				20/11		1035.00		5.00
14	1120.00	Septimal Major 7th	$M7^{7}$			40/21			1115.53		+4.47
15	1200.00	Unison (Octave)	U	2/1					1200		0

Table 16 16 Division Equal Temperament

16-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	75.00	Small Semitone	m2-		25/24				70.67		+4.33
2	150.00	Neutral 2nd	N2				12/11		150.64		-0.64
3	225.00	Septimal 2nd	$M2^{7}$			8/7			231.17		-6.17
4	300.00	Quasi-Tempered Minor 3rd	$m3^{qt}$			25/21			301.85		-1.85
5	375.00	Just Major 3rd	M3		5/4				386.31		-11.31
6	450.00	49th Harmonic Undertone	4^{49}			64/49			462.35		-12.35
7	525.00	11th Harmonic	4^{11}				11/8		551.32		-26.32
8	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
9	675.00	11th Harmonic Undertone	5^{11}				16/11		648.68		26.32
10	750.00	49th Harmonic	5^{49}			49/32			737.65		+12.35
11	825.00	Just Minor 6th	m6		8/5				813.69		+11.31
12	900.00	Quasi- Tempered Major 6th	$M6^{qt}$			42/25			898.15		+1.85
13	975.00	Harmonic 7th	$m7^{7}$			7/4			968.83		+6.17
14	1050.00	Neutral 7th	N7				11/6		1049.36		+0.64
15	1125.00	Diminished Octave	M7+		48/25				1129.33		-4.33
16	1200.00	Unison (Octave)	U	2/1					1200		0

Table 17 17 Division Equal Temperament

17-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	70.59	Small Semitone	m2-		25/24				70.67		-0.08
2	141.18	Large Semitone (13)	$m2^{13}$					13/12	138.57		+2.61
3	211.76	Just 2nd	M2	9/8					203.91		+7.85
4	282.35	Pythagorean Minor 3rd	$m3^{py}$	32/27					294.14		-11.79
5	352.94	Neutral 3rd (11)	$N3^{11}$				11/9		347.41		+5.53
6	423.53	Just Large 3rd	M3+		32/25				427.37		-3.84
7	494.12	Just Perfect 4th	4	4/3					498.04		-3.93
8	564.71	11th Harmonic	4^{11}				11/8		551.32		-13.39
9	635.29	11th Harmonic Undertone	5^{11}				16/11		648.68		-13.39
10	705.88	Just Perfect 5th	5	3/2					701.96		+3.92
11	776.47	Small Just Minor 6th	m6-		25/16				772.63		+3.84
12	847.06	Neutral 6th (11)	$N6^{11}$				18/11		852.59		-5.53
13	917.65	Pythagorean Major 6th	$M6^{py}$	27/16					905.87		+11.78
14	988.24	Minor 7th	m7	16/9					996.09		-7.85
15	1058.82	Small Major 7th	$M7^{13}$					24/13	1061.43		-2.61
16	1129.41	Diminished Octave	M7+		48/25				1129.33		0.08
17	1200.00	Unison (Octave)	U	2/1					1200		0

Table 18 18 Division Equal Temperament

18-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	66.67	Small Semitone	m2-		25/24				71.67		-4.00
2	133.33	Large Semitone	m2+		27/25				133.24		0.09
3	200.00	Just 2nd	M2	9/8					203.91		-3.91
4	266.67	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		-0.20
5	333.33	Neutral 3rd (11)	$N3^{11}$				11/9		347.41		-14.08
6	400.00	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		-7.82
7	466.67	21st Harmonic	4^{21}			21/16			470.78		+4.11
8	533.33	11th Harmonic	4^{11}				11/8		551.32		-17.99
9	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
10	666.67	11th Harmonic Undertone	5^{11}				16/11		648.68		17.99
11	733.33	21st Harmonic Undertone	5^{21}			32/21			729.22		+4.11
12	800.00	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		+7.82
13	866.67	Neutral 6th (11)	$N6^{11}$				18/11		852.59		+14.08
14	933.33	Septimal Major 6th	$M6^{7}$			12/7			933.13		0.20
15	1000.00	Minor 7th	m7	16/9					996.09		+3.91
16	1066.67	Small Major 7th	M7-		50/27				1066.76		-0.09
17	1133.33	Diminished Octave	M7+		48/25				1129.33		+4.00
18	1200.00	Unison (Octave)	U	2/1					1200		0

Table 19 19 Division Equal Temperament

19-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	63.16	Small Semitone	m2-		25/24				70.67		-7.51
2	126.32	Large Semitone	m2+		27/25				133.24		-6.92
3	189.47	Small Just 2nd	M2-		10/9				182.40		+7.07
4	252.63	Septimal Minor 3rd	m3^7			7/6			266.87		-14.24
5	315.79	Just Minor 3rd	m3		6/5				315.64		0.15
6	378.95	Just Major 3rd	M3		5/4				386.31		-7.36
7	442.11	"Supermajor" 3rd	M3^7			9/7			435.08		+7.03
8	505.26	Just Perfect 4th	4	4/3					498.04		+7.22
9	568.42	Septimal Augmented 4th	4^7			7/5			582.51		-14.09
10	631.58	Septimal Diminished 5th	5^7			10/7			617.49		+14.09
11	694.74	Just Perfect 5th	5	3/2					701.96		-7.23
12	757.89	Septimal Minor 6th	m6^7			14/9			764.92		-7.03
13	821.05	Just Minor 6th	m6		8/5				813.69		+7.36
14	884.21	Just Major 6th	M6		5/3				884.36		-0.15
15	947.37	Septimal Major 6th	M6^7			12/7			933.13		+14.24
16	1010.53	Large Minor 7th	m7+		9/5				1017.60		-7.07
17	1073.68	Small Major 7th	M7-		50/27				1066.76		+6.92
18	1136.84	Diminished Octave	M7+		48/25				1129.33		+7.51
19	1200.00	Unison (Octave)	U	2/1					1200		0

Table 20 20 Division Equal Temperament

20-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	60.00	33rd Harmonic	m2^{11}				33/32		53.27		+6.73
2	120.00	Just Semitone	m2		16/15				111.73		+8.27
3	180.00	Small Just 2nd	M2-		10/9				182.40		-2.40
4	240.00	Septimal 2nd	M2^7			8/7			231.17		+8.83
5	300.00	Quasi-Tempered Minor 3rd	m3^{qt}			25/21			301.85		-1.85
6	360.00	Neutral 3rd (13)	N3^{13}					16/13	359.47		0.53
7	420.00	Large Major 3rd (11)	M3^{11}				14/11		417.51		+2.49
8	480.00	21st Harmonic	4^{21}			21/16			470.78		9.22
9	540.00	11th Harmonic	4^{11}				11/8		551.32		-11.32
10	600.00	Just Augmented 4th	4+		45/32				590.22		+9.78
		(or) Just Diminished 5th	5-		64/45				609.78		-9.78
11	660.00	11th Harmonic Undertone	5^{11}				16/11		648.68		+11.32
12	720.00	21st Harmonic Undertone	5^{21}			32/21			729.22		-9.22
13	780.00	Small Minor 6th (11)	m6^{11}				11/7		782.49		-2.49
14	840.00	Neutral 6th (13)	N6^{13}					13/8	840.53		-0.53
15	900.00	Quasi-Tempered Major 6th	M6^{qt}			42/25			898.15		+1.85
16	960.00	Harmonic 7th	m7^7			7/4			968.83		-8.83
17	1020.00	Large Minor 7th	m7+		9/5				1017.60		+2.40
18	1080.00	Just Major 7th	M7		15/8				1088.27		-8.27
19	1140.00	33rd Harmonic Undertone	M7^{11}				64/33		1146.73		-6.73
20	1200.00	Unison (Octave)	U	2/1					1200		0

Table 21 21 Division Equal Temperament

21-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	57.14	33rd Harmonic	$m2^{11}$				33/32		53.27		+3.87
2	114.29	Just Semitone	m2		16/15				111.73		+2.56
3	171.43	Ptolemy's 2nd	$M2^{pt}$				11/10		165.00		+6.43
4	228.57	Septimal 2nd	$M2^{7}$			8/7			231.17		-2.60
5	285.71	Pythagorean Minor 3rd	$m3^{py}$	32/27					294.14		-8.43
6	342.86	Neutral 3rd (11)	$N3^{11}$				11/9		347.41		-4.55
7	400.00	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		-7.82
8	457.14	49th Harmonic Undertone	4^{49}			64/49			462.35		-5.21
9	514.29	Just Perfect 4th	4	4/3					498.04		+16.24
10	571.43	Septimal Augmented 4th	4^{7}			7/5			582.51		-11.08
11	628.57	Septimal Diminished 5th	5^{7}			10/7			617.49		+11.08
12	685.71	Just Perfect 5th	5	3/2					701.96		-16.24
13	742.86	49th Harmonic	5^{49}			49/32			737.65		+5.21
14	800.00	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		+7.82
15	857.14	Neutral 6th (11)	$N6^{11}$				18/11		852.59		+4.55
16	914.29	Pythagorean Major 6th	$M6^{py}$	27/16					905.87		+8.43
17	971.43	Harmonic 7th	$m7^{7}$			7/4			968.83		+2.60
18	1028.57	Large Minor 7th (11)	$m7^{11}$				20/11		1035.00		-6.43
19	1085.71	Just Major 7th	M7		15/8				1088.27		-2.56
20	1142.86	33rd Harmonic Undertone	$M7^{11}$				64/33		1147.73		-3.87
21	1200.00	Unison (Octave)	U	2/1					1200		0

Table 22 22 Division Equal Temperament

22-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	54.55	33rd Harmonic	$m2^{11}$				33/32		53.27		+1.28
2	109.09	Just Semitone	m2		16/15				111.73		-2.64
3	163.64	Ptolemy's 2nd	$M2^{pt}$				11/10		165.00		-1.36
4	218.18	Septimal 2nd	$M2^{7}$			8/7			231.17		-12.99
5	272.73	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		+5.86
6	327.27	Just Minor 3rd	m3		6/5				315.64		+11.63
7	381.82	Just Major 3rd	M3		5/4				386.31		-4.49
8	436.36	"Supermajor" 3rd	$M3^{7}$			9/7			435.08		+1.28
9	490.91	Just Perfect 4th	4	4/3					498.04		-7.14
10	545.45	11th Harmonic	4^{11}				11/8		551.32		-5.87
11	600.00	Just Augmented 4th (or)Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
12	654.55	11th Harmonic Undertone	5^{11}				16/11		648.68		+5.87
13	709.09	Just Perfect 5th	5	3/2					701.96		+7.14
14	763.64	Septimal Minor 6th	$m6^{7}$			14/9			764.92		-1.28
15	818.18	Just Minor 6th	m6		8/5				813.69		+4.49
16	872.73	Just Major 6th	M6		5/3				884.36		-11.63
17	927.27	Septimal Major 6th	$M6^{7}$			12/7			933.13		-6.86
18	981.82	Harmonic 7th	$m7^{7}$			7/4			968.83		+12.99
19	1036.36	Large Minor 7th (11)	$m7^{11}$				20/11		1035.00		+1.36
20	1090.91	Just Major 7th	M7		15/8				1088.27		+2.64
21	1145.45	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		-1.28
22	1200.00	Unison (Octave)	U	2/1					1200		0

Table 23 23 Division Equal Temperament

23-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	52.17	33rd Harmonic	$m2^{11}$				33/32		53.27		-1.10
2	104.35	17th Harmonic	$m2^{17}$					17/16	104.96		-0.61
3	156.52	Neutral 2nd	N2				12/11		150.64		+5.88
4	208.70	Just 2nd	M2	9/8					203.91		+4.79
5	260.87	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		-6.00
6	313.04	Just Minor 3rd	m3		6/5				315.64		-2.60
7	365.22	Neutral 3rd (13)	$N3^{13}$					16/13	359.47		5.75
8	417.39	Large Major 3rd (11)	$M3^{11}$				14/11		417.51		-0.12
9	469.57	21st Harmonic	4^{21}			21/16			470.78		-1.21
10	521.74	Just Perfect 4th	4	4/3					498.04		+23.69
11	573.91	Septimal Augmented 4th	4^{7}			7/5			582.51		-8.60
12	626.09	Septimal Diminished 5th	5^{7}			10/7			617.49		+8.60
13	678.26	Just Perfect 5th	5	3/2					701.96		-23.69
14	730.43	21st Harmonic Undertone	5^{21}			32/21			729.22		+1.21
15	782.61	Small Minor 6th (11)	$m6^{11}$				11/7		782.49		+0.12
16	834.78	Neutral 6th (13)	$N6^{13}$					13/8	840.53		-5.75
17	886.96	Just Major 6th	M6		5/3				884.36		+2.60
18	939.13	Septimal Major 6th	$M6^{7}$			12/7			933.13		+6.00
19	991.30	Minor 7th	m7	16/9					996.09		-4.79
20	1043.48	Neutral 7th	N7				11/6		1049.36		-5.88
21	1095.65	Major 7th (17)	$M7^{17}$					32/17	1095.05		+0.60
22	1147.83	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+1.10
23	1200.00	Unison (Octave)	U	2/1					1200		0

Table 24 24 Division Equal Temperament (Quartertones)

24-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	50.00	33rd Harmonic	$m2^{11}$				33/32		53.27		-3.27
2	100.00	17th Harmonic	$m2^{17}$					17/16	104.96		-4.96
3	150.00	Neutral 2nd	N2				12/11		150.64		-0.64
4	200.00	Just 2nd	M2	9/8					203.91		-3.91
5	250.00	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		-16.87
6	300.00	Quasi-Tempered Minor 3rd	$m3^{qt}$			25/21			301.85		-1.85
7	350.00	Neutral 3rd (11)	$N3^{11}$				11/9		347.41		+2.59
8	400.00	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		-7.82
9	450.00	49th Harmonic Undertone	4^{49}			64/49			462.35		-12.35
10	500.00	Just Perfect 4th	4	4/3					498.04		+1.96
11	550.00	11th Harmonic	4^{11}				11/8		551.32		-1.32
12	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
13	650.00	11th Harmonic Undertone	5^{11}				16/11		648.68		+1.32
14	700.00	Just Perfect 5th	5	3/2					701.96		-1.96
15	750.00	49th Harmonic	5^{49}			49/32			737.65		+12.35
16	800.00	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		+7.82
17	850.00	Neutral 6th (11)	$N6^{11}$				18/11		852.59		-2.59
18	900.00	Quasi-Tempered Major 6th	$M6^{qt}$			42/25			898.15		+1.85
19	950.00	Septimal Major 6th	$M6^{7}$			12/7			833.13		+16.87
20	1000.00	Minor 7th	m7	16/9					996.09		+3.91
21	1050.00	Neutral 7th	N7				11/6		1049.36		+0.64
22	1100.00	Major 7th (17)	$M7^{17}$					32/17	1095.05		+4.95
23	1150.00	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+3.27
24	1200.00	Unison (Octave)	U	2/1					1200		0

Table 25 25 Division Equal Temperament

25-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3- limit	5- limit	7- limit	11- limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	48.00	33rd Harmonic	$m2^{11}$				33/32		53.27		-5.27
2	96.00	Pythagorean Semitone	$m2^{py}$	256/243					90.23		+5.77
3	144.00	Large Semitone (13)	$m2^{13}$					13/12	138.57		+5.43
4	192.00	Small Just 2nd	M2-		10/9				182.40		+9.60
5	240.00	Septimal 2nd	$M2^{7}$			8/7			231.17		+8.83
6	288.00	Pythagorean Minor 3rd	$m3^{py}$	32/27					294.14		-6.14
7	336.00	Neutral 3rd (11)	$N3^{11}$				11/9		347.41		-11.41
8	384.00	Just Major 3rd	M3		5/4				386.31		-2.31
9	432.00	“Supermajor” 3rd	$M3^{7}$			9/7			435.08		-3.08
10	480.00	21st Harmonic	4^{21}			21/16			470.78		+9.22
11	528.00	11th Harmonic	4^{11}				11/8		551.32		-23.32
12	576.00	Septimal Augmented 4th	4^{7}			7/5			582.51		-6.51
13	624.00	Septimal Diminished 5th	5^{7}			10/7			617.49		+6.51
14	672.00	11th Harmonic Undertone	5^{11}				16/11		648.68		+23.32
15	720.00	21st Harmonic Undertone	5^{21}			32/21			729.22		-9.22
16	768.00	Septimal Minor 6th	$m6^{7}$			14/9			764.92		+3.08
17	816.00	Just Minor 6th	m6		8/5				813.69		+2.31
18	864.00	Neutral 6th (11)	$N6^{11}$				18/11		852.59		+11.41
19	912.00	Pythagorean Major 6th	$M6^{py}$	27/16					905.87		+6.13
20	960.00	Harmonic 7th	$m7^{7}$			7/4			968.83		-8.83
21	1008.00	Large Minor 7th	m7+		9/5				1017.60		-9.60
22	1056.00	Small Major 7th (13)	$M7^{13}$					24/13	1061.43		-5.43
23	1104.00	Pythagorean Major 7th	$M7^{py}$	243/128					1109.78		-5.78
24	1152.00	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+5.27
25	1200.00	Unison (Octave)	U	2/1					1200		0

Table 26 26 Division Equal Temperament

26-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3- limit	5- limit	7- limit	11- limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	46.15	33rd Harmonic	$m2^{11}$				33/32		53.27		-7.12
2	92.31	Pythagorean Semitone	$m2^{py}$	256/243					90.23		+2.08
3	138.46	Large Semitone (13)	$m2^{13}$					13/12	138.57		-0.11
4	184.62	Small Just 2nd	M2-		10/9				182.40		+2.22
5	230.77	Septimal 2nd	$M2^{7}$			8/7			231.17		-0.40
6	276.92	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		+10.05
7	323.08	Just Minor 3rd	m3		6/5				315.64		+7.44
8	369.23	Neutral 3rd (13)	$N3^{13}$					16/13	359.47		9.76
9	415.38	Large Major 3rd (11)	$M3^{11}$				14/11		417.51		-2.13
10	461.54	49th Harmonic Undertone	4^{49}			64/49			462.35		-0.81
11	507.69	Just Perfect 4th	4	4/3					498.04		+9.65
12	553.85	11th Harmonic	4^{11}				11/8		551.32		+2.53
13	600.00	Just Augmented 4th or Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
14	646.15	11th Harmonic Undertone	5^{11}				16/11		648.68		-2.53
15	692.31	Just Perfect 5th	5	3/2					701.96		-9.65
16	738.46	49th Harmonic	5^{49}			49/32			737.65		+0.81
17	784.62	Small Minor 6th (11)	$m6^{11}$				11/7		782.49		+2.13
18	830.77	Neutral 6th (13)	$N6^{13}$					13/8	840.53		-9.76
19	876.92	Just Major 6th	M6		5/3				884.36		-7.44
20	923.08	Septimal Major 6th	$M6^{7}$			12/7			933.13		-10.05
21	969.23	Harmonic 7th	$m7^{7}$			7/4			968.83		+0.40
22	1015.38	Large Minor 7th	m7+		9/5				1017.60		-2.22
23	1061.54	Small Major 7th (13)	$M7^{13}$					24/13	1061.43		+0.11
24	1107.69	Pythagorean Major 7th	$M7^{py}$	243/128					1109.78		-2.09
25	1153.85	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+7.12
26	1200.00	Unison (Octave)	U	2/1					1200		0

Table 27 27 Division Equal Temperament

27-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3- limit	5- limit	7- limit	11- limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	44.44	33rd Harmonic	$m2^{11}$				33/32		53.27		-8.83
2	88.89	Pythagorean Semitone	m2-	256/243					90.23		-1.34
3	133.33	Large Semitone	m2+		27/25				133.24		+0.09
4	177.78	Small Just 2nd	M2-		10/9				182.40		-4.62
5	222.22	Septimal 2nd	$M2^{7}$			8/7			231.17		-8.95
6	266.67	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		-0.20
7	311.11	Just Minor 3rd	m3		6/5				315.64		-4.53
8	355.56	Neutral 3rd (13)	$N3^{13}$					16/13	359.47		-3.91
9	400.00	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		-7.82
10	444.44	“Supermajor” 3rd	$M3^{7}$			9/7			435.08		+9.36
11	488.89	Just Perfect 4th	4	4/3					498.04		-9.16
12	533.33	11th Harmonic	4^{11}				11/8		551.32		-17.99
13	577.78	Septimal Augmented 4th	4^{7}			7/5			582.51		-4.73
14	622.22	Septimal Diminished 5th	5^{7}			10/7			617.49		+4.73
15	666.67	11th Harmonic Undertone	5^{11}				16/11		648.68		17.99
16	711.11	Just Perfect 5th	5	3/2					701.96		+9.15
17	755.56	Septimal Minor 6th	$m6^{7}$			14/9			764.92		-9.36
18	800.00	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		+7.82
19	844.44	Neutral 6th (13)	$N6^{13}$					13/8	840.53		+3.91
20	888.89	Just Major 6th	M6		5/3				884.36		+4.53
21	933.33	Septimal Major 6th	$M6^{7}$			12/7			933.13		+0.20
22	977.78	Harmonic 7th	$m7^{7}$			7/4			968.83		+8.95
23	1022.22	Large Minor 7th	m7+		9/5				1017.60		+4.62
24	1066.67	Small Major 7th	M7-		50/27				1066.76		-0.09
25	1111.11	Pythagorean Major 7th	$M7^{py}$	243/128					1109.78		+1.33
26	1155.56	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+8.83
27	1200.00	Unison (Octave)	U	2/1					1200		0

Table 28 28 Division Equal Temperament

28-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	42.86	33rd Harmonic	$m2^{11}$				33/32		53.27		-10.41
2	85.71	Subminor 2nd	$m2^{7}$			21/20			84.47		+1.24
3	128.57	Large Semitone	m2+		27/25				133.24		-4.67
4	171.43	Ptolemy's 2nd	$M2^{pt}$				11/10		165.00		+6.43
5	214.29	Just 2nd	M2	9/8					203.91		+10.48
6	257.14	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		-9.73
7	300.00		$m3^{py}$			25/21			301.85		-1.85
8	342.86	Neutral 3rd (11)	$N3^{11}$				11/9		347.41		-4.55
9	385.71	Just Major 3rd	M3		5/4				386.31		-0.66
10	428.57	Just Large 3rd	M3+		32/25				427.37		+1.20
11	471.43		4^{49}			21/16			470.78		+0.65
12	514.29	Just Perfect 4th	4	4/3					498.04		+16.24
13	557.14	11th Harmonic	4^{11}				11/8		551.32		+5.82
14	600.00	Just Augmented 4th or Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
15	642.86	11th Harmonic Undertone	5^{11}				16/11		648.68		-5.82
16	685.71	Just Perfect 5th	5	3/2					701.96		-16.24
17	728.57	49th Harmonic	5^{49}			32/21			729.22		-0.65
18	771.43	Small Just Minor 6th	m6-		25/16				772.63		-1.20
19	814.29	Just Minor 6th	m6		8/5				813.69		+0.60
20	857.14	Neutral 6th (11)	$N6^{11}$				18/11		852.59		+4.55
21	900.00		$M6^{py}$			42/25			898.15		+1.85
22	942.86	Septimal Major 6th	$M6^{7}$			12/7			933.13		+9.73
23	985.71	Minor 7th	m7	16/9					996.09		-10.38
24	1028.57	Large Minor 7th (11)	$m7^{11}$				20/11		1035.00		-6.43
25	1071.43	Small Major 7th	M7-		50/27				1066.76		+4.67
26	1114.29	Septimal Major 7th	$M7^{7}$			40/21			1115.53		-1.24
27	1157.14	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+10.41
28	1200.00	Unison (Octave)	U	2/1					1200		0

Table 29 29 Division Equal Temperament

29-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	41.38	33rd Harmonic	$m2^{11}$				33/32		53.27		-11.89
2	82.76	Subminor 2nd	$m2^{7}$			21/20			84.47		-1.71
3	124.14	Large Semitone	m2+		27/25				133.24		-9.10
4	165.52	Ptolemy's 2nd	$M2^{pt}$				11/10		165.00		+0.52
5	206.90	Just 2nd	M2	9/8					203.91		+2.99
6	248.28	Septimal 2nd	$M2^{7}$			8/7			231.17		+17.11
7	289.66	Pythagorean Minor 3rd	$m3^{py}$	32/27					294.14		-4.48
8	331.03	Just Minor 3rd	m3		6/5				315.64		+15.39
9	372.41	Neutral 3rd (13)	$N3^{13}$					16/13	359.47		-12.94
10	413.79	Large Major 3rd (11)	$M3^{11}$				14/11		417.51		-3.72
11	455.17	49th Harmonic Undertone	4^{49}			64/49			462.35		-7.18
12	496.55	Just Perfect 4th	4	4/3					498.04		-1.49
13	537.93	11th Harmonic	4^{11}				11/8		551.32		-13.39
14	579.31	Septimal Augmented 4th	4^{7}			7/5			582.51		-3.20
15	620.69	Septimal Diminished 5th	5^{7}			10/7			617.49		+3.20
16	662.07	11th Harmonic Undertone	5^{11}				16/11		648.68		+13.39
17	703.45	Just Perfect 5th	5	3/2					701.96		+1.48
18	744.83	49th Harmonic	5^{49}			49/32			737.65		+7.18
19	786.21	Small Minor 6th (11)	$m6^{11}$				11/7		782.49		+3.72
20	827.59	Neutral 6th (13)	$N6^{13}$					13/8	840.53		-12.94
21	868.97	Just Major 6th	M6		5/3				884.36		-15.39
22	910.34	Pythagorean Major 6th	$M6^{py}$	27/16					905.87		+4.47
23	951.72	Harmonic 7th	$m7^{7}$			7/4			968.83		-17.11
24	993.10	Minor 7th	m7	16/9					996.09		-2.99
25	1034.48	Large Minor 7th (11)	$m7^{11}$				20/11		1035.00		-0.52
26	1075.86	Small Major 7th	M7-		50/27				106676		+9.10
27	1117.24	Septimal Major 7th	$M7^{7}$			40/21			1115.53		+1.71
28	1158.62	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+11.89
29	1200.00	Unison (Octave)	U	2/1					1200		0

Table 30 30 Division Equal Temperament

30-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	40.00	33rd Harmonic	$m2^{11}$				33/32		53.27		-13.27
2	80.00	Subminor 2nd	$m2^{7}$			21/20			84.47		-4.47
3	120.00	Just Semitone	m2		16/15				111.73		+8.27
4	160.00	Ptolomy’s 2nd	$M2^{pt}$				11/10		165.00		-5.00
5	200.00	Just 2nd	M2	9/8					203.91		-3.91
6	240.00	Septimal 2nd	$M2^{7}$			8/7			231.17		+8.83
7	280.00	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		+13.13
8	320.00	Just Minor 3rd	m3		6/5				315.64		+4.36
9	360.00	Neutral 3rd (13)	$N3^{13}$					16/13	359.47		+0.53
10	400.00	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		-7.82
11	440.00	“Supermajor” 3rd	$M3^{7}$			9/7			435.08		+4.92
12	480.00	21st Harmonic	4^{49}			21/16			470.78		+9.22
13	520.00	Just Perfect 4th	4	4/3					498.04		+21.96
14	560.00	11th Harmonic	4^{11}				11/8		551.32		+8.68
15	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
16	640.00	11th Harmonic Undertone	5^{11}				16/11		648.68		-8.68
17	680.00	Just Perfect 5th	5	3/2					701.96		-21.96
18	720.00	21st Harmonic Undertone	5^{21}			32/21			729.22		-9.22
19	760.00	Septimal Minor 6th	$m6^{7}$			14/9			764.92		-4.92
20	800.00	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		+7.82
21	840.00	Neutral 6th (13)	$N6^{13}$					13/8	840.53		-0.53
22	880.00	Just Major 6th	M6		5/3				884.36		-4.36
23	920.00	Septimal Major 6th	$M6^{7}$			12/7			933.13		-13.13
24	960.00	Harmonic 7th	$m7^{7}$			7/4			968.83		-8.83
25	1000.00	Minor 7th	m7	16/9					996.09		+3.91
26	1040.00	Large Minor 7th (11)	$m7^{11}$				20/11		1035.00		+5.00
27	1080.00	Just Major 7th	M7		15/8				1088.27		-8.27
28	1120.00	Septimal Major 7th	$M7^{7}$			40/21			1115.53		+4.47
29	1160.00	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+13.27
30	1200.00	Unison (Octave)	U	2/1					1200		0

Table 31 31 Division Equal Temperament

31-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	38.71	33rd Harmonic	$m2^{11}$				33/32		53.27		-14.56
2	77.42	Small Semitone	m2-		25/24				70.67		+6.75
3	116.13	Just Semitone	m2		16/15				111.73		+4.40
4	154.84	Neutral 2nd	N2				12/11		150.64		+4.20
5	193.55	Just 2nd	M2	9/8					203.91		-10.36
6	232.26	Septimal 2nd	$M2^{7}$			8/7			231.17		+1.09
7	270.97	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		+4.10
8	309.68	Just Minor 3rd	m3		6/5				315.64		-5.96
9	348.39	Neutral 3rd (11)	$N3^{11}$				11/9		347.41		+0.98
10	387.10	Just Major 3rd	M3		5/4				386.31		+0.78
11	425.81	Just Large 3rd	M3+		32/25				427.37		-1.56
12	464.52	49th Harmonic Undertone	4^{49}			64/49			462.35		+2.17
13	503.23	Just Perfect 4th	4	4/3					498.04		+5.18
14	541.94	11th Harmonic	4^{11}				11/8		551.32		-9.38
15	580.65	Septimal Augmented 4th	4^{7}			7/5			582.51		-1.86
16	619.35	Septimal Diminished 5th	5^{7}			10/7			617.49		+1.86
17	658.06	11th Harmonic Undertone	5^{11}				16/11		648.68		+9.38
18	696.77	Just Perfect 5th	5	3/2					701.96		-5.19
19	735.48	49th Harmonic	5^{49}			49/32			737.65		-2.17
20	774.19	Small Just Minor 6th	m6-		25/16				772.63		+1.56
21	812.90	Just Minor 6th	m6		8/5				813.69		-0.79
22	851.61	Neutral 6th (11)	$N6^{11}$				18/11		852.59		-0.98
23	890.32	Just Major 6th	M6		5/3				884.36		+5.96
24	929.03	Septimal Major 6th	$M6^{7}$			12/7			933.13		-4.10
25	967.74	Harmonic 7th	$m7^{7}$			7/4			968.83		-1.09
26	1006.45	Minor 7th	m7	16/9					996.09		+10.36
27	1045.16	Neutral 7th	N7				11/6		1049.36		-4.20
28	1083.87	Just Major 7th	M7		15/8				1088.27		-4.40
29	1122.58	Diminished Octave	M7+		48/25				1129.33		-6.75
30	1161.29	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+14.56
31	1200.00	Unison (Octave)	U	2/1					1200		0

Table 32 32 Division Equal Temperament

32-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	37.50	33rd Harmonic or '1/4-Tone'	m2^{11}				33/32		53.27		-15.77
2	75.00	Small Semitone	m2-		25/24				70.67		+4.33
3	112.50	Just Semitone	m2		16/15				111.73		+0.77
4	150.00	Neutral 2nd	N2				12/11		150.64		-0.64
5	187.50	Small Just 2nd	M2-		10/9				182.40		+5.10
6	225.00	Septimal 2nd	M2^{7}			8/7			231.17		-6.17
7	262.50	Septimal Minor 3rd	m3^{7}			7/6			266.87		-4.37
8	300.00	Quasi-Tempered Minor 3rd	m3^{qt}			25/21			301.85		-1.85
9	337.50	Neutral 3rd (11)	N3^{11}				11/9		347.41		-9.91
10	375.00	Just Major 3rd	M3		5/4				386.31		-11.31
11	412.50	Pythagorean Major 3rd	M3^{py}	81/64					407.82		+4.68
12	450.00	49th Harmonic Undertone	4^{49}			64/49			462.35		-12.35
13	487.50	Just Perfect 4th	4	4/3					498.04		-10.54
14	525.00	11th Harmonic	4^{11}				11/8		551.32		-26.32
15	562.50	11th Harmonic	4^{11}				11/8		551.32		+11.18
16	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
17	637.50	11th Harmonic Undertone	5^{11}				16/11		648.68		-11.19
18	675.00	11th Harmonic Undertone	5^{11}				16/11		648.68		+26.32
19	712.50	Just Perfect 5th	5	3/2					701.96		+10.54
20	750.00	49th Harmonic	5^{49}			49/32			737.65		+12.35
21	787.50	Pythagorean Minor 6th	m6^{py}	128/81					792.18		-4.68
22	825.00	Just Minor 6th	m6		8/5				813.69		+11.31
23	862.50	Neutral 6th (11)	N6^{11}				18/11		852.59		+9.91
24	900.00	Quasi-Tempered Major 6th	M6^{qt}			42/25			898.15		+1.85
25	937.50	Septimal Major 6th	M6^{7}			12/7			933.13		+4.37
26	975.00	Harmonic 7th	m7^{7}			7/4			968.83		+6.17
27	1012.50	Large Minor 7th	m7+		9/5				1017.60		-5.10
28	1050.00	Neutral 7th	N7				11/6		1049.36		+0.64
29	1087.50	Just Major 7th	M7		15/8				1088.27		-0.77
30	1125.00	Diminished Octave	M7+		48/25				1129.33		-4.33
21	1162.50	33rd Harmonic Undertone	M7^{11}				64/33		1146.73		+15.77
32	1200.00	Unison (Octave)	U	2/1					1200		0

Table 33 33 Division Equal Temperament

33-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	36.36	33rd Harmonic or ‘1/4-Tone’	$m2^{11}$				33/32		53.27		-16.91
2	72.73	Small Semitone	m2-		25/24				70.67		+2.06
3	109.09	Just Semitone	m2		16/15				111.73		-2.64
4	145.45	Neutral 2nd	N2				12/11		150.64		-5.19
5	181.82	Small Just 2nd	M2-		10/9				182.40		-0.58
6	218.18	Septimal 2nd	$M2^7$			8/7			231.17		-12.99
7	254.55	Septimal Minor 3rd	$m3^7$			7/6			266.87		-12.32
8	290.91	Pythagorean Minor 3rd	$m3^{py}$	32/27					294.14		-3.23
9	327.27	Just Minor 3rd	m3		6/5				315.64		+11.63
10	363.64	Neutral 3rd (13)	$N3^{13}$					16/13	359.47		+4.17
11	400.00	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		-7.82
12	436.36	“Supermajor” 3rd	$M3^7$			9/7			435.08		+1.28
13	472.73	21st Harmonic	4^{21}			21/16			470.78		+1.95
14	509.09	Just Perfect 4th	4	4/3					498.04		+11.05
15	545.45	11th Harmonic	4^{11}				11/8		551.32		-5.87
16	581.82	Septimal Augmented 4th	4^7			7/5			582.51		-0.69
17	618.18	Septimal Diminished 5th	5^7			10/7			617.49		+0.69
18	654.55	11th Harmonic Undertone	5^{11}				16/11		648.68		+5.87
19	690.91	Just Perfect 5th	5	3/2					701.96		-11.06
20	727.27	21st Harmonic Undertone	5^{21}			32/21			729.22		-1.95
21	763.64	Septimal Minor 6th	$m6^7$			14/9			764.92		-1.28
22	800.00	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		+7.82
23	836.36	Neutral 6th (13)	$N6^{13}$					13/8	840.53		-4.17
24	872.73	Just Major 6th	M6		5/3				884.36		-11.63
25	909.09	Pythagorean Major 6th	$M6^{py}$	27/16					905.87		+3.22
26	945.45	Septimal Major 6th	$M6^7$			12/7			933.13		+12.32
27	981.82	Harmonic 7th	$m7^7$			7/4			968.83		+12.99
28	1018.18	Large Minor 7th	m7+		9/5				1017.60		+0.58
29	1054.55	Neutral 7th	N7				11/6		1049.36		+5.19
30	1090.91	Just Major 7th	M7		15/8				1088.27		+2.64
31	1127.27	Diminished Octave	M7+		48/25				1129.33		-2.06
32	1163.64	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+16.91
33	1200.00	Unison (Octave)	U	2/1					1200		0

Table 34 34 Division Equal Temperament

34-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13+ limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	35.29	33rd Harmonic or ‘1/4-Tone’	m2^{11}				33/32		53.27		-17.98
2	70.59	Small Semitone	m2-		25/24				70.67		-0.08
3	105.88	17th Harmonic	m2^{17}					17/16	104.96		+0.92
4	141.18	Large Semitone (13)	m2^{13}					13/12	138.57		+2.61
5	176.47	Small Just 2nd	M2-		10/9				182.40		-5.93
6	211.76	Just 2nd	M2	9/8					203.91		+7.85
7	247.06	Septimal 2nd	M2^{7}			8/7			231.17		15.89
8	282.35	Pythagorean Minor 3rd	m3^{py}	32/27					294.14		-11.79
9	317.65	Just Minor 3rd	m3		6/5				315.64		+2.01
10	352.94	Neutral 3rd (11)	N3^{11}				11/9		347.41		+5.53
11	388.24	Just Major 3rd	M3		5/4				386.31		+1.93
12	423.53	Just Large 3rd	M3+		32/25				427.37		-3.84
13	458.82	49th Harmonic Undertone	4^{49}			64/49			462.35		-3.53
14	494.12	Just Perfect 4th	4	4/3					498.04		-3.93
15	529.41	11th Harmonic	4^{11}				11/8		551.32		-21.91
16	564.71	11th Harmonic	4^{11}				11/8		551.32		+13.39
17	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
18	635.29	11th Harmonic Undertone	5^{11}				16/11		648.68		-13.39
19	670.59	11th Harmonic Undertone	5^{11}				16/11		648.68		21.94
20	705.88	Just Perfect 5th	5	3/2					701.96		+3.92
21	741.18	49th Harmonic	5^{49}			49/32			737.65		+3.53
22	776.47	Small Just Minor 6th	m6-		25/16				772,63		+3.84
23	811.76	Just Minor 6th	m6		8/5				813.69		-1.93
24	847.06	Neutral 6th (11)	N6^{11}				18/11		852.59		-5.53
25	882.35	Just Major 6th	M6		5/3				884.36		-2.01
26	917.65	Pythagorean Major 6th	M6^{py}	27/16					905.87		+11.78
27	952.94	Harmonic 7th	m7^{7}			7/4			968.83		-15.89
28	988.24	Minor 7th	m7	16/9					996.09		-7.85
29	1023.53	Large Minor 7th	m7+		9/5				1017.60		+5.93
30	1058.82	Small Major 7th (13)	M7^{13}					24/13	1061.43		-2.61
31	1094.12	Major 7th (17)	M7^{17}					32/17	1095.05		-0.93
32	1129.41	Diminished Octave	M7+		48/25				1129.33		+0.08
33	1164.71	33rd Harmonic Undertone	M7^{11}				64/33		1146.73		+17.98
34	1200.00	Unison (Octave)	U	2/1					1200		0

Table 35 35 Division Equal Temperament

35-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13+ limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	34.29	33rd Harmonic or '1/4-Tone'	m2^{11}				33/32		53.27		-18.98
2	68.57	Small Semitone	m2-		25/24				70.67		-2.10
3	102.86	17th Harmonic	m2^{17}					17/16	104.96		-2.10
4	137.14	Large Semitone (13)	m2^{13}					13/12	138.57		-1.43
5	171.43	Ptolemy's 2nd	M2^{pt}				11/10		165.00		+6.43
6	205.71	Just 2nd	M2	9/8					203.91		+1.80
7	240.00	Septimal 2nd	M2^{7}			8/7			231.17		+8.83
8	274.29	Septimal Minor 3rd	m3^{7}			7/6			266.87		+7.42
9	308.57	Quasi-Tempered Minor 3rd	m3^{qt}			25/21			301.85		-6.72
10	342.86	Neutral 3rd (11)	N3^{11}				11/9		347.41		-4.55
11	377.14	Just Major 3rd	M3		5/4				386.31		-9.17
12	411.43	Pythagorean Major 3rd	M3^{py}	81/64					407.82		+3.61
13	445.71	"Supermajor" 3rd	M3^{7}			9/7			435.08		+10.63
14	480.00	21st Harmonic	4^{21}			21/16			470.78		+9.22
15	514.29	Just Perfect 4th	4	4/3					498.04		+16.24
16	548.57	11th Harmonic	4^{11}				11/8		551.32		-2.75
17	582.86	Septimal Augmented 4th	4^{7}			7/5			582.51		+0.35
18	617.14	Septimal Diminished 5th	5^{7}			10/7			617.49		-0.35
19	651.43	11th Harmonic Undertone	5^{11}				16/11		648.68		+2.75
20	685.71	Just Perfect 5th (v. flat)	5	3/2					701.96		-16.24
21	720.00	21st Harmonic Undertone	5^{21}			32/21			729.22		-9.22
22	754.29	Septimal Minor 6th	m6^{7}			14/9			764.92		-10.63
23	788.57	Pythagorean Minor 6th	m6^{py}	128/81					792.18		-3.61
24	822.86	Just Minor 6th	m6		8/5				813.69		+9.17
25	857.14	Neutral 6th (11)	N6^{11}				18/11		852.59		+4.55
26	891.43	Quasi-Tempered Major 6th	M6^{qt}			42/25			898.15		-6.72
27	925.71	Septimal Major 6th	M6^{7}			12/7			933.13		-7.42
28	960.00	Harmonic 7th	m7^{7}			7/4			968.83		-8.83
29	994.29	Minor 7th	m7	16/9					996.09		-1.80
30	1028.57	Large Minor 7th (11)	m7^{11}				20/11		1035.00		-6.43
31	1062.86	Small Major 7th (13)	M7^{13}					24/13	1061.43		+1.43
32	1097.14	Major 7th (17)	M7^{17}					32/17	1095.05		+2.09
33	1131.43	Diminished Octave	M7+		48/25				1129.33		+2.10
34	1165.71	33rd Harmonic Undertone	M7^{11}				64/33		1146.73		18.98
35	1200.00	Unison (Octave)	U	2/1					1200		0

Table 36 36 Division Equal Temperament

36-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13 + Limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	33.33	33rd Harmonic or ‘1/4-Tone’	$m2^{11}$				33/32		53.27		-19.94
2	66.67	Small Semitone	m2-		25/24				70.67		-4.00
3	100.00	17th Harmonic	$m2^{17}$					17/16	104.96		-4.96
4	133.33	Large Semitone	m2+		27/25				133.24		+0.09
5	166.67	Ptolemy’s 2nd	$M2^{pt}$				11/10		165.00		+1.67
6	200.00	Just 2nd	M2	9/8					203.91		-3.91
7	233.33	Septimal 2nd	$M2^{7}$			8/7			231.17		+2.16
8	266.67	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		-0.20
9	300.00	Quasi-Tempered Minor 3rd	$m3^{qt}$			25/21			301.85		-1.85
10	333.33	Neutral 3rd (11)	$N3^{11}$				11/9		347.41		-14.08
11	366.67	Neutral 3rd (13)	$N3^{13}$					16/13	359.47		+7.20
12	400.00	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		-7.82
13	433.33	“Supermajor” 3rd	$M3^{7}$			9/7			435.08		-1.75
14	466.67	21st Harmonic	4^{21}			21/16			470.78		-4.11
15	500.00	Just Perfect 4th	4	4/3					498.04		+1.96
16	533.33	11th Harmonic	4^{11}				11/8		551.32		-17.99
17	566.67	11th Harmonic	4^{11}				11/8		551.32		+15.35
18	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
19	633.33	11th Harmonic Undertone	5^{11}				16/11		648.68		-15.35
20	666.67	11th Harmonic Undertone	5^{11}				16/11		648.68		+17.99
21	700.00	Just Perfect 5th	5	3/2					701.96		-1.95
22	733.33	21st Harmonic Undertone	5^{21}			32/21			729.22		+4.11
23	766.67	Septimal Minor 6th	$m6^{7}$			14/9			764.92.		+1.75
24	800.00	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		+7.82
25	833.33	Neutral 6th (13)	$N6^{13}$					13/8	840.53		-7.20
26	866.67	Neutral 6th (11)	$N6^{11}$				18/11		852.59.		+14.08
27	900.00	Quasi-Tempered Major 6th	$M6^{qt}$			42/25			898.15		+1.85
28	933.33	Septimal Major 6th	$M6^{7}$			12/7			933.13		+0.20
29	966.67	Harmonic 7th	$m7^{7}$			7/4			968.83		-2.16
30	1000.00	Minor 7th	m7	16/9					996.09		+3.91
31	1033.33	Large Minor 7th (11)	$m7^{11}$				20/11		1035.00		-1.67
32	1066.67	Small Major 7th	M7-		50/27				1066.76		-0.09
33	1100.00	Major 7th (17)	$M7^{17}$					32/17	1095.05		+4.95
34	1133.33	Diminished Octave	M7+		48/25				1129.33		+4.00
35	1166.67	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+19.94
36	1200.00	Unison (Octave)	U	2/1					1200		0

Table 37 37 Division Equal Temperament

37-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3- limit	5- limit	7- limit	11- limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	32.43	33rd Harmonic or ‘1/4-Tone’	$m2^{11}$				33/32		53.27		-20.84
2	64.86	Small Semitone	m2-		25/24				70.67		-5.81
3	97.30	Pythagorean Semitone	$m2^{py}$	256/243					90.23		+7.07
4	129.73	Large Semitone	m2+		27/25				133.24		-3.51
5	162.16	Ptolemy’s 2nd	$M2^{pt}$				11/10		165.00		-2.84
6	194.59	Just 2nd	M2	9/8					203.91		-9.32
7	227.03	Septimal 2nd	$M2^{7}$			8/7			231.17		-4.14
8	259.46	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		-7.41
9	291.89	Pythagorean Minor 3rd	$m3^{py}$	32/27					294.14		-2.25
10	324.32	Just Minor 3rd	m3		6/5				315.64		+8.68
11	356.76	Neutral 3rd (13)	$N3^{13}$					16/13	359.47		-2.71
12	389.19	Just Major 3rd	M3		5/4				386.31		+2.88
13	421.62	Large Major 3rd (11)	$M3^{11}$				14/11		417.51		+4.11
14	454.05	49th Harmonic Undertone	4^{49}			64/49			462.35		-8.30
15	486.49	Just Perfect 4th	4	4/3					498.04		-11.56
16	518.92	Just Perfect 4th	4	4/3					498.04		20.87
17	551.35	11th Harmonic	4^{11}				11/8		551.32		+0.03
18	583.78	Septimal Augmented 4th	4^{7}			7/5			582.51		+1.27
19	616.22	Septimal Diminished 5th	5^{7}			10/7			617.49		-1.27
20	648.65	11th Harmonic Undertone	5^{11}				16/11		648.68		-0.03
21	681.08	?	?						737.65		-20.87
22	713.51	Just Perfect 5th	5	3/2					701.96		+11.56
23	745.95	Just Perfect 5th	5	3/2					701.96		+8.30
24	778.38	Small Minor 6th (11)	$m6^{11}$				11/7		782.49		-4.11
25	810.81	Just Minor 6th	m6		8/5				813.69		-2.88
26	843.24	Neutral 6th (13)	$N6^{13}$					13/8	840.53		+2.71
27	875.68	Just Major 6th	M6		5/3				884.36		-8.68
28	908.11	Pythagorean Major 6th	$M6^{py}$	27/16					905.87		+2.24
29	940.54	Septimal Major 6th	$M6^{7}$			12/7			933.13		+7.41
30	972.97	Harmonic 7th	$m7^{7}$			7/4			968.83		+4.14
31	1005.41	Minor 7th	m7	16/9					996.09		+9.32
32	1037.84	Large Minor 7th (11)	$m7^{11}$				20/11		1035.00		+2.84
33	1070.27	Small Major 7th	M7-		50/27				1066.76		+3.51
34	1102.70	Pythagorean Major 7th	$M7^{py}$	243/128					1109.78.		-7.08
35	1135.14	Diminished Octave	M7+		48/25				1129.33		+5.81
36	1167.57	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		20.84
37	1200.00	Unison (Octave)	U	2/1					1200		0

Table 38 38 Division Equal Temperament

38-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3- limit	5- limit	7- limit	11- limit	13 + limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	31.58	33rd Harmonic or ‘1/4-Tone’	m2^{11}				33/32		53.27		-21.69
2	63.16	Small Semitone	m2-		25/24				70.67		-7.51
3	94.74	Pythagorean Semitone	m2^{py}	256/243					90.23		+4.51
4	126.32	Large Semitone	m2+		27/25				133.24		-6.92
5	157.89	Ptolemy’s 2nd	M2^{pt}				11/10		165.00		-7.11
6	189.47	Small Just 2nd	M2-		10/9				182.40		+7.07
7	221.05	Septimal 2nd	M2^{7}			8/7			231.17		-10.12
8	252.63	Septimal Minor 3rd	m3^{7}			7/6			266.87		-14.24
9	284.21	Pythagorean Minor 3rd	m3^{py}	32/27					294.14		-9.93
10	315.79	Just Minor 3rd	m3		6/5				315.64		+0.15
11	347.37	Neutral 3rd (11)	N3^{11}				11/9		347.41		-0.04
12	378.95	Just Major 3rd	M3		5/4				386.31		-7.36
13	410.53	Pythagorean Major 3rd	M3^{py}	81/64					407.82		+2.71
14	442.11	“Supermajor” 3rd	M3^{7}			9/7			435.08		+7.03
15	473.68	21st Harmonic	4^{21}			21/16			470.78		+2.90
16	505.26	Just Perfect 4th	4	4/3					498.04		+7.22
17	536.84	11th Harmonic	4^{11}				11/8		551.32		-14.48
18	568.42	Septimal Augmented 4th	4^{7}			7/5			582.51		-14.09
19	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
20	631.58	Septimal Diminished 5th	5^{7}			10/7			617.49		+14.09
21	663.16	11th Harmonic Undertone	5^{11}				16/11		648.68		+14.48
22	694.74	Just Perfect 5th	5	3/2					701.96		-7.22
23	726.32	21st Harmonic Undertone	5^{21}			32/21			729.22		-2.90
24	757.89	Septimal Minor 6th	m6^{7}			14/9			764.92		-7.03
25	789.47	Pythagorean Minor 6th	m6^{py}	128/81					792.18		-2.71
26	821.05	Just Minor 6th	m6		8/5				813.69		+7.36
27	852.63	Neutral 6th (11)	N6^{11}				18/11		852.59		+0.04
28	884.21	Just Major 6th	M6		5/3				884.36		-0.15
29	915.79	Pythagorean Major 6th	M6^{py}	27/16					905.97		+9.92
30	947.37	Septimal Major 6th	M6^{7}			12/7			933.13		+14.24
31	978.95	Harmonic 7th	m7^{7}			7/4			968.83		+10.12
32	1010.53	Large Minor 7th	m7+		9/5				1017.60		-7.07
33	1042.11	Large Minor 7th	m7^{11}				20/11		1035.00		+7.11
34	1073.68	Small Major 7th	M7-		50/27				1066.76		+6.92
35	1105.26	Pythagorean Major 7th	M7^{py}	243/128					1109.78		-4.52
36	1136.84	Diminished Octave	M7+		48/25				1129.33		+7.51
37	1168.42	33rd Harmonic Undertone	M7^{11}				64/33		1146.73		+21.69
38	1200.00	Unison (Octave)	U	2/1					1200		0

Table 39 39 Division Equal Temperament

39-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3- limit	5- limit	7- limit	11- limit	13+ limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	30.77	33rd Harmonic or '1/4-Tone'	m2^{11}				33/32		53.27		-22.50
2	61.54	33rd Harmonic or '1/4-Tone'	m2^{11}				33/32		53.27		+8.27
3	92.31	Pythagorean Semitone	m2^{py}	256/243					90.23		+2.08
4	123.08	Large Semitone	m2+		27/25				133.24		-10.16
5	153.85	Neutral 2nd	N2				12/11		150.64		+3.21
6	184.62	Small Just 2nd	M2-		10/9				182.40		+2.22
7	215.38	Just 2nd	M2	9/8					203.91		+11.47
8	246.15	Septimal 2nd	M2^{7}			8/7			231.17		+14.98
9	276.92	Septimal Minor 3rd	m3^{7}			7/6			266.87		+10.05
10	307.69	Quasi-Tempered Minor 3rd	m3^{qt}			25/21			301.85		+5.84
11	338.46	Neutral 3rd (11)	N3^{11}				11/9		347.41		-8.95
12	369.23	Neutral 3rd (13)	N3^{13}					16/13	359.47		+9.76
13	400.00	Pythagorean Major 3rd	M3^{py}	81/64					407.82		-7.82
14	430.77	Just Large 3rd	M3+		32/25				427.37		+3.40
15	461.54	49th Harmonic Undertone	4^{49}			64/49			462.35		-0.81
16	492.31	Just Perfect 4th	4	4/3					498.04		-5.74
17	523.08	Just Perfect 4th	4	4/3					498.04		+25.03
18	553.85	11th Harmonic	4^{11}				11/8		551.32		+2.53
19	584.62	Septimal Augmented 4th	4^{7}			7/5			582.51		+2.11
20	615.38	Septimal Diminished 5th	5^{7}			10/7			617.49		-2.11
21	646.15	11th Harmonic Undertone	5^{11}				16/11		648.68		-2.53
22	676.92	Just Perfect 5th	5	3/2					701.96		-25.03
23	707.69	Just Perfect 5th	5	3/2					701.96		+5.74
24	738.46	49th Harmonic	5^{49}			49/32			737.65		+0.81
25	769.23	Small Just Minor 6th	m6-		25/16				772.63		-3.40
26	800.00	Pythagorean Minor 6th	m6^{py}	128/81					792.18		+7.82
27	830.77	Neutral 6th (13)	N6^{13}					13/8	840.53		-9.76
28	861.54	Neutral 6th (11)	N6^{11}				18/11		852.59		+8.95
29	892.31	Quasi-Tempered Major 6th	M6^{qt}			42/25			898.15		-5.84
30	923.08	Septimal Major 6th	M6^{7}			12/7			933.13		-10.05
31	953.85	Harmonic 7th	m7^{7}			7/4			968.83		-14.98
32	984.62	Minor 7th	m7	16/9					996.09		-11.47
33	1015.38	Large Minor 7th	m7+		9/5				1017.60		-2.22
34	1046.15	Neutral 7th	N7				11/6		1049.36		-3.21
35	1076.92	Small Major 7th	M7-		50/27				1066.76		+10.16
36	1107.69	Pythagorean Major 7th	M7^{py}	243/128					1109.78		-2.09
37	1138.46	33rd Harmonic Undertone	M7^{11}				64/33		1146.73		-8.27
38	1169.23	33rd Harmonic Undertone	M7^{11}				64/33		1146.73		+22.50
39	1200.00	Unison (Octave)	U	2/1					1200		0

Table 40 40 Division Equal Temperament

40-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3- limit	5- limit	7- limit	11- limit	13+ limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	30.00	33rd Harmonic or '1/4-Tone'	$m2^{11}$				33/32		53.27		-23.27
2	60.00	33rd Harmonic	$m2^{11}$				33/32		53.27		+6.73
3	90.00	Pythagorean Semitone	$m2^{py}$	256/243					90.23		0.23
4	120.00	Just Semitone	m2		16/15				111.73		+8.27
5	150.00	Neutral 2nd	N2				12/11		150.64		-0.64
6	180.00	Small Just 2nd	M2-		10/9				182.40		-2.40
7	210.00	Just 2nd	M2	9/8					203.91		+6.09
8	240.00	Septimal 2nd	$M2^{7}$			8/7			231.17		+8.83
9	270.00	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		+3.13
10	300.00	Quasi-Tempered Minor 3rd	$m3^{qt}$			25/21			301.85		-1.85
11	330.00	Just Minor 3rd	m3		6/5				315.64		+14.36
12	360.00	Neutral 3rd (13)	$N3^{13}$					16/13	359.47		+0.53
13	390.00	Just Major 3rd	M3		5/4				386.31		+3.69
14	420.00	Large Major 3rd (11)	$M3^{11}$				14/11		417.51		+2.49
15	450.00	49th Harmonic Undertone	4^{49}			64/49			462.35		-12.35
16	480.00	21st Harmonic	4^{21}			21/16			470.78		+9.22
17	510.00	Just Perfect 4th	4	4/3					498.04		+11.96
18	540.00	11th Harmonic	4^{11}				11/8		551.32		-11.32
19	570.00	Septimal Augmented 4th	4^{7}			7/5			582.51		-12.51
20	600.00	Just Augmented 4th (or) Just Diminished 5th	4+ 5-		45/32 64/45				590.22 609.78		+9.78 -9.78
21	630.00	Septimal Diminished 5th	5^{7}			10/7			617.49		+12.51
22	660.00	11th Harmonic Undertone	5^{11}				16/11		648.68		+11.32
23	690.00	Just Perfect 5th	5	3/2					701.96		-11.95
24	720.00	21st Harmonic Undertone	5^{21}			32/21			729.22		-9.22
25	750.00	49th Harmonic	5^{49}			49/32			737.65		+12.35
26	780.00	Small Minor 6th (11)	$m6^{11}$				11/7		782.49		-2.49
27	810.00	Just Minor 6th	m6		8/5				813.6		-3.69
28	840.00	Neutral 6th (13)	$N6^{13}$					13/8	840.53		-0.53
29	870.00	Just Major 6th	M6		5/3				884.36		-14.36
30	900.00	Quasi-Tempered Major 6th	$M6^{qt}$			42/25			898.15		+1.85
31	930.00	Septimal Major 6th	$M6^{7}$			12/7			933.13		-3.13
32	960.00	Harmonic 7th	$m7^{7}$			7/4			968.83		-8.83
33	990.00	Minor 7th	m7	16/9					996.09		-6.09
34	1020.00	Large Minor 7th	m7+		9/5				1017.60		+2.40
35	1050.00	Neutral 7th	N7				11/6		1049.36		+0.64
36	1080.00	Just Major 7th	M7		15/8				1088.27		-8.27
37	1110.00	Pythagorean Major 7th	$M7^{py}$	243/128					1109.78		+0.22
38	1140.00	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		-6.73
39	1170.00	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+23.27
40	1200.00	Unison (Octave)	U	2/1					1200		0

Table 41 41 Division Equal Temperament

41-ET		Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Cents	Interval	Abbr	3- limit	5- limit	7- limit	11- limit	13+ limit	Cents	Zone Score	Diff. Cents
0	0	Unison	U	1/1					0		0
1	29.27	33rd Harmonic or '1/4-Tone'	$m2^{11}$				33/32		53.27		-24.00
2	58.54	33rd Harmonic or '1/4-Tone'	$m2^{11}$				33/32		53.27		+5.27
3	87.80	Pythagorean Semitone	$m2^{py}$	256/243					90.23		-2.43
4	117.07	Just Semitone	m2		16/15				111.73		+5.34
5	146.34	Neutral 2nd	N2				12/11		150.64		-4.30
6	175.61	Small Just 2nd	M2-		10/9				182.40		-6.79
7	204.88	Just 2nd	M2	9/8					203.91		+0.97
8	234.15	Septimal 2nd	$M2^{7}$			8/7			231.17		+2.98
9	263.41	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		-3.46
10	292.68	Pythagorean Minor 3rd	$m3^{py}$	32/27					294.14		-1.46
11	321.95	Just Minor 3rd	m3		6/5				315.64		+6.31
12	351.22	Neutral 3rd (11)	$N3^{11}$				11/9		347.41		+3.81
13	380.49	Just Major 3rd	M3		5/4				386.31		-5.82
14	409.76	Pythagorean Major 3rd	$M3^{py}$	81/64					407.82		+1.94
15	439.02	"Supermajor" 3rd	$M3^{7}$			9/7			435.08		+3.94
16	468.29	21st Harmonic	4^{21}			21/16			470.78		-2.49
17	497.56	Just Perfect 4th	4	4/3					498.04		-0.48
18	526.83	11th Harmonic	4^{11}				11/8		551.32		-24.49
19	556.10	11th Harmonic	4^{11}				11/8		551.32		+4.78
20	585.37	Septimal Augmented 4th	4^{7}			7/5			582.51		+2.86
21	614.63	Septimal Diminished 5th	5^{7}			10/7			617.49		-2.86
22	643.90	11th Harmonic Undertone	5^{11}				16/11		648.68		-4.78
23	673.17	11th Harmonic Undertone	5^{11}				16/11		648.68		+24.49
24	702.44	Just Perfect 5th	5	3/2					701.955		0.48
25	731.71	21st Harmonic Undertone	5^{21}			32/21			729.22		+2.49
26	760.98	Septimal Minor 6th	$m6^{7}$			14/9			764.92		-3.94
27	790.24	Pythagorean Minor 6th	$m6^{py}$	128/81					792.18		-1.94
28	819.51	Just Minor 6th	m6		8/5				813.69		+5.82
29	848.78	Neutral 6th (11)	$N6^{11}$				18/11		852.59		-3.81
30	878.05	Just Major 6th	M6		5/3				884.36		-6.31
31	907.32	Pythagorean Major 6th	$M6^{py}$	27/16					905.87		+1.45
32	936.59	Septimal Major 6th	$M6^{7}$			12/7			933.13		+3.46
33	965.85	Harmonic 7th	$m7^{7}$			7/4			968.83		-2.98
34	995.12	Minor 7th	m7	16/9					996.09		-0.97
35	1024.39	Large Minor 7th	m7+		9/5				1017.60		+6.79
36	1053.66	Neutral 7th	N7				11/6		1049.36		+4.30
37	1082.93	Just Major 7th	M7		15/8				1088.27		-5.34
38	1112.20	Pythagorean Major 7th	$M7^{py}$	243/128					1109.78		+2.42
39	1141.46	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		-5.27
40	1170.73	33rd Harmonic Undertone	$M7^{11}$				64/33		1146.73		+24.00
41	1200.00	Unison (Octave)	U	2/1					1200		0

Table 43 Harry Partch's 43-Division JI Scale

Compared to 'Interval Zones' of Appendix I (B)

Partch 43 JI			Interval Zone		Closest Ratio from APPENDIX 1 (A)							
Scale Deg.	Ratio	Cents	Interval	Abbr	3-limit	5-limit	7-limit	11-limit	13+ limit	Cents	Zone Score	Diff. Cents
0	1/1	0	Unison	U	1/1					0		
1	81/80	21.51	'1/8th or 1/6th Tone'	U^{+}						18.0 - 35.0		N/A
2	33/32	53.27	33rd Harm. Or '¼ -Tone'	$m2^{11}$				33/32		53.27		0
3	21/20	84.47	Subminor 2nd	$m2^{7}$	21/20					84.47		0
4	16/15	111.73	Just Semitone	m2		16/15				111.73		0
5	12/11	150.64	Neutral 2nd	N2				12/11		150.64		0
6	11/10	165.00	Ptolemy's 2nd	$M2^{pt}$				11/10		165.00		0
7	10/9	182.40	Small Just 2nd	M2-		10/9				182.40		0
8	9/8	203.91	Just 2nd	M2	9/8					203.91		0
9	8/7	231.17	Septimal 2nd	$M2^{7}$			8/7			231.17		0
10	7/6	266.87	Septimal Minor 3rd	$m3^{7}$			7/6			266.87		0
11	32/27	294.14	Pythagorean Minor 3rd	$m3^{py}$	32/27					294.14		0
12	6/5	315.64	Just Minor 3rd	m3		6/5				315.64		0
13	11/9	347.41	Neutral 3rd (11)	$N3^{11}$				11/9		347.41		0
14	5/4	386.31	Just Major 3rd	M3		5/4				386.31		0
15	14/11	417.51	Large Major 3rd (11)	$M3^{11}$				14/11		417.51		0
16	9/7	435.08	"Supermajor" 3rd	$M3^{7}$			9/7			435.08		0
17	21/16	470.78	21st Harmonic	4^{21}			21/16			470.78		0
18	4/3	498.045	Just Perfect 4th	4	4/3					498.045		0
19	27/20	519.55	?	?						?		?
20	11/8	551.32	11th Harmonic	4^{11}				11/8		551.32		0
21	7/5	582.51	Septimal Augmented 4th	4^{7}			7/5			582.51		0
22	10/7	617.49	Septimal Diminished 5th	5^{7}			10/7			617.49		0
23	16/11	648.68	11th Harm. Undertone	5^{11}				16/11		648.68		0
24	40/27	680.45	?	?						?		?
25	3/2	701.955	Just Perfect 5th	5	3/2					701.955		0
26	32/21	729.22	21st Harm. Undertone	5^{21}			32/21			729.22		0
27	14/9	764.92	Septimal Minor 6th	$m6^{7}$			14/9			764.92		0
28	11/7	782.49	Small Minor 6th (11)	$m6^{11}$				11/7		782.49		0
29	8/5	813.69	Just Minor 6th	m6		8/5				813.69		0
30	18/11	852.59	Neutral 6th (11)	$N6^{11}$				18/11		852.59		0
31	5/3	884.36	Just Major 6th	M6		5/3				884.36		0
32	27/16	905.87	Pythagorean Major 6th	$M6^{py}$	27/16					905.87		0
33	12/7	933.13	Septimal Major 6th	$M6^{7}$			12/7			933.13		0
34	7/4	968.83	Harmonic 7th	$m7^{7}$			7/4			968.83		0
35	16/9	996.09	Minor 7th	m7	16/9					996.09		0
36	9/5	1017.60	Large Minor 7th	m7+		9/5				1017.60		0
37	20/11	1035.00	Large Minor 7th (11)	$m7^{11}$				20/11		1035.00		0
38	11/6	1049.36	Neutral 7th	N7				11/6		1049.36		0
39	15/8	1088.27	Just Major 7th	M7		15/8				1088.27		0
40	40/21	1115.53	Septimal Major 7th	$M7^{7}$			40/21			1115.53		0
41	64/33	1146.73	33rd Harm. Undertone	$M7^{11}$				64/33		1146.73		0
42	160/81	1178.49	Small Octave	U^{-}						1166 - 1182		N/A
43	2/1	1200	Unison (Octave)	U	2/1					1200		0

APPENDIX II TABLE 42 (d)

ERV WILSON - 1 3 5 7 9 11 - eikosany

INTERVALS IN THE SCALE

ROTATIONS OF THE SCALE - TREATING EACH SCALE DEGREE IN TURN AS '1/1'

	0 steps		1 step		2 steps		3 steps		4 steps		5 steps		6 steps		7 steps		8 steps		9 steps		10 steps	
Rotation on	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents
33/32	1/1	0.00	45/44	38.91	35/33	101.87	9/8	203.91	7/6	266.87	105/88	305.78	5/4	386.31	14/11	417.51	21/16	470.78	15/11	536.95	63/44	621.42
135/128	1/1	0.00	28/27	62.96	11/10	165.00	154/135	227.97	7/6	266.87	11/9	347.41	56/45	378.60	77/60	431.88	4/3	498.05	7/5	582.51	77/54	614.28
35/32	1/1	0.00	297/280	102.04	11/10	165.00	9/8	203.91	33/28	284.45	6/5	315.64	99/80	368.91	9/7	435.08	27/20	519.55	11/8	551.32	99/70	600.09
297/256	1/1	0.00	28/27	62.96	35/33	101.87	10/9	182.40	112/99	213.60	7/6	266.87	40/33	333.04	14/11	417.51	35/27	449.27	4/3	498.05	140/99	599.91
77/64	1/1	0.00	45/44	38.91	15/14	119.44	12/11	150.64	9/8	203.91	90/77	270.08	27/22	354.55	5/4	386.31	9/7	435.08	15/11	536.95	108/77	585.72
315/256	1/1	0.00	22/21	80.54	16/15	111.73	11/10	165.00	8/7	231.17	6/5	315.64	11/9	347.41	44/35	396.18	4/3	498.05	48/35	546.82	88/63	578.58
165/128	1/1	0.00	56/55	31.19	21/20	84.47	12/11	150.64	63/55	235.10	7/6	266.87	6/5	315.64	14/11	417.51	72/55	466.28	4/3	498.05	7/5	582.51
21/16	1/1	0.00	33/32	53.27	15/14	119.44	9/8	203.91	55/48	235.68	33/28	284.45	5/4	386.31	9/7	435.08	55/42	466.85	11/8	551.32	10/7	617.49
693/512	1/1	0.00	80/77	66.17	12/11	150.64	10/9	182.40	8/7	231.17	40/33	333.04	96/77	381.81	80/63	413.58	4/3	498.05	320/231	564.21	10/7	617.49
45/32	1/1	0.00	21/20	84.47	77/72	116.23	11/10	165.00	7/6	266.87	6/5	315.64	11/9	347.41	77/60	431.88	4/3	498.05	11/8	551.32	7/5	582.51
189/128	1/1	0.00	55/54	31.77	22/21	80.54	10/9	182.40	8/7	231.17	220/189	262.94	11/9	347.41	80/63	413.58	55/42	466.85	4/3	498.05	88/63	578.58
385/256	1/1	0.00	36/35	48.77	12/11	150.64	432/385	199.41	8/7	231.17	6/5	315.64	96/77	381.81	9/7	435.08	72/55	466.28	48/35	546.82	108/77	585.72
99/64	1/1	0.00	35/33	101.87	12/11	150.64	10/9	182.40	7/6	266.87	40/33	333.04	5/4	386.31	14/11	417.51	4/3	498.04	15/11	536.95	140/99	599.91
105/64	1/1	0.00	36/35	48.77	22/21	80.54	11/10	165.00	8/7	231.17	33/28	284.45	6/5	315.64	44/35	396.18	9/7	435.08	4/3	498.04	99/70	600.09
27/16	1/1	0.00	55/54	31.77	77/72	116.23	10/9	182.40	55/48	235.68	7/6	266.87	11/9	347.41	5/4	386.31	35/27	449.27	11/8	551.32	77/54	614.28
55/32	1/1	0.00	21/20	84.47	12/11	150.64	9/8	203.91	63/55	235.10	6/5	315.64	27/22	354.55	14/11	417.51	27/20	519.55	7/5	582.51	63/44	621.42
231/128	1/1	0.00	80/77	66.17	15/14	119.44	12/11	150.64	8/7	231.17	90/77	270.08	40/33	333.04	9/7	435.08	4/3	498.04	15/11	536.95	10/7	617.49
15/8	1/1	0.00	33/32	53.27	21/20	84.47	11/10	165.00	9/8	203.91	7/6	266.87	99/80	368.91	77/60	431.88	21/16	470.78	11/8	551.32	7/5	582.51
495/256	1/1	0.00	56/55	31.19	16/15	111.73	12/11	150.64	112/99	213.60	6/5	315.64	56/45	378.60	14/11	417.51	4/3	498.04	224/165	529.24	7/5	582.51
63/32	1/1	0.00	22/21	80.54	15/14	119.44	10/9	182.40	33/28	284.45	11/9	347.41	5/4	386.31	55/42	466.85	4/3	498.04	11/8	551.32	10/7	617.49

11 steps		12 steps		13 steps		14 steps		15 steps		16 stesp		17 steps		18 steps		19 steps		20 steps	
Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents	Ratios	Cents
35/24	653.18	3/2	701.96	35/32	803.82	18/11	852.59	5/3	884.36	7/4	968.83	20/11	1035.00	15/8	1088.27	21/11	1119.46	2/1	1200.00
22/15	663.05	14/9	764.92	8/5	813.69	44/27	845.45	77/45	929.92	16/9	996.09	11/6	1049.36	28/15	1080.56	88/45	1161.09	2/1	1200.00
3/2	701.96	54/35	750.73	11/7	782.49	33/20	866.96	12/7	933.13	99/56	986.40	9/5	1017.60	66/35	1098.13	27/14	1137.04	2/1	1200.00
16/11	648.68	40/27	680.45	14/9	764.92	160/99	831.09	5/3	884.36	56/33	915.55	16/9	996.09	20/11	1035.00	560/297	1097.96	2/1	1200.00
10/7	617.49	3/2	701.96	120/77	768.12	45/28	821.40	18/11	852.59	12/7	933.13	135/77	972.03	20/11	1035.00	27/14	1137.04	2/1	1200.00
22/15	663.05	32/21	729.22	11/7	782.49	8/5	813.69	176/105	894.22	12/7	933.13	16/9	996.09	66/35	1098.13	88/45	1161.09	2/1	1200.00
16/11	648.68	3/2	701.96	84/55	733.15	8/5	813.69	18/11	852.59	56/33	915.55	9/5	1017.60	28/15	1080.56	21/11	1119.46	2/1	1200.00
165/112	670.76	3/2	701.96	11/7	782.49	45/28	821.40	5/3	884.36	99/56	986.40	11/6	1049.36	15/8	1088.27	55/28	1168.81	2/1	1200.00
16/11	648.68	32/21	729.22	120/77	768.12	160/99	831.09	12/7	933.13	16/9	996.09	20/11	1035.00	40/21	1115.53	64/33	1146.73	2/1	1200.00
22/15	663.05	3/2	701.95	14/9	764.92	33/20	866.96	77/45	929.92	7/4	968.83	11/6	1049.36	28/15	1080.56	77/40	1133.83	2/1	1200.00
10/7	617.49	40/27	680.45	11/7	782.49	44/27	845.45	5/3	884.36	110/63	964.90	16/9	996.09	11/6	1049.36	40/21	1115.53	2/1	1200.00
16/11	648.68	54/35	750.73	8/5	813.69	18/11	852.59	12/7	933.13	96/55	964.32	9/5	1017.60	144/77	1083.77	108/55	1168.23	2/1	1200.00
3/2	701.95	14/9	764.92	35/32	803.82	5/3	884.36	56/33	915.55	7/4	968.83	20/11	1035.00	21/11	1119.46	35/18	1151.23	2/1	1200.00
22/15	663.05	3/2	701.95	11/7	782.49	8/5	813.69	33/20	866.96	12/7	933.13	9/5	1017.60	11/6	1049.36	66/35	1098.13	2/1	1200.00
35/24	653.18	55/36	733.72	14/9	764.92	77/48	818.19	5/3	884.36	7/4	968.83	385/216	1000.59	11/6	1049.36	35/18	1151.23	2/1	1200.00
3/2	701.95	84/55	733.15	63/40	786.42	18/11	852.59	189/110	937.06	7/4	968.83	9/5	1017.60	21/11	1119.46	108/55	1168.23	2/1	1200.00
16/11	648.68	3/2	701.95	120/77	768.12	18/11	852.59	5/3	884.36	12/7	933.13	20/11	1035.00	144/77	1083.77	40/21	1115.53	2/1	1200.00
231/160	635.79	3/2	701.95	63/40	786.42	77/48	818.19	33/20	866.96	7/4	968.83	9/5	1017.60	11/6	1049.36	77/40	1133.83	2/1	1200.00
16/11	648.68	84/55	733.15	14/9	764.92	8/5	813.69	56/33	915.55	96/55	964.32	16/9	996.09	28/15	1080.56	64/33	1146.73	2/1	1200.00
3/2	701.95	55/36	733.72	11/7	782.49	5/3	884.36	12/7	933.13	110/63	964.90	11/6	1049.36	40/21	1115.53	55/28	1168.81	2/1	1200.00

APPENDIX III

APPENDIX III (a) The order of and degree to which 11 'primary' consonant intervals (and their inversions) are approximated by all the Equal-Temperaments between 5 and 53 octave divisions inclusive.

Order and degree of approximation to the Just Fifth (3/2 or 701.9 cents) and for the Just Fourth (4/3 or 498.1cents)				
***n*-ET**	**Step (cents)**	**Scale Degree**	**Cents**	**Deviation from 3/2**
53	22.6	31	701.9	-0.1
41	29.3	24	702.4	0.5
29	41.4	17	703.4	1.5
12	100.0	7	700.0	-2.0
24	50.0	14	700.0	-2.0
36	33.3	21	700.0	-2.0
48	25.0	28	700.0	-2.0
46	26.1	27	704.3	2.4
17	70.6	10	705.9	3.9
34	35.3	20	705.9	3.9
51	23.5	30	705.9	3.9
43	27.9	25	697.7	-4.3
31	38.7	18	696.8	-5.2
39	30.8	23	707.7	5.7
50	24.0	29	696.0	-6.0
22	54.5	13	709.1	7.1
44	27.3	26	709.1	7.1
19	63.2	11	694.7	-7.2
38	31.6	22	694.7	-7.2
49	24.5	29	710.2	8.2
45	26.7	26	693.3	-8.6
27	44.4	16	711.1	9.2
26	46.2	15	692.3	-9.6
52	23.1	30	692.3	-9.6
32	37.5	19	712.5	10.5
33	36.4	19	690.9	-11.0
37	32.4	22	713.5	11.6
40	30.0	23	690.0	-12.0
42	28.6	25	714.3	12.3
47	25.5	27	689.4	-12.6
21	57.1	12	685.7	-16.2
7	171.4	4	685.7	-16.2
14	85.7	8	685.7	-16.2
28	42.9	16	685.7	-16.2
35	34.3	20	685.7	-16.2
5	240.0	3	720.0	18.0
10	120.0	6	720.0	18.0
15	80.0	9	720.0	18.0
20	60.0	12	720.0	18.0
25	48.0	15	720.0	18.0
30	40.0	18	720.0	18.0
23	52.2	13	678.3	-23.7
16	75.0	9	675.0	-27.0
18	66.7	11	733.3	31.4
9	133.3	5	666.7	-35.3
13	92.3	8	738.5	36.5
11	109.1	6	654.5	-47.4
8	150.0	5	750.0	48.0
6	200.0	4	800.0	98.0

Order and degree of approximation to the Just Major Third (5/4 or 386.3 cents) and for the Just Minor Sixth (8/5 or 813.7 cents)				
***n*-ET**	**Step (cents)**	**Scale Degree**	**Cents**	**Deviation from 5/4**
28	42.9	9	385.7	-0.6
31	38.7	10	387.1	0.8
53	22.6	17	384.9	-1.4
34	35.3	11	388.2	1.9
50	24.0	16	384.0	-2.3
25	48.0	8	384.0	-2.3
37	32.4	12	389.2	2.9
47	25.5	15	383.0	-3.3
40	30.0	13	390.0	3.7
43	27.9	14	390.7	4.4
44	27.3	14	381.8	-4.5
22	54.5	7	381.8	-4.5
46	26.1	15	391.3	5.0
49	24.5	16	391.8	5.5
41	29.3	13	380.5	-5.8
52	23.1	17	392.3	6.0
38	31.6	12	378.9	-7.4
19	63.2	6	378.9	-7.4
35	34.3	11	377.1	-9.2
51	23.5	16	376.5	-9.8
48	25.0	15	375.0	-11.3
32	37.5	10	375.0	-11.3
16	75.0	5	375.0	-11.3
45	26.7	14	373.3	-13.0
6	200.0	2	400.0	13.7
9	133.3	3	400.0	13.7
12	100.0	4	400.0	13.7
15	80.0	5	400.0	13.7
18	66.7	6	400.0	13.7
21	57.1	7	400.0	13.7
24	50.0	8	400.0	13.7
27	44.4	9	400.0	13.7
30	40.0	10	400.0	13.7
33	36.4	11	400.0	13.7
36	33.3	12	400.0	13.7
39	30.8	13	400.0	13.7
42	28.6	14	400.0	13.7
29	41.4	9	372.4	-13.9
26	46.2	8	369.2	-17.1
13	92.3	4	369.2	-17.1
23	52.2	7	365.2	-21.1
20	60.0	6	360.0	-26.3
10	120.0	3	360.0	-26.3
17	70.6	5	352.9	-33.4
14	85.7	5	428.6	42.3
7	171.4	2	342.9	-43.4
11	109.1	4	436.4	50.1
8	150.0	3	450.0	63.7
5	240.0	2	480.0	93.7

Order and degree of approximation to the Harmonic 7th (7/4 or 968.8 cents) and for the Septimal Second (8/7 or 231.2 cents)				
***n*-ET**	**Step (cents)**	**Scale Degree**	**Cents**	**Deviation from 7/4**
26	46.2	21	969.2	0.4
52	23.1	42	969.2	0.4
31	38.7	25	967.7	-1.1
47	25.5	38	970.2	1.4
36	33.3	29	966.7	-2.1
21	57.1	17	971.4	2.6
42	28.6	34	971.4	2.6
41	29.3	33	965.9	-2.9
46	26.1	37	965.2	-3.6
51	23.5	41	964.7	-4.1
37	32.4	30	973.0	4.2
53	22.6	43	973.6	4.8
16	75.0	13	975.0	6.2
32	37.5	26	975.0	6.2
48	25.0	39	975.0	6.2
43	27.9	35	976.7	7.9
5	240.0	4	960.0	-8.8
10	120.0	8	960.0	-8.8
15	80.0	12	960.0	-8.8
20	60.0	16	960.0	-8.8
25	48.0	20	960.0	-8.8
30	40.0	24	960.0	-8.8
35	34.3	28	960.0	-8.8
40	30.0	32	960.0	-8.8
45	26.7	36	960.0	-8.8
50	24.0	40	960.0	-8.8
27	44.4	22	977.8	9.0
38	31.6	31	978.9	10.1
49	24.5	40	979.6	10.8
11	109.1	9	981.8	13.0
22	54.5	18	981.8	13.0
33	36.4	27	981.8	13.0
44	27.3	36	981.8	13.0
39	30.8	31	953.8	-15.0
34	35.3	27	952.9	-15.9
28	42.9	23	985.7	16.9
29	41.4	23	951.7	-17.1
24	50.0	19	950.0	-18.8
17	70.6	14	988.2	19.4
19	63.2	15	947.4	-21.4
23	52.2	19	991.3	22.5
14	85.7	11	942.9	-25.9
6	200.0	5	1000.0	31.2
12	100.0	10	1000.0	31.2
18	66.7	15	1000.0	31.2
9	133.3	7	933.3	-35.5
13	92.3	10	923.1	-45.7
7	171.4	6	1028.6	59.8
8	150.0	6	900.0	-68.8

Order and degree of approximation to the Just Second (9/8 or 203.9 cents) and for the Minor Seventh (16/9 or 996.1 cents)				
***n*-ET**	**Step (cents)**	**Scale Degree**	**Cents**	**Deviation from 9/8**
53	22.6	9	203.8	-0.1
47	25.5	8	204.3	0.4
41	29.3	7	204.9	1.0
35	34.3	6	205.7	1.8
29	41.4	5	206.9	3.0
52	23.1	9	207.7	3.8
6	200.0	1	200.0	-3.9
12	100.0	2	200.0	-3.9
18	66.7	3	200.0	-3.9
24	50.0	4	200.0	-3.9
30	40.0	5	200.0	-3.9
36	33.3	6	200.0	-3.9
42	28.6	7	200.0	-3.9
48	25.0	8	200.0	-3.9
23	52.2	4	208.7	4.8
46	26.1	8	208.7	4.8
40	30.0	7	210.0	6.1
17	70.6	3	211.8	7.9
34	35.3	6	211.8	7.9
51	23.5	9	211.8	7.9
49	24.5	8	195.9	-8.0
43	27.9	7	195.3	-8.6
37	32.4	6	194.6	-9.3
45	26.7	8	213.3	9.4
31	38.7	5	193.5	-10.4
28	42.9	5	214.3	10.4
39	30.8	7	215.4	11.5
25	48.0	4	192.0	-11.9
50	24.0	8	192.0	-11.9
44	27.3	7	190.9	-13.0
11	109.1	2	218.2	14.3
22	54.5	4	218.2	14.3
33	36.4	6	218.2	14.3
19	63.2	3	189.5	-14.4
38	31.6	6	189.5	-14.4
32	37.5	5	187.5	-16.4
27	44.4	5	222.2	18.3
13	92.3	2	184.6	-19.3
26	46.2	4	184.6	-19.3
16	75.0	3	225.0	21.1
20	60.0	3	180.0	-23.9
21	57.1	4	228.6	24.7
7	171.4	1	171.4	-32.5
14	85.7	2	171.4	-32.5
5	240.0	1	240.0	36.1
10	120.0	2	240.0	36.1
15	80.0	3	240.0	36.1
8	150.0	1	150.0	-53.9
9	133.3	2	266.7	62.8

APPENDIX III (a) continued (The order of and degree to which 11 'primary' consonant intervals (and their inversions) are approximated by all the Equal-Temperaments between 5 and 53 octave divisions inclusive).

Order and degree of approximation to the Just Minor Third (6/5 or 315.6 cents) and for the Just Major Sixth (5/3 or 884.4 cents)

n-ET	Step (cents)	Scale Degree	Cents	Deviation from 6/5
19	63.2	5	315.8	0.2
38	31.6	10	315.8	0.2
42	28.6	11	314.3	-1.3
53	22.6	14	317.0	1.4
34	35.3	9	317.6	2.0
23	52.2	6	313.0	-2.6
46	26.1	12	313.0	-2.6
49	24.5	13	318.4	2.8
50	24.0	13	312.0	-3.6
15	80.0	4	320.0	4.4
30	40.0	8	320.0	4.4
45	26.7	12	320.0	4.4
27	44.4	7	311.1	-4.5
31	38.7	8	309.7	-5.9
41	29.3	11	322.0	6.4
35	34.3	9	308.6	-7.0
26	46.2	7	323.1	7.5
52	23.1	14	323.1	7.5
39	30.8	10	307.7	-7.9
43	27.9	11	307.0	-8.6
37	32.4	10	324.3	8.7
47	25.5	12	306.4	-9.2
48	25.0	13	325.0	9.4
51	23.5	13	305.9	-9.7
11	109.1	3	327.3	11.7
22	54.5	6	327.3	11.7
33	36.4	9	327.3	11.7
44	27.3	12	327.3	11.7
40	30.0	11	330.0	14.4
29	41.4	8	331.0	15.4
8	150.0	2	300.0	-15.6
12	100.0	3	300.0	-15.6
16	75.0	4	300.0	-15.6
20	60.0	5	300.0	-15.6
24	50.0	6	300.0	-15.6
28	42.9	7	300.0	-15.6
32	37.5	8	300.0	-15.6
36	33.3	9	300.0	-15.6
18	66.7	5	333.3	17.7
25	48.0	7	336.0	20.4
7	171.4	2	342.9	27.3
14	85.7	4	342.9	27.3
21	57.1	6	342.9	27.3
17	70.6	4	282.4	-33.2
13	92.3	3	276.9	-38.7
10	120.0	3	360.0	44.4
9	133.3	2	266.7	-48.9
5	240.0	1	240.0	-75.6
6	200.0	2	400.0	84.4

Order and degree of approximation to the Septimal Minor Third (7/6 or 266.9 cents) and for the Septimal Major Sixth (12/7 or 933.1 cents)

n-ET	Step (cents)	Scale Degree	Cents	Deviation from 7/6
9	133.3	2	266.7	-0.2
18	66.7	4	266.7	-0.2
27	44.4	6	266.7	-0.2
36	33.3	8	266.7	-0.2
45	26.7	10	266.7	-0.2
49	24.5	11	269.4	2.5
50	24.0	11	264.0	-2.9
40	30.0	9	270.0	3.1
41	29.3	9	263.4	-3.5
31	38.7	7	271.0	4.1
32	37.5	7	262.5	-4.4
53	22.6	12	271.7	4.8
22	54.5	5	272.7	5.8
44	27.3	10	272.7	5.8
23	52.2	5	260.9	-6.0
46	26.1	10	260.9	-6.0
35	34.3	8	274.3	7.4
37	32.4	8	259.5	-7.4
51	23.5	11	258.8	-8.1
48	25.0	11	275.0	8.1
14	85.7	3	257.1	-9.8
28	42.9	6	257.1	-9.8
42	28.6	9	257.1	-9.8
13	92.3	3	276.9	10.0
26	46.2	6	276.9	10.0
39	30.8	9	276.9	10.0
52	23.1	12	276.9	10.0
47	25.5	10	255.3	-11.6
43	27.9	10	279.1	12.2
33	36.4	7	254.5	-12.4
30	40.0	7	280.0	13.1
19	63.2	4	252.6	-14.3
38	31.6	8	252.6	-14.3
17	70.6	4	282.4	15.5
34	35.3	8	282.4	15.5
24	50.0	5	250.0	-16.9
29	41.4	6	248.3	-18.6
21	57.1	5	285.7	18.8
25	48.0	6	288.0	21.1
5	240.0	1	240.0	-26.9
10	120.0	2	240.0	-26.9
15	80.0	3	240.0	-26.9
20	60.0	4	240.0	-26.9
8	150.0	2	300.0	33.1
12	100.0	3	300.0	33.1
16	75.0	4	300.0	33.1
11	109.1	2	218.2	-48.7
6	200.0	1	200.0	-66.9
7	171.4	2	342.9	76.0

Order and degree of approximation to the 11th Harmonic (11/8 or 551.3 cents) and for the 11th Harmonic Undertone (16/11 or 648.7 cents)

n-ET	Step (cents)	Scale Degree	Cents	Deviation from 11/8
37	32.4	17	551.4	0.1
50	24.0	23	552.0	0.7
24	50.0	11	550.0	-1.3
48	25.0	22	550.0	-1.3
13	92.3	6	553.8	2.5
26	46.2	12	553.8	2.5
39	30.8	18	553.8	2.5
52	23.1	24	553.8	2.5
35	34.3	16	548.6	-2.7
46	26.1	21	547.8	-3.5
41	29.3	19	556.1	4.8
28	42.9	13	557.1	5.8
11	109.1	5	545.5	-5.8
22	54.5	10	545.5	-5.8
33	36.4	15	545.5	-5.8
44	27.3	20	545.5	-5.8
43	27.9	20	558.1	6.8
53	22.6	24	543.4	-7.9
42	28.6	19	542.9	-8.4
15	80.0	7	560.0	8.7
30	40.0	14	560.0	8.7
45	26.7	21	560.0	8.7
31	38.7	14	541.9	-9.4
51	23.5	23	541.2	-10.1
47	25.5	22	561.7	10.4
32	37.5	15	562.5	11.2
20	60.0	9	540.0	-11.3
40	30.0	18	540.0	-11.3
49	24.5	23	563.3	12.0
29	41.4	13	537.9	-13.4
17	70.6	8	564.7	13.4
34	35.3	16	564.7	13.4
38	31.6	17	536.8	-14.5
36	33.3	17	566.7	15.4
19	63.2	9	568.4	17.1
9	133.3	4	533.3	-18.0
18	66.7	8	533.3	-18.0
27	44.4	12	533.3	-18.0
21	57.1	10	571.4	20.1
23	52.2	11	573.9	22.6
25	48.0	11	528.0	-23.3
16	75.0	7	525.0	-26.3
7	171.4	3	514.3	-37.0
14	85.7	6	514.3	-37.0
6	200.0	3	600.0	48.7
8	150.0	4	600.0	48.7
10	120.0	5	600.0	48.7
12	100.0	6	600.0	48.7
5	240.0	2	480.0	-71.3

Order and degree of approximation to the Septimal Augmented Fourth (7/5 or 582.5 cents) and for the Septimal Diminished Fifth (10/7 or 617.5 cents)

n-ET	Step (cents)	Scale Degree	Cents	Deviation from 7/5
35	34.3	17	582.9	0.4
33	36.4	16	581.8	-0.7
37	32.4	18	583.8	1.3
31	38.7	15	580.6	-1.9
39	30.8	19	584.6	2.1
41	29.3	20	585.4	2.9
29	41.4	14	579.3	-3.2
43	27.9	21	586.0	3.5
45	26.7	22	586.7	4.2
27	44.4	13	577.8	-4.7
47	25.5	23	587.2	4.7
49	24.5	24	587.8	5.3
52	23.1	25	576.9	-5.6
51	23.5	25	588.2	5.7
53	22.6	26	588.7	6.2
25	48.0	12	576.0	-6.5
50	24.0	24	576.0	-6.5
48	25.0	23	575.0	-7.5
23	52.2	11	573.9	-8.6
46	26.1	22	573.9	-8.6
44	27.3	21	572.7	-9.8
21	57.1	10	571.4	-11.1
42	28.6	20	571.4	-11.1
40	30.0	19	570.0	-12.5
19	63.2	9	568.4	-14.1
38	31.6	18	568.4	-14.1
36	33.3	17	566.7	-15.8
6	200.0	3	600.0	17.5
8	150.0	4	600.0	17.5
10	120.0	5	600.0	17.5
12	100.0	6	600.0	17.5
14	85.7	7	600.0	17.5
16	75.0	8	600.0	17.5
18	66.7	9	600.0	17.5
20	60.0	10	600.0	17.5
22	54.5	11	600.0	17.5
24	50.0	12	600.0	17.5
26	46.2	13	600.0	17.5
28	42.9	14	600.0	17.5
30	40.0	15	600.0	17.5
32	37.5	16	600.0	17.5
34	35.3	17	600.0	17.5
17	70.6	8	564.7	-17.8
15	80.0	7	560.0	-22.5
13	92.3	6	553.8	-28.7
11	109.1	5	545.5	-37.0
9	133.3	4	533.3	-49.2
7	171.4	3	514.3	-68.2
5	240.0	2	480.0	-102.5

APPENDIX III (a) continued (The order of and degree to which 11 'primary' consonant intervals (and their inversions) are approximated by all the Equal-Temperaments between 5 and 53 octave divisions inclusive).

Order and degree of approximation to the Small Just Second (10/9 or 182.4 cents) and for the Large Minor Seventh (9/5 or 1017.6 cents)				
***n*-ET**	**Step (cents)**	**Scale Degree**	**Cents**	**Deviation from 10/9**
46	26.1	7	182.6	0.2
33	36.4	5	181.8	-0.6
53	22.6	8	181.1	-1.3
13	92.3	2	184.6	2.2
26	46.2	4	184.6	2.2
39	30.8	6	184.6	2.2
52	23.1	8	184.6	2.2
20	60.0	3	180.0	-2.4
40	30.0	6	180.0	-2.4
47	25.5	7	178.7	-3.7
45	26.7	7	186.7	4.3
27	44.4	4	177.8	-4.6
32	37.5	5	187.5	5.1
51	23.5	8	188.2	5.8
34	35.3	5	176.5	-5.9
41	29.3	6	175.6	-6.8
19	63.2	3	189.5	7.1
38	31.6	6	189.5	7.1
48	25.0	7	175.0	-7.4
44	27.3	7	190.9	8.5
25	48.0	4	192.0	9.6
50	24.0	8	192.0	9.6
7	171.4	1	171.4	-11.0
14	85.7	2	171.4	-11.0
21	57.1	3	171.4	-11.0
28	42.9	4	171.4	-11.0
35	34.3	5	171.4	-11.0
42	28.6	6	171.4	-11.0
49	24.5	7	171.4	-11.0
31	38.7	5	193.5	11.1
37	32.4	6	194.6	12.2
43	27.9	7	195.3	12.9
36	33.3	5	166.7	-15.7
29	41.4	4	165.5	-16.9
6	200.0	1	200.0	17.6
12	100.0	2	200.0	17.6
18	66.7	3	200.0	17.6
24	50.0	4	200.0	17.6
30	40.0	5	200.0	17.6
22	54.5	3	163.6	-18.8
15	80.0	2	160.0	-22.4
23	52.2	3	156.5	-25.9
17	70.6	3	211.8	29.4
8	150.0	1	150.0	-32.4
16	75.0	2	150.0	-32.4
11	109.1	2	218.2	35.8
9	133.3	1	133.3	-49.1
5	240.0	1	240.0	57.6
10	120.0	2	240.0	57.6

Order and degree of approximation to the "Supermajor" Third (9/7 or 435.1 cents) and for the Septimal Minor Sixth (14/9 or 764.9 cents)				
***n*-ET**	**Step (cents)**	**Scale Degree**	**Cents**	**Deviation from 9/7**
47	25.5	17	434.0	-1.1
11	109.1	4	436.4	1.3
22	54.5	8	436.4	1.3
33	36.4	12	436.4	1.3
44	27.3	16	436.4	1.3
36	33.3	13	433.3	-1.8
25	48.0	9	432.0	-3.1
50	24.0	18	432.0	-3.1
52	23.1	19	438.5	3.4
41	29.3	15	439.0	3.9
39	30.8	14	430.8	-4.3
30	40.0	11	440.0	4.9
53	22.6	19	430.2	-4.9
49	24.5	18	440.8	5.7
14	85.7	5	428.6	-6.5
28	42.9	10	428.6	-6.5
42	28.6	15	428.6	-6.5
19	63.2	7	442.1	7.0
38	31.6	14	442.1	7.0
46	26.1	17	443.5	8.4
45	26.7	16	426.7	-8.4
31	38.7	11	425.8	-9.3
27	44.4	10	444.4	9.3
48	25.0	17	425.0	-10.1
35	34.3	13	445.7	10.6
43	27.9	16	446.5	11.4
17	70.6	6	423.5	-11.6
34	35.3	12	423.5	-11.6
51	23.5	18	423.5	-11.6
37	32.4	13	421.6	-13.5
8	150.0	3	450.0	14.9
16	75.0	6	450.0	14.9
24	50.0	9	450.0	14.9
32	37.5	12	450.0	14.9
40	30.0	15	450.0	14.9
20	60.0	7	420.0	-15.1
23	52.2	8	417.4	-17.7
26	46.2	9	415.4	-19.7
29	41.4	11	455.2	20.1
21	57.1	8	457.1	22.0
13	92.3	5	461.5	26.4
18	66.7	7	466.7	31.6
6	200.0	2	400.0	-35.1
9	133.3	3	400.0	-35.1
12	100.0	4	400.0	-35.1
15	80.0	5	400.0	-35.1
5	240.0	2	480.0	44.9
10	120.0	4	480.0	44.9
7	171.4	3	514.3	79.2

Order and degree of approximation to the Just Semitone (16/15 or 111.7 cents) and for the Just Major Seventh (15/8 or 1088.3 cents)				
***n*-ET**	**Step (cents)**	**Scale Degree**	**Cents**	**Deviation from 16/15**
43	27.9	4	111.6	-0.1
32	37.5	3	112.5	0.8
53	22.6	5	113.2	1.5
21	57.1	2	114.3	2.6
42	28.6	4	114.3	2.6
11	109.1	1	109.1	-2.6
22	54.5	2	109.1	-2.6
33	36.4	3	109.1	-2.6
44	27.3	4	109.1	-2.6
52	23.1	5	115.4	3.7
31	38.7	3	116.1	4.4
45	26.7	4	106.7	-5.0
41	29.3	4	117.1	5.4
34	35.3	3	105.9	-5.8
51	23.5	5	117.6	5.9
23	52.2	2	104.3	-7.4
46	26.1	4	104.3	-7.4
10	120.0	1	120.0	8.3
20	60.0	2	120.0	8.3
30	40.0	3	120.0	8.3
40	30.0	4	120.0	8.3
50	24.0	5	120.0	8.3
35	34.3	3	102.9	-8.8
47	25.5	4	102.1	-9.6
49	24.5	5	122.4	10.7
39	30.8	4	123.1	11.4
12	100.0	1	100.0	-11.7
24	50.0	2	100.0	-11.7
36	33.3	3	100.0	-11.7
48	25.0	4	100.0	-11.7
29	41.4	3	124.1	12.4
37	32.4	3	97.3	-14.4
19	63.2	2	126.3	14.6
38	31.6	4	126.3	14.6
25	48.0	2	96.0	-15.7
28	42.9	3	128.6	16.9
13	92.3	1	92.3	-19.4
26	46.2	2	92.3	-19.4
9	133.3	1	133.3	21.6
18	66.7	2	133.3	21.6
27	44.4	3	133.3	21.6
14	85.7	1	85.7	-26.0
17	70.6	2	141.2	29.5
15	80.0	1	80.0	-31.7
16	75.0	1	75.0	-36.7
8	150.0	1	150.0	38.3
7	171.4	1	171.4	59.7
6	200.0	1	200.0	88.3
5	240.0	1	240.0	128.3

APPENDIX III (b) Largest single deviation of any interval within an *n*-ET from the 11 chosen 'primary' intervals shown for 5 to 53-ET and 5 to 41-ET

Largest single deviation of any interval within an *n*-ET from the 11 chosen just intervals			
***n*-ET (5 to 53)**	**Deviation**	***n*-ET (5 to 41)**	**Deviation**
41	6.8	**41**	6.8
53	7.9	**31**	11.1
46	8.6	**33**	14.3
52	10.0	**37**	14.4
31	11.1	**38**	14.6
51	11.6	**40**	14.9
48	11.7	**39**	15.0
50	11.9	**36**	15.8
49	12.0	**35**	16.2
47	12.6	**34**	17.5
43	12.9	**32**	17.5
45	13.0	**28**	17.5
44	13.0	**30**	18.0
42	13.7	**29**	18.6
33	14.3	**24**	18.8
37	14.4	**22**	18.8
38	14.6	**26**	19.7
40	14.9	**19**	21.4
39	15.0	**27**	21.6
36	15.8	**25**	23.3
35	16.2	**23**	25.9
34	17.5	**20**	26.9
32	17.5	**21**	27.3
28	17.5	**18**	31.6
30	18.0	**17**	33.4
29	18.6	**15**	36.1
24	18.8	**16**	36.7
22	18.8	**14**	42.3
26	19.7	**13**	45.7
19	21.4	**12**	48.7
27	21.6	**11**	50.1
25	23.3	**10**	57.6
23	25.9	**9**	62.8
20	26.9	**8**	68.8
21	27.3	**7**	79.2
18	31.6	**6**	98.0
17	33.4	**5**	128.3
15	36.1		
16	36.7		
14	42.3		
13	45.7		
12	48.7		
11	50.1		
10	57.6		
9	62.8		
8	68.8		
7	79.2		
6	98.0		
5	128.3		

APPENDIX III

APPENDIX III (c) Order of summed deviations of each of the 11 chosen intervals, shown for between 5 and 53-ET, and for between 5 and 41-ET

Total deviation for all 11 intervals (5 to 53-ET)	
***n*-ET**	**Total deviation**
53	34.3
41	43.8
46	52.4
52	54.8
31	63.5
50	63.7
47	67.9
45	75.1
48	78.9
43	80.8
49	82.5
51	82.7
44	83.1
42	83.2
35	83.9
37	85.5
39	86.4
33	87.1
40	97.5
36	97.9
34	101.3
22	102.4
38	110.8
27	113.2
32	113.9
30	118.9
19	124.8
26	125.3
28	127.2
24	133.9
29	135.5
25	140.7
23	162.8
21	170.1
20	174.1
18	204.4
17	214.9
15	228.3
12	230.1
16	242.1
13	246.5
14	251.9
11	267.7
10	337.5
9	369.3
8	434.9
6	505.3
7	510.3
5	663.7

Total deviation for all 11 intervals (5 to 41-ET)	
***n*-ET**	**Total deviation**
41	43.8
31	63.5
35	83.9
37	85.5
39	86.4
33	87.1
40	97.5
36	97.9
34	101.3
22	102.4
38	110.8
27	113.2
32	113.9
30	118.9
19	124.8
26	125.3
28	127.2
24	133.9
29	135.5
25	140.7
23	162.8
21	170.1
20	174.1
18	204.4
17	214.9
15	228.3
12	230.1
16	242.1
13	246.5
14	251.9
11	267.7
10	337.5
9	369.3
8	434.9
6	505.3
7	510.3
5	663.7

APPENDIX III (d) Order of summed deviations of the chosen 11 intervals *multiplied* by the number of divisions per octave, shown for between 5 and 53-ET, and for between 5 and 41-ET

Total deviation (5 to 53-ET) relative to divisions per octave: Harmonicity/Manageability	
***n*-ET**	**Total Deviation *multiplied* by *n***
41	1796.1
53	1819.1
31	1967.5
22	2252.8
19	2370.8
46	2408.7
12	2760.7
52	2847.7
33	2872.8
35	2937.9
11	2944.8
6	3032.1
27	3055.5
37	3163.6
50	3182.8
47	3192.4
13	3204.9
24	3212.5
26	3257.8
5	3318.7
9	3323.9
39	3369.3
10	3375.5
45	3377.5
15	3425.2
34	3444.5
43	3473.8
8	3479.6
20	3482.9
42	3495.7
25	3518.6
36	3525.2
14	3527.2
28	3561.5
30	3568.4
7	3571.8
21	3572.4
32	3646.2
17	3653.3
44	3657.6
18	3679.4
23	3744.5
48	3785.0
16	3872.9
40	3898.2
29	3928.4
49	4041.1
38	4211.7
51	4218.8

Total deviation (5 to 41-ET) relative to divisions per octave: Harmonicity/Manageability	
***n*-ET**	**Total Deviation *multiplied* by *n***
41	1796.1
31	1967.5
22	2252.8
19	2370.8
12	2760.7
33	2872.8
35	2937.9
11	2944.8
6	3032.1
27	3055.5
37	3163.6
13	3204.9
24	3212.5
26	3257.8
5	3318.7
9	3323.9
39	3369.3
10	3375.5
15	3425.2
34	3444.5
8	3479.6
20	3482.9
25	3518.6
36	3525.2
14	3527.2
28	3561.5
30	3568.4
7	3571.8
21	3572.4
32	3646.2
17	3653.3
18	3679.4
23	3744.5
16	3872.9
40	3898.2
29	3928.4
38	4211.7

APPENDIX III

APPENDIX III

APPENDIX III

APPENDIX III

APPENDIX IV - *This appendix was co-authored by Paul Hahn.*[1]

‘Consistency’, ‘completeness’ and ‘diameter’ of *n*-division equal-temperaments

Informally, a system of equal-temperament is ‘consistent’ or ‘inconsistent’ to the extent that intervals formed by JI ratios are consistently approximated by their individual *and* combinatory nearest equivalents in the temperament. The purpose of quantifying ‘consistency’ is to predict the relative compositional usefulness of differing *n*-ETs. The validity of analysing ETs in terms of consistency rests to some extent on the premise that small integer ratio intervals are more consonant than more complex ratio intervals, and, as discussed earlier, that intervals formed by simpler ratios to some extent determine or correspond to the location (if not necessarily the intervallic ‘centre’) of ‘primary’ intervallic zones for harmonic tones.

In Erlich’s original formulation[2] (and also in Hahn’s work), consistency is relative to an *m*-limit - that is, each temperament is compared against a finite set of *m*-limit interval ratios between 1/1 and 2/1 (as explained below). Arguably, the definition of ‘consistency’ need not necessarily be restricted in this way. In a broader conception of consistency, alternative finite ‘comparison sets’ might take the role of the *m*-limit. For example, combinations or subsets of intervals within an *m*-limit, or sets of intervals defined in terms of some other harmonicity functions, might be adopted. For simplicity, following Erlich and Hahn, consistency is presented here with reference to an *m*-limit; some extensions of this principle are briefly suggested later.

The intention here is to communicate the essential idea of consistency rather than to formulate its most concise mathematical expression. In addition, two concepts related to consistency which were both originally formulated by Paul Hahn - ‘completeness’ and ‘diameter’ - are also introduced.

Some definitions:

The <u>primary *m*-limit</u> is the set of all interval ratios between 1/1 and 2/1 of the form x/y (and their inversions ($2y/x$)) where m, x and y are integers, and $x, y \leq m \geq 1$. For the purposes of this exposition, octave equivalence is assumed for this and all subsequent definitions.

The <u>secondary *m*-limit</u> is the set of all interval ratios between 1/1 and 2/1 of the form x/y (and inversions) which may be generated by adding together any *two* intervals of the primary *m*-limit, but which are *not* members of the primary *m*-limit.

Similarly, the <u>tertiary *m*-limit</u> is the set of all interval ratios between 1/1 and 2/1 of the form x/y (and inversions) which may be generated by adding together any *three* intervals of the primary *m*-limit, and which are *neither* members of the primary *m*-limit *nor* of the secondary *m*-limit.

The <u>extended *m*-limit</u> is the (infinite) set of interval ratios between 1/1 and 2/1 which may be generated by adding or subtracting *any number* of intervals from the primary *m*-limit (thus including members of the primary *m*-limit).

A <u>triad</u> is a combination of 3 intervals a, b and z, such that $a + b = z$; a <u>tetrad</u> is a combination of 4 intervals a, b, c and z, such that $a + b + c = z$; and so on.

A <u>primary *m*-limit triad</u> is one in which a, b and z are <u>all</u> primary *m*-limit intervals.

[1] I am also grateful for advice from Paul Erlich, John Chalmers and Manuel Op de Coul.

[2] To date, the only published reference to consistency is Erlich’s definition in a footnote: ‘… a tuning is not *consistent* within a given limit … if for some odd numbers a, b, and c less than or equal to the limit, the best approximation of b:a plus the best approximation of c:b does not equal the best approximation of c:a …’. Paul Erlich, ‘Tuning, tonality and twenty-two-tone temperament’, *Xenharmonikôn* 17, Spring 1998, p. 13, note 8. However, Erlich, Hahn and others have discussed the concept on the Mills Tuning list (tuning@eartha.mills.edu) a number of times.

A secondary *m*-limit triad is one in which at least one of *a*, *b* and *z* is a secondary *m*-limit interval, but none are tertiary or higher. (Analogous definitions are assumed for the tertiary *m*-limit, and for tetrads and pentads).

"⇔" means: "is best approximated by";
"~" means: "the nearest approximation in *n*-ET of ".

A definition of consistency (paraphrasing Hahn) can now be given as:

***n*-ET is consistent at the *m*-limit to level-*p* iff all the nearest approximations within *n*-ET to primary, secondary, tertiary… (ie., level-*p*) intervals are distributive for interval addition and subtraction within the *m*-limit - that is, if :**

$$\sim a + \sim b = \sim(a + b) = \sim z$$

and *a*, *b*, *z* comprise a level-*p* *m*-limit triad.

Consistency relative to the *primary m-limit* is known as Level 1 consistency (or just plain 'consistency'); consistency relative to the *secondary m-limit* is known as Level 2 consistency; consistency relative to the *tertiary m-limit* is known as Level 3 consistency; and so on.

m-limit Interval Sets

Primary ratio sets up to the 11-limit

The primary 3-limit comprises the set of intervals: {3/2 (4/3)}.

(The octave equivalents of the intervals of this set also belong to the primary 3-limit and are also considered primary; a similar condition is assumed for each subsequent definition.)

The primary 5-limit comprises the set of intervals: {3/2 (4/3), 5/3 (6/5), 5/4 (8/5)}.

The primary 7-limit comprises the set of intervals: {3/2 (4/3), 5/3 (6/5), 5/4 (8/5), 7/4 (8/7), 7/5 (10/7), 7/6 (12/7)}.

The primary 9-limit comprises the set of intervals: {3/2 (4/3), 5/3 (6/5), 5/4 (8/5), 7/4 (8/7), 7/5 (10/7), 7/6 (12/7), 9/5 (10/9), 9/7 (14/9), 9/8 (16/9)}

The primary 11-limit comprises the set of intervals: {3/2 (4/3), 5/4 (8/5), 5/3 (6/5), 7/4 (8/7), 7/5 (10/7), 7/6 (12/7), 9/8 (16/9), 9/5 (10/9), 9/7 (14/9), 11/8 (16/11), 11/6 (12/11), 11/7 (14/11), 11/9 (18/11), 11/10 (20/11)}

And so on.

Each primary m_α-limit includes the intervals of any m_β-limit where $m_\alpha \geq m_\beta$. Thus, if an *n*-ET is level-*p* consistent at the m_α-limit, it will be at least level-*p* consistent at the m_β-limit.

Secondary ratio sets up to the 7-limit

The secondary 3-limit comprises the set of intervals: {9/8 (16/9)}.

The secondary 5-limit comprises the set of intervals: {9/5 (10/9), 9/8 (16/9), 16/15 (15/8), 25/16 (32/25), 25/18 (36/25), 25/24 (48/25)}.

The secondary 7-limit comprises the set of intervals: {9/5 (10/9), 9/7 (14/9), 9/8 (16/9), 15/14 (28/15), 16/15 (15/8), 21/20 (40/21), 25/16 (32/25), 25/18 (36/25), 25/21 (42/25), 25/24 (48/25), 21/16 (32/21), 25/14 (28/25), 35/24 (48/35), 35/32 (64/35), 35/18 (36/35), 49/25 (50/49), 49/30 (60/49), 49/32 (64/49), 49/36 (72/49), 49/40 (80/49), 49/48 (96/49)}.

And so on.

Tertiary ratio sets up to the 5-Limit

The tertiary 3-limit comprises the set of intervals: {27/16 (32/27)}.

The tertiary 5-limit comprises the set of intervals: {25/16 (32/25), 25/18 (36/25), 25/24 (48/25), 27/16 (32/27), 27/20 (40/27), 27/25 (50/27), 45/32 (64/45), 75/64 (128/75), 125/64 (128/125), 125/72 (144/125), 125/96 (192/125), 125/108 (216/125)}. And so on.

Note that the secondary 3-limit intervals also belong to the primary 9-Limit; the primary 25-Limit is the smallest primary *m-limit* which includes all the secondary 5-Limit intervals; and so on. The value of considering secondary and tertiary ratio sets for consistency is controversial, since the extent to which complex ratios are aurally distinct or can be said to locate unique intervallic characters is dubious. The reason for including them in this analysis is given below.

Level-1 Triadic Consistency at the 3-Limit

Since there are no *primary* 3-limit-*triads* it is impossible for an *n*-ET to be level-1 *in*consistent at the 3-limit. This is all that really need be said about level-1 consistency at the 3-Limit, but if further explanation is required see the note below.[3]

Level-1 Triadic Consistency at the 5-Limit

Strictly speaking, six *primary 5-limit-triads* exist. These are written below in the form [*a*, *b*, *z*]:

{[5/4, 6/5, 3/2], [6/5, 5/4, 3/2], [6/5, 4/3, 8/5], [5/4, 4/3, 5/3], [4/3, 5/4, 5/3], [4/3, 6/5, 8/5]}.

However, since these six triads comprise JI major and minor triads in root, first and second inversion, it can be seen that the last four are octave equivalent to one of the first two; in addition, the JI minor triad is the inversion of the JI major triad. Therefore, to analyse for 'consistency' only *one* of these triads need be considered.

Therefore, 12-ET is *level-1* consistent at the 5-limit, because:

[5/4, 6/5, 3/2]: 5/4 ⇔ 4 steps; 6/5 ⇔ 3 steps; 3/2 ⇔ 7 steps; and 4 + 3 = 7

and because every interval included in this calculation belongs to the set of primary 5-limit intervals.[4]

Similarly, 15-ET is level-1 consistent at the 5-limit because:

[5/4, 6/5, 3/2] : 5/4 ⇔ 5 steps; 6/5 ⇔ 4 steps; 3/2 ⇔ 9 steps; and 5 + 4 = 9.

---

[3]As a special case, allow 2/1 to be a primary (as opposed to 'zero-ary) interval in the 3-Limit, and thus a component of a 'triad'. Then two 'triads' exist - these are written below in the form [*a*, *b*, *z*]: {[4/3, 3/2, 2/1], [3/2, 4/3, 2/1]}. Thus, for example (in this special sense) 12-ET is *level-1* consistent at the 3-limit, because:

[4/3, 3/2, 2/1] : 4/3 ⇔ 5 steps; 3/2 ⇔ 7 steps; 2/1 ⇔ 12 steps; and 5 + 7 = 12
[3/2, 4/3, 2/1] : 3/2 ⇔ 7 steps; 4/3 ⇔ 5 steps; 2/1 ⇔ 12 steps; and 7 + 5 = 12

Similarly, 11-ET is *level-1* consistent at the 3-limit, because:

[4/3, 3/2, 2/1] : 4/3 ⇔ 5 steps; 3/2 ⇔ 6 steps; 2/1 ⇔ 11 steps; and 5 + 6 = 11
[3/2, 4/3, 2/1] : 3/2 ⇔ 6 steps; 4/3 ⇔ 5 steps; 2/1 ⇔ 12 steps; and 6 + 5 = 11

In fact, since each of the *primary 3-limit*-'*triads*' is the inversion of the other, we only need to have considered one of them to show consistency. Similarly, because all *n*-ETs are symmetrical about the tritone, the nearest approximation of 4/3 will always be an identical distance from 4/3 as is the distance from the nearest approximation to 3/2, and the two intervals will be best approximated in each *n*-ET by an *n*-complementary number of scale steps. It can therefore be seen that (in the slightly arbitrary sense specified) *all n*-ETs are *level-1* consistent at the primary 3-limit. Practically speaking, however, in Level-1 it is only necessary to consider the 5-Limit and above.

[4] Similarly, it can be shown that:
[6/5, 5/4, 3/2] : 6/5 ⇔ 3 steps; 5/4 ⇔ 4 steps; 3/2 ⇔ 7 steps; and 3 + 4 = 7
[4/3, 5/4, 5/3] : 4/3 ⇔ 5 steps: 5/4 ⇔ 4 steps; 5/3 ⇔ 9 steps; and 5 + 4 = 9
[4/3, 6/5, 8/5] : 4/3 ⇔ 5 steps: 6/5 ⇔ 3 steps; 8/5 ⇔ 8 steps; and 5 + 3 = 8
etc., but this is not strictly necessary. Analogous statements are assumed in all further calculations of this type.

However, 14-ET is **NOT** level-1 consistent at the 5-limit because:

[5/4, 6/5, 3/2] : 5/4 ⇔ 5 steps; 6/5 ⇔ 4 steps; 3/2 ⇔ 8 steps; and 5 + 4 ≠ 8.

A table showing level-1 *m-limit* consistency of systems from 2-ET to 41-ET is shown in Figure 7, page 60 of the main text.

<u>Level-1 Triadic Consistency at the 7-Limit</u>

Similarly, assuming octave and inversional equivalence, only four of the *primary 7-limit-triads* need be considered:

{[5/4, 6/5, 3/2], [5/4, 7/5, 7/4], [3/2, 7/6, 7/4], [6/5, 7/6, 7/5]}.

Thus, for example, 12-ET is *level-1* consistent at the 7-limit, because:

[5/4, 6/5, 3/2]: 5/4 ⇔ 4 steps; 6/5 ⇔ 3 steps; 3/2 ⇔ 7 steps; and 4 + 3 = 7
[5/4, 7/5, 7/4]: 5/4 ⇔ 4 steps; 7/5 ⇔ 6 steps; 7/4 ⇔ 10 steps; and 4 + 6 = 10
[3/2, 7/6, 7/4]: 3/2 ⇔ 7 steps; 7/6 ⇔ 3 steps; 7/4 ⇔ 10 steps; and 7 + 3 = 10
[6/5, 7/6, 7/5]: 6/5 ⇔ 3 steps; 7/6 ⇔ 3 steps; 7/5 ⇔ 6 steps; and 3 + 3 = 6

but, 24-ET is **NOT** level-1 consistent at the 7-limit because:

[5/4, 6/5, 3/2]: 5/4 ⇔ 8 steps; 6/5 ⇔ 6 steps; 3/2 ⇔ 14 steps; and 8 + 6 = 14
[3/2, 7/6, 7/4]: 3/2 ⇔ 14 steps; 7/6 ⇔ 5 steps; 7/4 ⇔ 19 steps; and 14 + 5 = 19
[5/4, 7/5, 7/4]: 5/4 ⇔ 8 steps; 7/5 ⇔ 12 steps; 7/4 ⇔ 19 steps; and 8 + 12 ≠ 19
[6/5, 7/6, 7/5]: 6/5 ⇔ 6 steps; 7/6 ⇔ 5 steps; 7/5 ⇔ 12 steps; and 6 + 5 ≠ 12

Note that consistency is defined by Erlich and Hahn such that if one or more of these calculations is inconsistent, then the *n*-ET is inconsistent at that Limit.

<u>Level-1 Tetradic Consistency at the 3 and 5-Limits</u>

No *primary 3-limit-tetrachords* or *primary 5-limit-tetrachords* exist.

<u>Level-1 *Tetradic* Consistency at the 7-Limit</u>

According to Erlich's and Hahn's accounts of consistency, *any chord structure larger than a triad (ie., tetrad, pentad etc.), will have the same consistency rating as for triads*, given that the component intervals conform to the same given limit. Thus, if a, b, c, z and z_1 are all primary *m-limit* intervals, then it will be true that:

$$\begin{array}{llll} \textit{if} & (\sim a + \sim b) & = \sim(a + b) & = \sim z \\ \textit{and} & (\sim z + \sim c) & = \sim(z + c) & = \sim z_1 \\ \textit{then} & (\sim a + \sim b + \sim c) & = \sim(a + b + c) & = \sim z_1 \end{array}$$

To examine this in slightly more detail, consider the following. Assuming octave and inversional equivalence, only one *primary 7-limit-tetrad* need be considered (expressed here in the form [*a*, *b*, *c*, *z*]):

[5/4, 6/5, 7/6, 7/4], (ie., the 'just dominant seventh chord').

(Note, for example, that in the 'just minor seventh chord' [6/5, 5/4, 7/6, 7/4], each 'consecutive' interval belongs to the primary 7-Limit, but it is not a *primary* 7-limit-tetrad because the interval between its 2^{nd} and 4^{th} note is 35/24 - that is, a *secondary* 7-limit interval. This chord is therefore a *secondary* 7-limit-tetrad).

In order to evaluate whether a tetrad is consistent (in Erlich's and Hahn's strict sense), not only the 'consecutive' intervals but also every implicit intervallic combination within the tetrad must be compared to its nearest *m-limit* equivalent.

For example, 12-ET is *level-1 tetradically* consistent at the 7-limit, not only because:

> [5/4, 6/5, 7/6, 7/4]: 5/4 ⇔ 4 steps; 6/5 ⇔ 3 steps; 7/6 ⇔ 3 steps; 7/4 ⇔ 10 steps; and, 4 + 3 +3 = 10;

but also because:

> [5/4, 6/5, 3/2]: 5/4 ⇔ 4 steps; 6/5 ⇔ 3 steps; 3/2 ⇔ 7 steps; and 4 + 3 = 7
> [5/4, 7/5, 7/4]: 5/4 ⇔ 4 steps; 7/5 ⇔ 6 steps; 7/4 ⇔ 10 steps; and 4 + 6 = 10
> [3/2, 7/6, 7/4]: 3/2 ⇔ 7 steps; 7/6 ⇔ 3 steps; 7/4 ⇔ 10 steps; and 7 + 3 = 10
> [6/5, 7/6, 7/5]: 6/5 ⇔ 3 steps; 7/6 ⇔ 3 steps; 7/5 ⇔ 6 steps; and 3 + 3 = 6

- that is, exactly the same reasons as why 12-ET is *level-1 triadically* consistent at the 7-limit.

Thus, 24-ET is **NOT** level-1 *tetradically* consistent at the 7-Limit because, although:

> [5/4, 6/5, 7/6, 7/4]: 5/4 ⇔ 8 steps; 6/5 ⇔ 6 steps; 7/6 ⇔ 5 steps; 7/4 ⇔ 19 steps; and, 8 + 6 +5 = 19;

at the same time:

> [5/4, 7/5, 7/4]: 5/4 ⇔ 8 steps; 7/5 ⇔ 12 steps; 7/4 ⇔ 19 steps; and 8 + 12 ≠ 19
> [6/5, 7/6, 7/5]: 6/5 ⇔ 6 steps; 7/6 ⇔ 5 steps; 7/5 ⇔ 12 steps; and 6 + 5 ≠ 12

To put it another way, the 7-Limit primary *tetrad* in 24-ET results in a 7/5 of <u>11</u> steps; but the 7-Limit primary *triads* have a 7/5 of <u>12</u> steps - so 24-ET is tetradically inconsistent.

Whether the consistency or inconsistency of the 'consecutive' intervals of a tetrad or pentad, (that is, discounting the 'inner' triads) implies a distinct (musically significant) criterion for distinguishing between *n*-ETs is considered below.

An algorithm for determining p-level consistency

There are many triads to consider for *p*-level consistency (*p*>1), and an algorithmic approach can provide the required results without analysing every triad individually.

In any *n*-ET, relative to the intervals which comprise a specific *m*-limit, there is one particular scale-degree (and its inversion) which deviates from the JI ratio it most nearly approximates by a greater margin than any other scale-degree.[5] The primary 5-limit comprises the set {3/2 (4/3), 5/3 (6/5), 5/4 (8/5)}, and in 12-ET (for example) the third scale-degree (300 cents) deviates by a greater distance from 6/5 than the 7th scale-degree deviates from 3/2 or the 4th scale-degree from 5/4. Therefore, the greatest possible deviation from any given number of *combinations* of two or more primary 5-limit intervals in 12-ET will arise from the *repetition* of that 'worst approximation'.

Thus, the tertiary 5-Limit interval formed by three consecutive 6/5's (ie., 216/125 or 946.8 cents) diverges from the 9th scale degree in 12-ET by three times as much as 6/5 diverges from the 3rd scale-degree (15.64 cents). The quaternary 5-Limit interval formed by four 6/5s (1296/625 or 1262.4 cents) diverges by 62.4 cents from the 12th scale-degree (the octave) - and is closer to 13 semitones. Therefore, according to this criterion, 12-ET is *not* level-4 consistent at the 5-limit. But 12-ET *is* level-3 consistent at the 5-limit since we know that no other possible combination of scale-degrees in 12-ET will deviate from its corresponding combinatory *m*-limit equivalent by more than 216/125 diverges from 9 semitones.

Taking this principle one stage further, Hahn has devised an algorithm to show whether any *n*-ET is level-*p* consistent at the *m*-limit. The algorithm finds the greatest possible deviation occurring between a *pair* of approximations within the *m*-limit, and derives *p*-level consistency from the number of repetitions of this deviation which results in inconsistency. It may be paraphrased as follows:

(i) Choose the *n*-ET and the *m*-limit:

eg., 12-ET in the 7-limit (1)

(ii) Take the set of odd harmonics up to *m* (omitting 1/1), here expressed as JI ratios, and in cents:

3/2, 5/4, 7/4 (2a)
701.955, 386.31, 968.83 (2b)

(iii) Find the closest approximations in *n*-ET of the elements of (2), expressed in cents:

700, 400, 1000 (3)

(iv) Find which in (3) is the greatest positive deviation and which the greatest negative deviation from the corresponding values of (2b):

- 1.955 (700 - 701.955) (4a)
+ 31.17 (1000 - 968.83) (4b)

(v) Add the two absolute values of (4) and divide the result into half the step-size of *n*-ET - the general expression is: (1200 ÷ 2*n*) ÷ ((ABS(4a)) + (ABS(4b))) :

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[5] If two or more scale-degrees within the tritone deviate *equally* this makes no difference to the analysis.

$$(1200 \div (2 \times 12)) \div (31.17 + 1.955) = (50 \div 33.132) = 1.509 \qquad (5)$$

(vi) The integer part of (5) is the consistency level of *n*-ET in the *m*-limit

Therefore 12-ET is level 1 consistent in the 7-limit (6)

Some further examples are worked in the footnote below.[6] A table showing the *p*-level *m-limit* consistency of 2-ET to 41-ET inclusive is given at the end of this APPENDIX.

Consistency at level-1 confirms the consonance (or accuracy for higher limits) of the most simple ratio chords within a given limit. The reason for analysing consistency at level-2 and above is controversial, but Hahn has argued at least for level-2 consistency in terms of the accuracy of voice leading. Consider a simple progression from one 'just dominant seventh chord' in close root position (4:5:6:7) to another in close first inversion - for example: C:E:G:Bb to C#:E:G:A. In Table 1, two ways of taking this progression in JI are shown, firstly so that the pitch of G remains constant (transposition of the chord by 7/6), and secondly so that E remains constant (transposition by 6/5). In addition, the closest approximations to these progressions are shown for 12, 21 and 22-ET.[7]

Comparison of voice leading between JI and *n*-ET between two 'dominant 7th' chords (root position followed by first inversion) in which the roots are separated by a Septimal m3 and a Just m3.						
JI Chord	*JI (G constant)*	*JI (E constant)*	*12-ET*	*21-ET*	*22-ET*	*JI Chord*
Bb (7)	48/49 (-35.7)	20/21 (-84.5)	-1 (-100)	-1 (-57.1)	-1 -54.5	A (8)
G (6)	1/1 (0)	35/36 (-48.8)	0 (0)	0 (0)	0 (0)	G (7)
E (5)	36/35 (+48.8)	1/1 (0)	0 (0)	0 (0)	+1 +54.5	E (6)
C (4)	15/14 (+119.4)	25/24 (70.7)	+1 (+100)	+2 (+114.3)	+2 +109.1	C# (5)

Table 1 : Comparison between JI and some equal-temperaments regarding voice leading in a familiar tonal harmonic progression.

Note that in both the JI progressions the E to G relation changes from 6/5 to 7/6, but that in 12- and 21-ET E and G remain constant, or to put it another way, the quarter-tone interval

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[6] <u>Consider 41-ET in the 9-limit</u> (1)
3/2, 5/4, 7/4, 9/8 (2a)
701.955, 386.31, 968.83, 203.91 (2b)
702.44, 380.49, 965.85, 204.88 (3)
-5.82 (380.49 - 386.31) (4a)
0.97 (204.88 - 203.91) (4b)
$(1200 \div (2 \times 41)) \div (5.82 + 0.97) = (14.634 \div 6.79) = 2.155$ (5)
Therefore, 41-ET is level 2 consistent in the 9-limit (6)

<u>Consider 11-ET in the 5-limit</u> (1)
3/2, 5/4 (2a)
701.955, 386.31 (2b)
654.55, 436.36 (3)
- 47.405 (654.55 - 701.955) (4a)
+ 50.05 (436.36 - 386.31) (4b)
$(1200 \div (2 \times 11)) \div (47.405 + 50.05) = (54.545 \div 97.455) = 0.56$ (5)
Therefore 11-ET is level 0 consistent (ie., inconsistent) in the 5-limit (6)

[7] This is not to deny that harmonic progressions involving the dominant seventh chord in 12-ET are extremely effective - but this is not necessarily a counter example to these observations about consistency. Similarly, it has been argued that the dominant 7th chord in 12-ET is particularly effective *because it is less smooth* than a 'just dominant seventh'. But the relevance of this analysis does not rest on whether the JI chords or the JI progressions are their 'ideal' realisations: the point is rather to obtain a notion of the 'consistency' of the intervallic structures likely to arise in an *n*-ET.

36/35 ‘vanishes’, whether it is positive in the 7/6 progression or negative in the 6/5 progression. In 12-ET this is expected since the interval of 3 steps is the nearest to both 6/5 and 7/6. 21-ET is more complicated because 6 scale-steps (342.86) is marginally closer to 6/5 than 5 scale-steps (285.71 cents), but 5 scale-steps is recognisable as a minor third, whereas 6 scale-steps is not. In contrast, 22-ET gives a better approximation of the progression, especially of the 7/6 version.

Hahn also points out the ‘inconsistency’ in the 12- and 21-ET realisations of this progression that the 36/35 vanishes whereas the smaller interval 49/48 does not. As Hahn puts it:

> Representing the movement of one consonant identity to another without producing (what are to me) bizarre perceptual artifacts requires level-2 consistency.[8]

In an ‘ideal’ system, which would of course have to provide ‘voice-leading’ of far greater complexity than the above example, it would seem preferable to obtain (a degree of) ‘consistency’ not merely in isolated chords, but in progressions, tonal and otherwise.[9] Clearly, Level-1 consistency is not in itself adequate to provide consistent approximations of the secondary intervals occurring therein. But different progressions give different results, and there is no absolute reason to privilege the analysis of some progressions over others. However, while Level-2 consistency or above may indicate generally that sequences of individual pitches and chord progressions will be harmonically flexible and will better remain ‘in tune’ than Level-1 consistent systems, the significance of this is perhaps only relevant to performance on instruments of *fixed intonation*.

Erlich has commented that the notion of higher-level consistency

> is equivalent to a more stringent criterion on the maximum deviation of intervals from JI (namely, level-N consistency means that no interval will deviate by more than 1/2N scale degrees, if N >= 1.5).[10]

In this sense, ‘higher level’ consistency depends on nothing more than that all the deviations from *m*-limit intervals within *n*-ET are small. It remains arguable whether higher-level consistency actually demonstrates intrinsic (or audible) musical properties beyond this initial fact.

Developing the idea of ‘consistency’?

It would appear that consistency analysis might be developed along the following lines:

1) Following the definition given by Erlich and Hahn, at any given limit and level, an *n*-ET is either consistent or inconsistent - there is no half-way house. As the ‘short-cut’ algorithm shows, whether an *n*-ET is consistent depends on whether just one interval combination is consistent. For the purpose of choosing a provisional alternative tuning standard this is useful, but only if the intention is that all *inconsistent* systems (relative to an *m*-limit) should henceforth to be disregarded, since the method does not

[8] Paul Hahn, Tuning Digest 1407.

[9] John Chalmers has mentioned that higher-level consistency may be important for modulation in the sense that, while a progression in JI might form a circle, leading back to the chord or note from which it started, in certain *n*-ETs the ‘best equivalent’ progression may form a spiral. The higher the consistency *level*, the less this is likely to occur. Personal communication, August 1998.

[10] Personal correspondence with the author.

rate the 'degree' of inconsistency. If this were measured, then consistent systems might also be compared in terms of the 'degree of inconsistency' at higher levels.

If, on the other hand, inconsistency does not rule a system out, then it would be useful to know, for any *n*-ET, the likelihood of inconsistent combinations occurring in compositions using inconsistent systems.[11] For this we need to quantify the *proportion* of how many triads (or tetrads etc.) are consistent or inconsistent in those systems. And, consistent or inconsistent, we need to combine this knowledge with the relative accuracy to just ratios (or interval zones, etc.,) of the combinations in each limit.

In each step of the 'longhand-derivation' given above, each *p*-level triad (or tetrad etc.) was individually tested for consistency. And at any given limit or level, an 'inconsistent' system is one in which at least one triad is inconsistent, but others may be consistent. For instance, in the example given above which shows 24-ET to be inconsistent at the primary 7-Limit, two triads are consistent and two inconsistent.

As a further measure of consistency, therefore, we may consider the extent to which *m*-limit intervals remain consistently expressed within *n*-ET considering all possible intervallic combinations (and approximations) within that limit. Since the triads shown in the above examples include all such possibilities, the *proportional* consistency of:

triads in 12-ET at the primary 7-limit is 100%;
triads in 24-ET at the primary 7-limit is 50%;
tetrads in 12-ET at the primary 7-limit is 100%;
tetrads in 24-ET at the primary 7-limit is 60%; (etc).[12]

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[11] That is - unless the analysis is restricted to level-1, and exclusively tonal music is envisaged, in which the most consonant chords are likely to occur very often.

[12] Note that a distinction between 'secondary' *m-limit* triads which comprise one, two, or three *secondary m-limit* intervals does not have any consequence for consistency. For example, 5 different forms of secondary triads can be distinguished (where '*p*' and '*s*' represent primary and secondary intervals respectively):

[*p, p, p*]		*primary triad*
[*p, p, s*]		*pps-secondary* triad
[*p, s, p*]	(= [*s, p, p*])	*psp-secondary* triad
[*s, p, s*]	(= [*p, s, s*])	*pss-secondary* triad
[*s, s, p*]		*ssp-secondary* triad
[*s, s, s*]		*sss-secondary* triad

Eg. : pps-secondary 7-limit triads: [7/6, 4/3, 14/9], [7/6, 5/4, 35/24];
psp-secondary 7-limit triads: [21/20, 8/7, 6/5];
pss-secondary 7-limit triads : [3/2, 49/48, 49/32];
ssp-secondary 7-Limit triads: [15/14, 14/9, 5/3], [28/25, 15/14, 6/5];
sss-secondary 7-Limit triads: [21/20, 25/14, 15/8], [21/20, 12/7, 9/5];

But, as we saw earlier, the '1st and 2nd inversions' of any primary *m*-limit triad merely confirm the consistency result of the original triad. (This kind of permutation is called a 'rotation' to distinguish it from the stricter notion of octave inversion).

For example, in 24-ET, the primary triad [5/4, 7/5, 7/4] and its rotations are unanimously inconsistent:

[5/4, 7/5, 7/4] : 5/4 ⇔ 8 steps; 7/5 ⇔ 12 steps; 7/4 ⇔ 19 steps; and $8 + 12 \neq 19$
[7/5, 8/7, 8/5] : 7/5 ⇔ 12 steps; 8/7 ⇔ 5 steps; 8/5 ⇔ 16 steps; and $12 + 5 \neq 16$
[8/7, 5/4, 10/7] : 8/7 ⇔ 5 steps; 5/4 ⇔ 8 steps; 10/7 ⇔ 12 steps; and $5 + 8 \neq 12$

Similarly for the primary triad [6/5, 7/6, 7/5]:

2) The notion of the <u>*level*</u> of consistency follows from the idea of the 'closest approximation' in *n*-ET of an *m*-limit interval. In the initial explanation of Hahn's algorithm, we saw that intervallic inconsistency occurs when the combination of *t m*-limit ratios diverges by more than *half* a step in *n*-ET from the expected consistent combination of *n*-ET scale-steps. This corresponds to our intuitive notion of 'consistency' in everyday terms, but it is easy to imagine how a more stringent criterion (from the point of view of intonation) might be substituted. For example, in step (v) of the algorithm, the general expression might be changed to (1200 ÷ 3*n*) ÷ ((ABS(4a)) + (ABS(4b))), thus narrowing the intonationally acceptable limit of 'consistency'. Similarly, if, in the expression (1200 ÷ 2*n*), '2' is replaced with a variable *v* (which need not be an integer), and *v*>2, then the consistency of *n*-ET may be plotted as a function of *v*. Again, this might be applied to the range of approximations in the temperament, as opposed to applying it solely to the worst case.

3) A '<u>user-defined-limit</u>' may also be chosen, comprising a set of ratios which are not defined in terms of an *m*-limit (or not solely in terms of an *m*-limit). The criterion of inclusion might comprise the set of nearest JI intervals of an already existing or invented tuning system, or the set of all just ratios which it is thought important to approximate (which need not correspond directly to any particular limit); or certain integer-limits could be omitted - for example, the consistency of *n*-ETs in the 9-limit could be found, omitting all ratios involving '7'. Alternatively, a 'limit' may be imposed and evaluated in terms of whether an interval has (for example) a Euler 'GS' or Barlow ('harmonicity') value of less than some constant; or whether intervals and their combinations belong within harmonicity zones defined in terms of dissonance curves.

   The notion of consistency is an important contribution to the theory of alternative equal-temperaments, not only because it provides a means of evaluating the harmonic combinations available within a temperament in terms of JI, but also because the 'logic' of consistency might be applied in a number of ways. In this case, an *n*-ET

[6/5, 7/6, 7/5] : 6/5 ⇔ 6 steps; 7/6 ⇔ 5 steps; 7/5 ⇔ 12 steps; and 6 + 5 ≠ 12
[7/6, 10/7, 5/3] : 7/6 ⇔ 5 steps; 10/7 ⇔ 12 steps; 5/3 ⇔ 18 steps; and 5 + 12 ≠ 18
[10/7, 6/5, 12/7]: 10/7 ⇔ 12 steps; 6/5 ⇔ 6 steps; 12/7 ⇔ 19 steps; and 12 + 6 ≠ 19

Amongst secondary triads, rotations change the 'form' of the triad - for example, the '<u>pps</u>-secondary 7-limit triad' [7/6, 4/3, 14/9] has the rotations [4/3, 9/7, 12/7] and [9/7, 7/6, 3/2] (<u>pss</u> and <u>spp</u> respectively) - but their consistency is uniform.

For example in 24-ET:

[7/6, 4/3, 14/9]: 7/6 ⇔ 5 steps; 4/3 ⇔ 10 steps; 14/9 ⇔ 15 steps; and 5 + 10 = 15
[4/3, 9/7, 12/7]: 4/3 ⇔ 10 steps; 9/7 ⇔ 9 steps; 12/7 ⇔ 19 steps; and 10 + 9 = 19
[9/7, 7/6, 3/2] : 9/7 ⇔ 9 steps; 7/6 ⇔ 5 steps; 3/2 ⇔ 14 steps; and 9 + 5 = 14

Or, in 19-ET:

[7/6, 4/3, 14/9]: 7/6 ⇔ 4 steps; 4/3 ⇔ 8 steps; 14/9 ⇔ 12 steps; and 4 + 8 = 12
[4/3, 9/7, 12/7]: 4/3 ⇔ 8 steps; 9/7 ⇔ 7 steps; 12/7 ⇔ 15 steps; and 8 + 7 = 15
[9/7, 7/6, 3/2] : 9/7 ⇔ 7 steps; 7/6 ⇔ 4 steps; 3/2 ⇔ 11 steps; and 7 + 4 = 11

And, in 20-ET:

[7/6, 4/3, 14/9]: 7/6 ⇔ 4 steps; 4/3 ⇔ 8 steps; 14/9 ⇔ 13 steps; and 4 + 8 ≠ 13
[4/3, 9/7, 12/7]: 4/3 ⇔ 8 steps; 9/7 ⇔ 7 steps; 12/7 ⇔ 16 steps; and 8 + 7 ≠ 16
[9/7, 7/6, 3/2] : 9/7 ⇔ 7 steps; 7/6 ⇔ 4 steps; 3/2 ⇔ 12 steps; and 7 + 4 ≠ 12

could be evaluated from a variety of angles, providing a 'general consistency' rating averaged against diverse criteria. In terms of the JI approach alone, if a set of 'primary' independent intervallic ratios for harmonic tones is chosen as the 'ideal' set of intervals which the 'ideal' quasi-universal system ought to have, and which 'quasi-universal' instruments would ideally approximate - then a consistency analysis can be made in terms of that specific set, rather than in terms of the (not uncontroversial) notion of a 'Limit'.

Completeness and Diameter

Hahn has also formalised two further analytical concepts which are closely related to that of consistency - 'completeness' and 'diameter'. An *n*-ET is 'complete at an *m*-limit' if all the individual intervals of that *n*-ET can each be generated by some combination of the nearest approximations in *n*-ET to the intervals of the *m*-limit. The 'diameter' of an *n*-ET is the number of combined steps (within the *m*-limit) which is required to achieve this.[13]

Some examples will clarify this. The primary 5-limit comprises the set of intervals: {3/2 (4/3), 5/3 (6/5), 5/4 (8/5)}, and in 12-ET these intervals are best represented by 7, 5, 9, 3, 4, and 8 steps, respectively. Any number of scale steps within 12-ET may therefore be generated by combinations of these intervals (that is, purely in terms of scale steps, not JI ratios): scale-degree 1 (the 12-ET semitone) may be generated by 4 steps minus 3 steps; scale degree 2 by 5 steps minus 3 steps; and so on. In this case, to reach any scale-degree need never take more than a combination of two (nearest approximations of) 5-limit intervals. Therefore 12-ET is 'complete' at the 5-Limit and has a diameter of 2.

As an example of *in*completeness - in 24-ET the best approximations of the primary 5-Limit intervals are 14 10, 18, 6, 16 and 8 steps, respectively. Therefore, no possible combination of these intervals will generate the intervals in 24-ET which contain an odd number of steps. Therefore 24-ET is incomplete (and has no diameter) at the 5-Limit.

In Table 2 , completeness is only shown for those systems which are consistent in the *m*-limit at level-*p*. When a system is consistent but not complete, this is shown by the sign "$\not\in$". As can be seen, the majority of systems which are consistent are also complete. The diameter of *n*-ET is therefore the more significant parameter. The smaller the diameter in any given limit (especially in the lower limits) the more likely that an *n*-ET will be, as Hahn puts it, '"comprehensible" to the ear/mind'.[14]

	***3-* Limit**	***5-* Limit**	***7-* Limit**	***9-* Limit**	***11-* Limit**	***13-* Limit**	***15-* Limit**

[13] See http://library.wustl.edu/~manynote/complete.txt

[14] Paul Hahn, Tuning digest 1399. See: http://library.wustl.edu/~manynote/complete.txt The validity of these theories depends on a question which remains unanswered - that is, 'how' do we hear intervals? When we hear tonal music played on the piano, some persons claim to 'hear' enharmonic shifting of pitches - implying that, the reason 12-ET is a particularly successful system is because it is just *inaccurate* enough, relative to JI, to convince the listener they are hearing true (enough) harmonies, and that much of the pleasure of music in 12-ET derives from the semi-conscious changes of interpretation of pitch which the listener 'hears in' the music (or which the composer successfully leads the listener to hear). On this view, a system in which tonal harmony is more accurately approximated may not afford the desired enharmonic ambiguity.

2-ET	2/1						
3-ET	2/1	2/1					
4-ET	1/∉	1/1	1/1				
5-ET	6/2	1/1	1/1	1/1			
6-ET	1/∉	1/∉	1/1				
7-ET	5/3	1/2					
8-ET	1/2	1/2					
9-ET	1/2	1/2	1/2				
10-ET	3/∉	1/3	1/2				
11-ET	1/5						
12-ET	25/6	3/2	1/2	1/2			
13-ET	1/6						
14-ET	2/∉						
15-ET	2/∉	2/2	1/2				
16-ET	1/8	1/2	1/2				
17-ET	8/8						
18-ET	1/9	1/3	1/2				
19-ET	4/9	4/3	1/2	1/2			
20-ET	1/∉						
21-ET	1/∉						
22-ET	3/11	2/3	1/2	1/2	1/2		
23-ET	1/11	1/3					
24-ET	12/∉	1/∉					
25-ET	1/∉	1/3					
26-ET	2/13	1/4	1/2	1/2	1/2	1/2	
27-ET	2/13	1/4	1/2	1/2			
28-ET	1/∉	1/3					
29-ET	13/14	1/3	1/3	1/2	1/2	1/2	1/2
30-ET	1/∉	1/∉					
31-ET	3/15	3/4	3/2	1/2	1/2		
32-ET	1/16						
33-ET	1/16						
34-ET	4∉	4/4					
35-ET	1/∉	1/4	1/2				
36-ET	8/∉	1/∉	1/2				
37-ET	1/18	1/4	1/2				
38-ET	2/∉	2/∉					
39-ET	2/19	1/6					
40-ET	1/20						
41-ET	30/20	2/4	2/3	2/2	1/2	1/2	1/2

Table 2 : *Consistency*, *completeness* and *diameter* for systems from 2-ET to 41-ET.[15]

Notes to Table 2:

1. *n*-ET is *inconsistent* at the *m*-limit if the corresponding cell is blank.
2. *n*-ET is level-*p consistent* in the *m*-limit where *p* is the numerator.
3. *n*-ET has a *diameter* of *q* in the *m*-limit where *q* is the denominator.
4. If the denominator is "∉" then *n*-ET is *incomplete* at the relevant limit.
5. Some examples of reading the Table: 41-ET is consistent to level-30 at the 3-Limit, to Level 2 at the 5, 7, and 9-Limits, and to Level 1 at the 11, 13 and 15-Limits. 40-ET is level-1 consistent and has diameter 20 at the 3-Limit, but is *inconsistent* in any higher limit. 38-ET is level-2 consistent in both the 3 and 5 limits, but is *incomplete* in both.

[15] This table was created by Paul Hahn and adapted by the author. Hahn's own discussion of this and other related tables can be accessed at Hahn's webpages - http://library.wustl.edu/~manynote/music.html.

APPENDIX V (a)

A Fusion Model of Melodic Harmonicity

Aside from musical conditioning, why is it that some unaccompanied melodic intervals are more frequently used, and sound more ‘compelling’ than others? An intuitive explanation of this is that unaccompanied melody always sounds in a resonant space - so each note is ‘accompanied’ by the resonance of the tone or tones, and the aural memory, which preceded it. In this sense, melody is always harmonic.

A further (speculative) explanation is that the degree of compellingness depends on the extent that a second tone was already present (or ‘implied by’) the ‘fusion’ of tones present in the first tone. ‘Auditory fusion’ is the perception of two or more components of a complex sound, or indeed, two or more complex sounds, as one auditory entity - that is, a single complex tone.

> [T]he individual harmonics of [a] tone are accessible to the auditory system, and yet listeners normally do not hear them [as such]. Instead, listeners perceive a periodic complex tone as a single entity, characterised by a pitch and a tone colour.[1]

What causes us to hear a pitch (and to identify it as *this* rather than *that* pitch) depends on the absolute height of the fundamental *and*, depending on the nature of the tone, its spectral structure. To be precise, for higher sounds pitch is increasingly determined by the fundamental; for medium range sounds pitch is dependant on the full spectrum; for lower sounds pitch is determined by the partials, but the fundamental tone does not have crucial input. Thus, in the latter case, even if the fundamental is removed from the sound, we will still identify that sound as having the pitch of the fundamental. This is known as ‘virtual’ pitch.

Let us suppose, however, that we hear an idealised harmonic complex tone having a fundamental of C2, and in which many consecutive harmonics are present - leaving aside relative intensity and inharmonic components. In hearing the pitch as C2, we hear the ‘fusion’ of the components - C2 (1/1), C3 (2/1), G3 (3/1), C4 (4/1), E4 (5/1), G4 (6/1), Bb4 (7/1), C4 (8/1), D2 (9/1), and so on. The harmonic components of the sound are related to the fundamental in terms of a series of ratios which (generally) increase in complexity as we follow the harmonic series upward - (1/1), 2/1, 3/2, 2/1, 5/4, 3/2, 7/4, 2/1, 9/8 …. (given here in octave corrected form). Similarly, the series of ratios formed by harmonic components relative to the 2nd harmonic is 2/1, (1/1), 3/2, 2/1, 5/4, 3/2, 7/4, 2/1, 9/8 ….; the series formed relative to the 3rd harmonic is 3/2, 3/2, (1/1), 4/3, 5/3, 2/1, 7/6, 4/3, 3/2… APPENDIX VI (b) sets out these relations up to the 24th harmonic (an arbitrarily chosen cut-off point). The preponderance of interval classes is shown in APPENDIX VI (c), where the number of occurrences of each ratio implicit between harmonics is summed relative to the *n*th harmonic (omitting any ratio whose numerator is greater than 16). In APPENDIX VI (d) the order of preponderant interval classes is shown, for an arbitrary selection of harmonic cut-off points. We can easily see that however far up the harmonic series we go, the octave-adjusted ‘just’ intervals (2/1, 3/2, and 5/4) predominate in the ‘fabric’ of fusion. In addition, the lower harmonics tend (in general) to have greater intensity than higher harmonics.

Thus a second note, following C2, stands in relation not only to the fundamental of C2, but to the auditory fusion of all the harmonics. An intuitive hypothesis is that melodic ‘rightness’ therefore may derive not only from the harmonic relationship to the resonance of previous sounds, but also in the reiteration of the already present *structure* of a harmonic tone.

[1] W. M. Hartmann, *Signals, Sound and Sensation,* AIP Press, New York, 1997, p. 117.

APPENDIX V

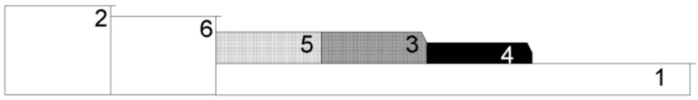
APPENDIX V (b)

Interval ratios implicit in an idealised complex harmonic tone (considered up to the 24th Harmonic)
(expressed as interval ratios within one octave).

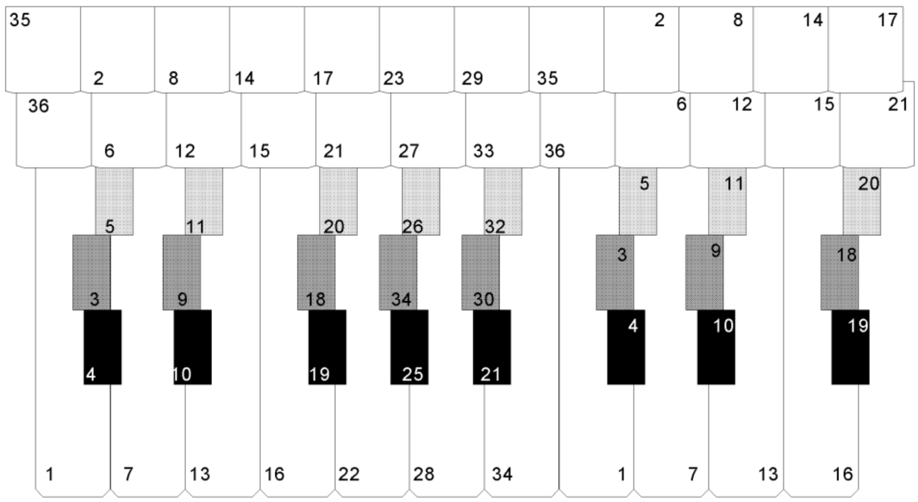
Harmonic Number(s)	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
1	2/1	3/2	2/1	5/4	3/2	7/4	2/1	9/8	5/4	11/8	3/2	13/8	7/4	15/8	2/1	17/16	9/8	19/16	5/4	21/16	11/8	23/16	3/2
2		3/2	2/1	5/4	3/2	7/4	2/1	9/8	5/4	11/8	3/2	13/8	7/4	15/8	2/1	17/16	9/8	19/16	5/4	21/16	11/8	23/16	3/2
3			4/3	5/3	2/1	7/6	4/3	3/2	5/3	11/6	2/1	13/12	7/6	5/4	4/3	17/12	3/2	19/12	5/3	7/4	11/6	23/12	2/1
4				5/4	3/2	7/4	2/1	9/8	5/4	11/8	3/2	13/8	7/4	15/8	2/1	17/16	9/8	19/16	5/4	21/16	11/8	23/16	3/2
5					6/5	7/5	8/5	9/5	2/1	11/10	6/5	13/10	7/5	3/2	8/5	17/10	9/5	19/10	2/1	21/20	11/10	23/20	6/5
6						7/6	4/3	3/2	5/3	11/6	2/1	13/12	7/6	5/4	4/3	17/12	3/2	19/12	5/3	7/4	11/6	23/12	2/1
7							8/7	9/7	10/7	11/7	12/7	13/7	2/1	15/14	8/7	17/14	9/7	19/14	10/7	3/2	11/7	23/14	12/7
8								9/8	5/4	11/8	3/2	13/8	7/4	15/8	2/1	17/16	9/8	19/16	5/4	21/16	11/8	23/16	3/2
9									10/9	11/9	4/3	13/9	14/9	5/3	16/9	17/9	2/1	19/18	10/9	7/6	11/9	23/18	4/3
10										11/10	6/5	13/10	7/5	3/2	8/5	17/10	9/5	19/10	2/1	21/10	11/10	23/20	6/5
11											12/11	13/11	14/11	15/11	16/11	17/11	18/11	19/11	20/11	21/11	2/1	23/11	12/11
12												13/12	7/6	5/4	4/3	17/12	3/2	19/12	5/3	7/4	11/6	23/12	2/1
13													14/13	15/13	16/13	17/13	18/13	19/13	20/13	21/13	22/13	23/13	24/13
14														15/14	8/7	17/14	9/7	19/14	10/7	3/2	11/7	23/14	12/7
15															16/15	17/15	18/15	19/15	4/3	7/5	22/15	23/15	8/5
16																17/16	9/8	19/16	5/4	21/16	11/8	23/16	3/2
17																	18/17	19/17	20/17	21/17	22/17	23/17	24/17
18																		19/18	10/9	7/6	11/9	23/18	4/3
19																			20/19	21/19	22/19	23/19	24/19
20																				21/20	11/10	23/20	6/5
21																					22/21	23/21	8/7
22																						23/22	12/11
23																							24/23

See sheets 1 -4

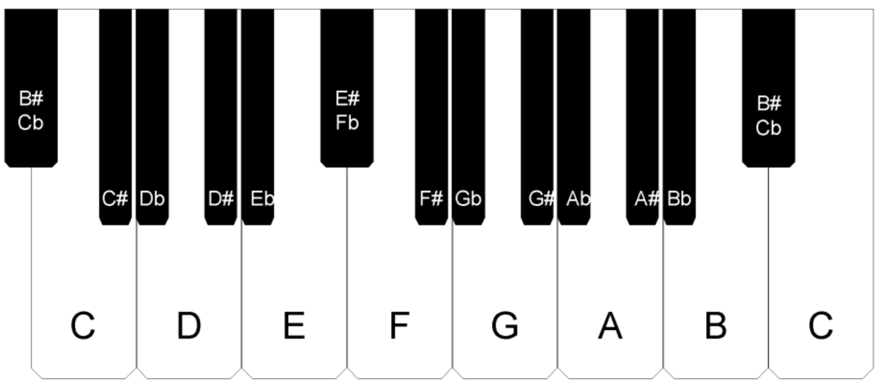
APPENDIX VI (a)



Approximate Side Elevation



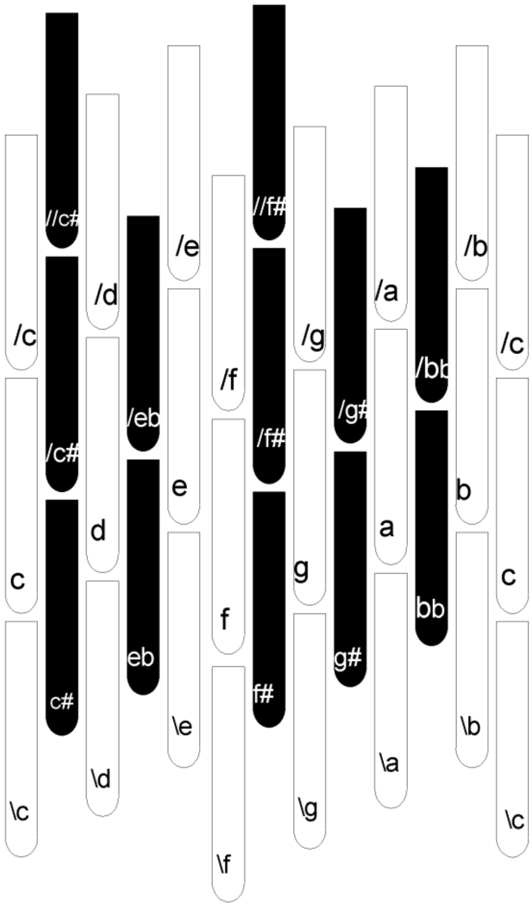
A 36-division keyboard design, taken from a sketch by Ezra Sims, dated 1977



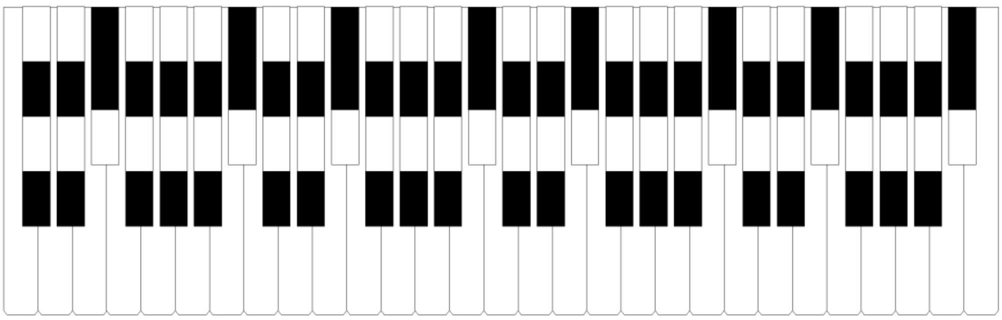
A 19-division keyboard design, taken from Joseph Yasser's "A Theory Of Evolving Tonality", p. 281

This APPENDIX was prepared by Joseph Sanger.

APPENDIX VI (b)



The layout of Bosanquet's Generalised Keyboard, taken from Helmholtz, "On the Sensations of Tone..." p.429



31-division Keyboard of the Enharmonic Harpsichord by Vito Trasuntino, taken from a photograph in the "New Groves Dictionary of Musical Instruments" p.710

This APPENDIX was prepared by Joseph Sanger.

APPENDIX VII

APPENDIX VII (a) **String distribution and totals for '7¼ Octave Pianos with 12 to 41 octave divisions (compared to conventional string distribution)**

Steinway Grand string distribution

7¼ Octave ***n*-ET**	*Single*	*Double*	*Triple*	*Total Strings*
	8/88	5/88	75/88	
12	8	10	225	243
13	9	11	244	263
14	9	12	263	284
15	10	13	281	304
16	11	13	300	324
17	11	14	319	344
18	12	15	338	365
19	13	16	356	385
20	13	17	375	405
21	14	18	394	425
22	15	18	413	446
23	15	19	431	466
24	16	20	450	486
25	17	21	469	506
26	17	22	488	527
27	18	23	506	547
28	19	23	525	567
29	19	24	544	587
30	20	25	563	608
31	21	26	581	628
32	21	27	600	648
33	22	28	619	668
34	23	28	638	689
35	23	29	656	709
36	24	30	675	729
37	25	31	694	749
38	25	32	713	770
39	26	33	731	790
40	27	33	750	810
41	27	34	769	830

Typical upright string distribution

7¼ Octave ***n*-ET**	*Single*	*Double*	*Triple*	*Total Strings*
	15/88	11/88	62/88	
12	15	22	186	223
13	16	24	202	242
14	18	26	217	260
15	19	28	233	279
16	20	29	248	297
17	21	31	264	316
18	23	33	279	335
19	24	35	295	353
20	25	37	310	372
21	26	39	326	390
22	28	40	341	409
23	29	42	357	427
24	30	44	372	446
25	31	46	388	465
26	33	48	403	483
27	34	50	419	502
28	35	51	434	520
29	36	53	450	539
30	38	55	465	558
31	39	57	481	576
32	40	59	496	595
33	41	61	512	613
34	43	62	527	632
35	44	64	543	650
36	45	66	558	669
37	46	68	574	688
38	48	70	589	706
39	49	72	605	725
40	50	73	620	743
41	51	75	636	762

Thinned string distribution

7¼ Octave ***n*-ET**	*Single*	*Double*	*Triple*	*Total Strings*
	27/88	24/88	37/88	
12	27	48	111	186
13	29	52	120	202
14	32	56	130	217
15	34	60	139	233
16	36	64	148	248
17	38	68	157	264
18	41	72	167	279
19	43	76	176	295
20	45	80	185	310
21	47	84	194	326
22	50	88	204	341
23	52	92	213	357
24	54	96	222	372
25	56	100	231	388
26	59	104	241	403
27	61	108	250	419
28	63	112	259	434
29	65	116	268	450
30	68	120	278	465
31	70	124	287	481
32	72	128	296	496
33	74	132	305	512
34	77	136	315	527
35	79	140	324	543
36	81	144	333	558
37	83	148	342	574
38	86	152	352	589
39	88	156	361	605
40	90	160	370	620
41	92	164	379	636

Singles and doubles only

7¼ Octave ***n*-ET**	*Single*	*Double*	*Total Strings*
	27/88	61/88	
12	27	122	149
13	29	132	161
14	32	142	174
15	34	153	186
16	36	163	199
17	38	173	211
18	41	183	224
19	43	193	236
20	45	203	248
21	47	214	261
22	50	224	273
23	52	234	286
24	54	244	298
25	56	254	310
26	59	264	323
27	61	275	335
28	63	285	348
29	65	295	360
30	68	305	373
31	70	315	385
32	72	325	397
33	74	336	410
34	77	346	422
35	79	356	435
36	81	366	447
37	83	376	459
38	86	386	472
39	88	397	484
40	90	407	497
41	92	417	509

Singles only

7¼ Octave ***n*-ET**	*Total Strings*
12	88
13	95
14	103
15	110
16	117
17	125
18	132
19	139
20	147
21	154
22	161
23	169
24	176
25	183
26	191
27	198
28	205
29	213
30	220
31	227
32	235
33	242
34	249
35	257
36	264
37	271
38	279
39	286
40	293
41	301

APPENDIX VII (b) **String distribution and totals for 6¼ Octave Pianos with 12 to 41 octave divisions (compared to conventional string distribution)**

Grand string distribution in unisons

6¼ Octave ***n*-ET**	*Single*	*Double*	*Triple*	*Total Strings*
	8/76	5/76	63/76	
12	8	10	189	207
13	9	11	205	224
14	9	12	221	242
15	10	13	236	259
16	11	13	252	276
17	11	14	268	293
18	12	15	284	311
19	13	16	299	328
20	13	17	315	345
21	14	18	331	362
22	15	18	347	380
23	15	19	362	397
24	16	20	378	414
25	17	21	394	431
26	17	22	410	449
27	18	23	425	466
28	19	23	441	483
29	19	24	457	500
30	20	25	473	518
31	21	26	488	535
32	21	27	504	552
33	22	28	520	569
34	23	28	536	587
35	23	29	551	604
36	24	30	567	621
37	25	31	583	638
38	25	32	599	656
39	26	33	614	673
40	27	33	630	690
41	27	34	646	707

Upright string distribution of unisons

6¼ Octave ***n*-ET**	*Single*	*Double*	*Triple*	*Total Strings*
	15/76	11/76	50/76	
12	15	22	150	187
13	16	24	163	203
14	18	26	175	218
15	19	28	188	234
16	20	29	200	249
17	21	31	213	265
18	23	33	225	281
19	24	35	238	296
20	25	37	250	312
21	26	39	263	327
22	28	40	275	343
23	29	42	288	358
24	30	44	300	374
25	31	46	313	390
26	33	48	325	405
27	34	50	338	421
28	35	51	350	436
29	36	53	363	452
30	38	55	375	468
31	39	57	388	483
32	40	59	400	499
33	41	61	413	514
34	43	62	425	530
35	44	64	438	545
36	45	66	450	561
37	46	68	463	577
38	48	70	475	592
39	49	72	488	608
40	50	73	500	623
41	51	75	513	639

Thinned string distribution

6¼ Octave ***n*-ET**	**Single**	**Double**	**Triple**	**Total Strings**
	27/76	24/76	25/76	
12	27	48	75	150
13	29	52	81	163
14	32	56	88	175
15	34	60	94	188
16	36	64	100	200
17	38	68	106	213
18	41	72	113	225
19	43	76	119	238
20	45	80	125	250
21	47	84	131	263
22	50	88	138	275
23	52	92	144	288
24	54	96	150	300
25	56	100	156	313
26	59	104	163	325
27	61	108	169	338
28	63	112	175	350
29	65	116	181	363
30	68	120	188	375
31	70	124	194	388
32	72	128	200	400
33	74	132	206	413
34	77	136	213	425
35	79	140	219	438
36	81	144	225	450
37	83	148	231	463
38	86	152	238	475
39	88	156	244	488
40	90	160	250	500
41	92	164	256	513

Singles and doubles only

6¼ Octave ***n*-ET**	**Single**	**Double**	**Total Strings**
	27/76	49/76	
12	27	98	125
13	29	106	135
14	32	114	146
15	34	123	156
16	36	131	167
17	38	139	177
18	41	147	188
19	43	155	198
20	45	163	208
21	47	172	219
22	50	180	229
23	52	188	240
24	54	196	250
25	56	204	260
26	59	212	271
27	61	221	281
28	63	229	292
29	65	237	302
30	68	245	313
31	70	253	323
32	72	261	333
33	74	270	344
34	77	278	354
35	79	286	365
36	81	294	375
37	83	302	385
38	86	310	396
39	88	319	406
40	90	327	417
41	92	335	427

Singles only

7¼ Octave ***n*-ET**	*Total Strings*
12	76
13	82
14	89
15	95
16	101
17	108
18	114
19	120
20	127
21	133
22	139
23	146
24	152
25	158
26	165
27	171
28	177
29	184
30	190
31	196
32	203
33	209
34	215
35	222
36	228
37	234
38	241
39	247
40	253
41	260

9. Bibliography

In addition to the *Journal of the Acoustical Society of America* and similar sources, this paper was greatly aided by the bibliographies contained in Fletcher and Rossing's *The Physics of Musical Instruments* (see below), and the vast bibliography of microtonality compiled by Brian Mclaren, Manuel Op de Coul, Franck Jedrzejewski and Dominique Devie - which is available on the internet at *ftp://ella.mills.edu/ccm/tuning/papers/.* (For anyone who does not have internet access, a considerable portion of the latter bibliography is included in Mclaren's 'A brief history of microtonality in the 20th Century' (see below)). In addition, two lesser known journals are of special relevance: *Experimental Musical Instruments* (EMI, Box 784, Nicasio, CA 94946, USA, or - http://www.thecombine.com/emi); *Xenharmonikôn*, (available through Frog Peak Music, Box 1052, Lebanon NH 03766, USA).

Anderson Julian, 'La Note Juste' (article celebrating the 90th anniversary of Giacinto Scelsi's birth), *The Musical Times*, Vol. Cxxxvi, No. 1823, January 1995, pp. 22-7.
Arnold Eric B. & **Weinrich** Gabriel, 'Acoustical Spectroscopy of Violins', *Journal of the Acoustical Society of America*, 72 (6), December, 1982, pp. 1739-46.
Askenfelt Anders, ed., *Five Lectures on the Acoustics of the Piano*, Royal Swedish Academy of Music, Stockholm, 1990.
Baines Anthony, ed., *Musical Instruments through the Ages*, Faber & Faber, 1961.
Baines Anthony, *Brass Instruments*, Faber & Faber, 1980.
Baines Anthony, *Woodwind Instruments and their History*, Dover, New York, 1991.
Baines Anthony, *The Oxford Companion to Musical Instruments*, Oxford University Press, 1992.
Baines Anthony, *Brass Instruments: Their history and development*, Dover, New York, 1993.
Balzano Gerald J., 'The Group theoretic Description of 12-Fold and Microtonal Pitch Systems', *Computer Music Journal*, 1980, Vol. 4, No. 4, pp. 66-84.
Backus John, *The Acoustical Foundations of Music*, John Murray, London, 1970.
Backus John & **Hundley** T.C., 'Harmonic Generation in the Trumpet', *Journal of the Acoustical Society of America*, Vol. 49, No. 2 Part 2, 1971, pp. 510-519.
Banek Reinhold & **Scoville** Jon, *Sound Designs: A handbook of musical instrument building*, Ten Speed Press, Berkeley California, 1995.
Barbour J. Murray, *Tuning and Temperament - A historical survey*, East Lansing, Michigan State College Press, 1951.
Barlow Clarence, *Bus Journey to Parametron - all about Çogluotobüsisletmesi*, Feedback Papers 21-23, Feedback-Studio-Verlag, Köln, 1980.
Barlow, Clarence 'Two essays on theory', *Computer Music Journal*, 11 (1), 1987, pp. 44-60.
Bartolozzi Bruno, *New Sounds for Woodwind*, trans. Reginald Smith Brindle, Oxford University Press, 1971.
Bate Philip, *The Oboe: An outline of its history, development, and construction*, Norton, New York, 1975.
Bate Philip, *The Trumpet and the Trombone*, Ernest Benn, London, 1978.
Bate Philip, *The Flute: A study of its history, development, and construction*, Norton, New York, 1979.
Beament James, *The Violin Explained - Components, Mechanism, Sound*, Clarendon Press, Oxford, 1997.
Beck John H, ed., *Encyclopedia of Percussion*, Garland, 1995.
Benade A.H., 'On the Mathematical Theory of Woodwind Finger Holes', *Journal of the Acoustical Society of America*, Vol. 32, No. 12, December 1960, pp. 1591-1608.
Benade A.H. & **Larson** C.O., 'Requirements and techniques for measuring the musical spectrum of the clarinet', *Journal of the Acoustical Society of America*, 78 (5), November 1985, pp. 1475-98.
Benade, A.H. & **Kouzoupis** S.N., 'The clarinet spectrum: theory and experiment', *Journal of the Acoustical Society of America*, 83 (1), January 1988, pp. 292-304.
Benade, A.H. & **Lutgen** S. J., 'The saxophone spectrum', *Journal of the Acoustical Society of America*, 83 (5), May 1988, pp. 1900-7.
Benade Arthur H., 'On the Tone Colour of Wind Instruments', *Musical Acoustics*, (?) pp 99-103.
Benade Arthur H., *Fundamentals of Musical Acoustics*, Dover, 1990.
Benade Arthur H., *Horns, Strings, and Harmony*, Dover, 1992.
Benade Arthur H., 'Woodwinds: The Evolutionary Path Since 1700', *Galpin Society Journal*, 47, 1994, pp. 63-110.
Bennetts Kathryn, **Boustead** Donald, & **Bowman** Peter, *The Quarter-Tone Recorder Manual*, Moeck Verlag Celle No. 2084, 1997.
Blackwood Easley, 'Modes and Chord Progressions in Equal Tunings', *Perspectives of New Music*, Vol. 29, No. 2, Summer 1991, pp. 166-201.
Blackwood Easley, *The Structure of Recognisable Diatonic Tunings*, Princeton University Press, Princeton New Jersey, 1985.
Böhm Theobald, *An Essay on the Construction of Flutes*, Rudall Carte, London, 1882.
Böhm Theobald, *The Flute and Flute Playing, in acoustical, technical and artistic aspects*, trans. Dayton C. Miller, Dover, New York, 1964.
Bolton P., 'Resonans', *Fellowship of Makers of Historical Instruments Quarterly*, April 1995.
Boomsliter Paul, & **Creel** Warren, 'Extended reference: an unrecognised dynamic in melody', *Journal of Music Theory*, Vol 7, 1963, pp. 2-22.
Born Georgina, *Rationalizing Culture: IRCAM, Boulez, and the institutionalization of the musical avante-garde*, University of California Press, 1995.
Bosanquet R.H.M., *Temperament -An elementary treatise on musical intervals and temperament*, Macmillan and Co., 1876, pp. 20-21.

Boulez Pierre, *Stocktakings from an Apprenticeship*, trans. Stephen Walsh, Clarendon Press, Oxford, 1991.
Brindley Giles, 'The Logical Bassoon', *The Galpin Society Journal*, XXI, 1968, pp. 152-161.
Brindley G.S., 'Logical woodwind instruments', *Proceedings of the Royal Institution of Great Britain*, Vol. 48, 1975, pp. 207-13.
Brindley Giles, **Waterhouse** William & **Warrack** John, 'The Case for a Logical Bassoon', transcription of radio discussion broadcast on BBC Radio 3, 4th January 1971, 21.05-21.35.
Brown A. Edgar, 'An experimental bassoon', *Proceedings of the Institute of Acoustics*, Vol. 19, Part 5, 1997, p. 279-83.
Brown Judith C., 'Frequency ratios of spectral components of musical sounds', *Journal of the Acoustical Society of America*, Vol. 99, No. 2, 1996, p. 1210-18.
Brown Judith C. & **Vaughn** Kathryn V., 'Pitch centre of string instrument vibrato tones', *Journal of the Acoustical Society of America*, Vol. 100 (3), September 1996, pp. 1728-1735.
Brown Wes, 'An experimental slide bass clarinet', *Experimental Musical Instruments*, Volume VIII, #3, March 1993, pp. 92-4.
Brymer, Jack. Clarinet, rev. ed. London: Macdonald and JaneUs, 1979. XXX????
Burns Edward M., & **Ward** W. Dixon, 'Categorical perception - phenomenon or epiphenomenon: evidence from experiments in the perception of melodic musical intervals', *Journal of the Acoustical Society of America*, 63 (2), Feb. 1978, pp. 456-68.
Busoni Ferrucio, 'Sketch of a New Esthetic of Music', in *Three Classics in the Aesthetic of Music*, Dover, New York, 1962.
Butler, David *The Musician's Guide to Perception and Cognition*, Schirmer, 1992.
Campbell Murray, & **Greated** Clive, *The Musician's Guide to Acoustics*, Dent, 1987.
Carlos Wendy, 'Tuning at the cross-roads', *Computer Music Journal*, Vol. 11, No1, Spring 1987, pp. 29-43.
Chadabe Joel, *Electric Sound - the past and promise of electronic music*, Prentice Hall, 1997.
Chalmers John H. Jr., *Divisions of the Tetrachord*, Frog Peak Music, Hanover NH, 1993.
Cohen Elizabeth, 'Some effects of inharmonic partials on interval perception', *Music Perception*, Spring 1984, Vol. 1, No. 3, pp. 323-349.
Conklin Harold A. Jr., 'Design and tone in the mechanoacoustic piano. Part II., Piano structure', *Journal of the Acoustical Society of America*, 100 (2), Pt. 1, August 1996, pp. 695-708.
Cook Nicholas, *Music Imagination and Culture*, Clarendon Press, Oxford, 1992.
Cranmer Margaret, 'Jankó (Paul von)', *The New Grove Dictionary of Musical Instruments,* MacMillan.
Crombie David, *Piano: Evolution Design and Performance*, Balafon 1995.
Daniels Arthur, 'Microtonality and Mean-Tone Temperament', *Journal of Music Theory*, 9, 1965, pp. 2-51.
Darreg I. & **McLaren** B., 'Biases in Xenharmonic Scales', *Xenharmonikôn 13*, Spring 1991, pp. 5-19.
Darreg Ivor, 'The quarter-tone question revisited', *Xenharmonikôn*, No. 2, 1974.
Darreg Ivor ,'Table of errors in equally-tempered intervals', *Xenharmonikôn* 10, 1987.
Davies Hugh, 'Vermeulen flute', *The New Grove Dictionary of Musical Instruments,* MacMillan.
Davies Hugh, 'Microtonal instruments', *The New Grove Dictionary of Musical Instruments,* MacMillan, pp. 653-9.
Davies John Booth, *The Psychology of Music*, Hutchinson & Co., 1978.
Deutsch Dianna, *The Psychology of Music*, Academic Press Inc., 1982
Dick Robert, *The Other Flute - A performance manual of contemporary techniques*, Multiple Breath Music Company, 1989.
Dillon James, 'Speculative instruments', *Eonta*, Vol. 2, No. 1, 1993/4, pp. 32-6.
Doty David, *The Just Intonation Primer*, Just Intonation Network, 1994.
Ehrlich Cyril, *The Piano: a history*, Dent, London, 1976.
Erlich Paul, 'Tuning, tonality and twenty-two-tone temperament', *Xenharmonikôn* 17, Spring 1998, pp. 12-40.
Farmer Gerald James, *Multiphonics and other Contemporary Clarinet Techniques,* Rochester, New York, Shall-u-mo Publications, 1982.
Fletcher N. H., 'Mode locking in nonlinearly excited inharmonic musical oscillators', *Journal of the Acoustic Society of America*, 64 (6), Dec. 1978, pp. 1566-9.
Fletcher Neville & **Rossing** Thomas, *The Physics of Musical Instruments,* Addison Wesley, 1993.
Fletcher Neville H., *Nonlinearity, complexity, and the sounds of musical instruments*, (forthcoming article).
Fokker Adriaan Daniël, *Selected Musical Compositions (1948-72)*, ed. Rudolf Rasch, Corpus Microtonale, The Diapason Press, Utrecht, 1987.
Fuks Leonardo & **Sundberg** Johan, 'Blowing pressures in reed woodwinds', *Proceedings of the Institute of Acoustics*, Vol. 19, Part 5, 1997, pp. 273-8.
Fuks Leonardo, 'Prediction and measurements of exhaled air effects in the pitch of wind instruments', *Proceedings of the Institute of Acoustics*, Vol. 19, Part 5, 1997, pp. 373-8.
Geary J.M., 'Consonance and dissonance of inharmonic tones', Journal of the Acoustical Society of America, 67 (5), May 1980, pp. 1785-9.
Gibson O. Lee, *Clarinet Acoustics*, Indiana University Press, 1994.
Gill, Dominic, ed., *The Book of the Piano*, Phaidon, Oxford, 1981.
Gilmore Bob, 'On Harry Partch's *Seventeen Lyrics by Li Po*', *Perspectives of New Music*, Vol. 40, No. 1, 1993, pp. 22-58.
Gilmore Bob, 'Changing the metaphor: ratio models of musical pitch in the work of Harry Partch, Ben Johnston, and James Tenney', *Perspectives of New Music*, Vol. 30-31, 1995, pp. 459-503.
Gilmore Bob, *Harry Partch - a biography*, Yale University Press, 1998.
Goehr Alexander, 'The Survival of the Symphony', *The Listener*, 19 November 1987, Vol. 118, No. 3038, and the following five issues (to 3043).
Goehr Alexander, *Finding the Key*, Faber and Faber, 1998.

Good Edwin M., *Giraffes, Black Dragons and Other Pianos, A technological history from Cristofori to the modern concert grand*, Stanford University Press, 1982.
Goossens, Leon & **Roxburgh** Edwin, *Oboe*, Kahn & Averill, London, 1993.
Gordon John W. & **Grey** John M., 'Perception of spectral modifications on orchestral instrument tones', *Computer Music Journal*, Vol. 2, No. 1, pp. 24-31.
Grady Kraig, 'Combination-product set patterns', *Xenharmonikôn*, IX, 1986, unpaged.
Gergory Robin, *The Horn: A comprehensive guide to the modern instrument & its music*, Faber & Faber, 19??.
Griffiths Paul, *Modern Music and After: Directions since 1945*, Clarendon Paperbacks, Oxford University Press, 1995.
Hartmann W.M., **McAdams** Stephen, **Gerzso** Andrew & **Boulez** Pierre, 'Discrimination of spectral density', *Journal of the Acoustical Society of America*, 79 (6), June 1986, pp. 1915-25.
Hartmann William Morris, 'On the origin of the enlarged melodic octave', *Journal of the Acoustical Society of America*, 93 (6), June 1993, pp. 3400-9.
Hartmann William M. & **Doty** Sandra L., 'On the pitches of the components of a complex tone', *Journal of the Acoustical Society of America*, (1), January 1996, pp. 567-78.
Hartmann William M., *Signals, Sound and Sensation*, AIP (American Institute of Physics) Press, Woodbury New York, 1997.
Harvey Paul, *Saxophone*, Kahn and Averill, London, 1995.
Heaton Roger, 'The contemporary clarinet', in *The Cambridge Companion to the Clarinet*, Cambridge University Press, ed. Colin Lawson.
Heim David Bruce, *Practical Tuning, Temperament and Conditioning for Hornists and Other Wind Instrumentalists: Understanding and attaining intonational flexibility in musical performance*, unpublished Masters thesis, University of Tulsa, 1990.
Helmholtz Hermann von, *On the Sensations of Tone as a Psychological basis for the Theory of Music*, (1877) trans. Alexander James Ellis (1885), Dover, 1985.
Herbert T. & **Wallace** J., eds., *The Cambridge Companion to Brass Instruments*, Cambridge University Press, 1997.
Hesse Horst-Peter, 'Breaking into a new world of sound: reflections on the ekmelic music of the Austrian composer Franz Richter Herf (1920-1989)', *Perspectives of New Music*, 1992,Vol. 1, pp. 212-234.
Hirschberg A., **Kergomard** J. & **Weinrich** G., eds., *Mechanics of Musical Instruments*, Springer Verlag, 1995.
Holliger Heinz, *Studies for playing Avant-Garde Music*, Pro Musica Nova, Breitkopf & Hartel, 1972.
Hopkin Bart, *Gravichords, Whirlies and Pyrophones: Experimental musical instruments* (Book and CD), Ellipsis Arts, 1996.
Hopkin Bart, *Musical Instrument Design - Practical Information for Instrument Design*, See Sharp Press, Tucson, Arizona, 1996.
Hopkin Bart et al., 'Computer control for acoustic instruments', *Experimental Musical Instruments*, Volume VII, #1, June 1991, pp. 1 & 8.
Hopkin Bart, 'A day in the patent library', *Experimental Musical Instruments*, Vol. VII, #1, June 1991, pp. 16-18.
Hopkins G.W., 'Schönberg and the 'logic' of atonality', *Tempo*, No. 94, 1970, pp. 15-20.
Horner Andrew and **Ayers** Lydia, 'Common tone adaptive tuning using genetic algorithms', *Journal of the Acoustical Society of America*, 100 (1), July, 1996, pp. 630-40;
Hotteterre Jacques, *Principles of the Flute, Recorder and Oboe*, trans. and ed. David Lasocki, F. A. Praeger, New York, 1968.
Houtsma Adrianus J. M., 'Pitch salience of various complex sounds', *Music Perception*, Spring 1984, Vol. 1, No. 3, pp. 296-307.
Houtsma A.J. M. & **Goldstein** J.L., 'The central origin of the pitch of complex tones: evidence from musical interval recognition', *Journal of the Acoustical Society of America*, Vol. 51, No. 2 (Part 2), 1972, 520-9.
Howard David M. & **Angus** James, *Acoustics and Psychoacoustics*, Focal Press, 1996.
Howell Thomas, *The Avant-Garde Flute: A Handbook for Composers and Flutists*, Berkeley, University of California Press, 1974.
Hutchins C.M, 'Founding a family of fiddles', *Physics Today*, 20, 1967, pp. 23-7.
Hutchins C.M, 'The new violin family', in *Sound Generation in Winds, Strings, Computers*, Royal Swedish Academy of Music, Stockholm, pp. 182-203.
Joppig Gunther, *The Oboe and the Bassoon*, trans. Alfred Clayton, Amadeus Press, Portland, Ore., 1988.
Jorgensen Owen H., *Tuning*, Michigan State University Press, East Lansing, 1991.
Kameoka A. & **Kuriyagawa** M., 'Consonance Theory Part I: consonance of dyads', *Journal of the Acoustical Society of America*, Vol. 45, No. 6, 1969, pp. 1451-9.
Kameoka A. & **Kuriyagawa** M., 'Consonance theory part II: consonance of complex tones and its calculation method', *Journal of the Acoustical Society of America*, Vol. 45, No. 6, 1969, pp. 1460-9.
Keislar Douglas, 'History and principles of microtonal keyboards', *Computer Music Journal*, Vol. 11, No. 1, Spring 1987, pp. 18-28.
Kergomard J. and **Meynial** X., 'Systèmes micro-intervalles pour les instruments de musique à vent avec trous latéraux', *Journale Acoustique* 1, 1988, pp. 255-70.
Kirck George T., 'Computer realization of extended just intonation compositions', *Computer Music Journal*, Vol. 11, No. 1, Spring 1987, pp. 69-75.
Kottick Edward L. & **Lucktenberg** George, *Early Keyboard Instruments in European Museums*, Indiana University Press, 1997.
Krantz Richard J. & **Douthett** Jack, 'A measure of the reasonableness of equal-tempered musical scales', *Journal of the Acoustical Society if America*, 95 (6), June 1994, pp. 3642-50.
Kroll Oskar, *The Clarinet*, translated Hilda Morris (ed. Anthony Baines), Batsford, London, 1968.
Lawson Colin, *The Cambridge Companion to the Clarinet*, Cambridge University Press, 1995.
Langwill Lyndesay Graham, *The Bassoon and Contrabassoon*, Norton, New York, 1965.
Leedy Douglas, 'Tuning Systems', in **Hitchcock** Wiley & **Sadie** Stanley, eds., *The New Grove Dictionary of American Music*, ??, 1986.

Leedy Douglas, ‘Giving number a voice’, *Xenharmonikôn* 15, 1993, pp. 56-9.
Levarie Siegmund & **Levy** Ernst, *Tone: A study in musical acoustics*, Kent State University Press, 1980.
Lindley Mark, *Lutes, Viols and Temperaments*, Cambridge University Press, date?
Lindley Mark & **Turner-Smith** Ronald, *Mathematical Models of Musical Scales - A new approach*, Verlag für systematische Musikwissenschaft, Bonn, 1993.
Lindley Mark, ‘Just intonation’, *The New Grove Dictionary of Musical Instruments*, MacMillan.
Lindley Mark, ‘Temperaments’, *The New Grove Dictionary of Musical Instruments*, MacMillan.
Lindley Mark, ‘J.S. Bach’s tunings’, *Musical Times*, 1986, pp. 721-5.
Loosen Franz, ‘Intonation of solo violin performance with reference to equally tempered, Pythagorean, and just intonations’, *Journal of the Acoustical Society of America*, 93 (1), January 1993, pp. 525-39.
Levenson Thomas, *Measure for Measure - how music and science together have explored the universe*, Oxford University Press, 1997.
Lewin David, *Generalized Musical Intervals and Transformations*, Yale University Press, New Haven and London, 19??
Mandelbaum M. Joel, ‘Multiple divisions of the octave and the tonal resources of 19-tone temperament: Chapter 11 - Joseph Yasser’, reprinted in *Xenharmonikôn* 4, 1975.
Maneri Joseph and **Van Duyne** Scott A.: *Preliminary Studies in the Virtual Pitch Continuum*, Accentuate Music, Plainview, NY, 1986 (not obtained).
Marcuse Sibyl, *Musical Instruments: A comprehensive dictionary*, Country Life, London, 1966.
Martin D.W. & Ward W.D., ‘Subjective evaluation of musical scale temperament in pianos’, Journal of the Acoustical Society of America, Vol. 333, No. 5, May 1961, pp. 582-5.
Mathieu W.A., *Harmonic Experience: Tonal harmony from its natural origins to its modern expression*, Inner Traditions International, Vermont, 1997.
Matthews M. V. and **Pierce** J.R., ‘The Bohlen-Pierce scale’, *Current Directions in Computer Music Research*, ed., M. V. Matthews and J.R. Pierce, MIT Press, 1991.
Matthews Max V., **Pierce** John R., **Reeves** Alyson & **Roberts** Linda A., ‘Theoretical and experimental explorations of the Bohlen-Pierce scale’, Journal of the Acoustical Society of America, 84 (4), October 1988, pp. 1214-22.
McClure A.R., ‘An extended meantone organ’, *The Organ*, 30, 1950-1, pp. 139-49.
McLaren B., ‘The uses and characteristics of non-octave scales’, *Xenharmonikôn*, 14, Spring 1993, pp. 12-22.
McLaren B., ‘The uses and characteristics of non-just non-equal-tempered scales’, *Xenharmonikôn*, 15, Autumn 1993, pp. 27-41.
Mclaren B., ‘A brief history of microtonality in the twentieth century’, *Xenharmonikôn* 17, Spring 1998, pp. 57-110.
McGill Howard, *Intelligent Woodwind*, unpublished dissertation, 1992, Cambridge University.
Meeus Nicolas, ‘Enharmonic Keyboard’, *The New Grove Dictionary of Musical Instruments*, MacMillan.
Meylan Raymond, *The Flute*, Trans Alfred Clayton, Amadeus Press, Portland, Ore., 1988.
Montagu Jeremy, *The World of Romantic and Modern Musical Instruments*, David & Charles, London, 1981.
Moreno Enrique**,** *Expanded Tunings in Contemporary Music: Theoretical Innovations and Practical Applications*, Edwin Melon Press, 1992.
Morley-Pegge R., *The French Horn: Some notes on the evolution of the instrument and its technique*, Ernest Benn, London, 1960.
Morris Gareth, *Flute Technique*, Oxford University Press, New York, 1991.
Myers Arnold, ‘Design, technology and manufacture since 1800’, in *Cambridge Companion to Brass Instruments*, edited T. Herbert & J. Wallace, Cambridge University Press, 1997.
Nederveen C.J., *Acoustical Aspects of Wind Instruments*, Frits Knuf, Amsterdam, 1969.
Neuwirth Erich, *Musical Temperaments* (Book with CD), Springer Verlag, 1997
Norbeck Edwin, ‘Computer-assisted woodwinds’, *Woodwind World*, December 1973, pp. 8-9 & 34.
Ouellette Fernand, *Edgar Varese: A musical biography*, trans. Derek Coltman, Caldar & Boyars, 1973.
Palmieri Robert, *Piano Information Guide: An aid to research,* Music research and information guides, Vol. 10, Garland, 1989.
Palmieri Robert, ed., *Encyclopaedia of the Piano*, Garland, 1996.
Parncutt Richard, *Harmony: A psychoacoustical approach*, Springer Verlag, Berlin, 1989.
Parncutt Richard & **Strasburger** Hans, ‘Applying psychoacoustics in composition: ‘harmonic’ progressions of ‘nonharmonic’ sonorities’, *Perspectives of New Music*, Vol. 32, No. 2, Summer 1994, pp. 88-129.
Partch Harry, *Genesis of a Music*: *An account of a creative work, its roots and its fulfillments*, Da Capo, New York, 1979.
Pay Anthony, ‘The mechanics of playing the clarinet’, in *The Cambridge Companion to the Clarinet*, Cambridge University Press, 1997.
Phelan James & **Brody** Mitchell D., *The Complete Guide to the Flute*, Conservatory Publication, Boston, 1980.
Pino David, *The Clarinet and Clarinet Playing*, Scribner's & Sons, New York, 1980.
Plomp R. and **Levelt** W.J.M., ‘Tonal consonance and critical bandwidth’, *Journal of the Acoustic Society of America*, 1965, Vol. 38, pp. 548-60.
Polansky Larry, ‘Paratactical tuning: An agenda for the use of computers in experimental just intonation’, *Computer Music Journal*, Vol. 11, No. 1, Spring 1987, pp. 61-8.
Pierce John R., *The Science of Musical Sound*, W.H. Freeman and Co., 1992.
Quantz Johann Joachim, *On Playing the Flute*, trans. Edward R. Reilly, Schirmer, New York, 1985.
Rasch Rudolf, ‘Theory of Helmholtz-Beat Frequencies’, *Music Perception*, Spring 1984, Vol. 1, No. 3, 308-22.
Rasch Rudolf, ed., *Adriaan Daniiel Fokker - Selected compositions,* Corpus Microtonale, vol. 1:, Utrecht, 1987.
Rasch Rudolf, ‘Review of Gardner Read, *20th-Century Microtonal Notation*’, *Perspectives of New Music*, Vol. 39, No. 1, 1992, pp. 258-262.

Rapoport Paul, 'The Structural Relationships of Fifths and Thirds in Equal Temperaments', *Journal of Music Theory*, Fall 1993, Vol. 37, No. 2, pp. 351-389.
Rapoport Paul, 'Just shape, nothing central', Musicworks 60, Fall, 1994, pp. 42-9.
Read Gardner, *Contemporary Instrumental Techniques*, Schirmer, New York, 1976.
Read Gardner, *20th Century Microtonal Notation*, Greenwood Press, 1990.
Reblitz Arthur A., *Piano Servicing, Tuning & Rebuilding*, The Vestal Press, New York, 1993.
Rehfeldt Philip, *New Directions for the Clarinet*, University of California Press, 1994.
Rendall F. Geoffrey, *The Clarinet: Some Notes Upon Its History and Construction*, Norton, New York, 1971.
Rensch Roslyn, *Harps and Harpists*, Duckworth, 1989.
Ripin Edwin M. et al., *Early Keyboard Instruments* - (*The New Grove musical instrument series*), Macmillan,1989.
Risset Jean-Claude & Mathews Max V., 'Analysis of musical instrument tones', *Physics Today*, Vol.22, No. 2, 1969, pp. 23-30.
Ritsma R.,'Frequencies dominant in the perception of the pitch of complex sounds', *Journal of the Acoustical Society of America*, Vol. 42, No. 1, 1967, pp. 191-98.
Roads Curtis, Editorial - Special Issue on Microtonality, *Computer Music Journal*, Vol. 11, No. 1, Spring 1987.
Roberts Linda A. and **Mathews** Max V., 'Intonation sensitivity for traditional and non-traditional chords', *Journal of the Acoustical Society of America*, 75, (3), March 1984, pp. 952-9.
Robinson Trevor, *The Amateur Wind Instrument Maker*, The University of Massachusetts Press, 1980.
Roederer J.G., *The Physics and Psychoacoustics of Music: An introduction*, Springer Verlag, New York, 1995.
Rossing Thomas D., *The Science of Sound*, Addison-Wesley Publishing Co., 1990.
Sadie Stanley, ed., *The New Grove Dictionary of Music and Musicians*, Macmillan, London,1980.
Sadie Stanley, ed., *The New Grove Dictionary of Musical Instruments and Instrument Makers*, Macmillan, London, 1984.
Salzedo Leonard & **Sharp** Peter E.M., 'The logical bassoon', *The Times Educational Supplement*, Friday, 7th February, 1969.
Schumacher Robert T., 'Measurements of some parameters of bowing', Journal of the Acoustical Society of America, 96 (4), October 1994, pp. 1985-1998.
Secor George, 'An approach to the construction of microtonal valved brass instruments - the French Horn', *Xenharmonikôn*, Vol. 3, No. 1, Spring 1976, pp. 1-3.
Sethares William A., *Tuning, Timbre, Spectrum, Scale*, Springer Verlag, 1997.
Sethares William A., 'Adaptive Tunings for Musical Scales', *Journal of the Acoustical Society of America*, 96 (1), July, 1994, pp. 10-18;
Sethares William A., 'Local consonance and the relationship between timbre and scale', *Journal of the Acoustic Society of America*, 94 (3), Sept. 1993, pp. 1218-1228.
Sethares Bill, 'Relating Timbre and Tuning', *Experimental Musical Instruments*, Volume IX, #2, December 1993, 62-9.
Schmidt Jim, 'The custom made flute', *Experimental Musical Instruments*, Vol. III, No. 5, February 1988, pp. 89-91.
Schönberg Arnold**, '**The future of orchestral instruments' (1924), in *Style and Idea*, edited by Leonard Stein, Faber & Faber, 1975, pp. 322-6.
Schuck and **Young**, 'Observations on the vibrations of piano strings', *Journal of the Acoustical Society of America*, 15, 1943, pp. 1-11.
Siepman Jeremy, *The Piano*, Everyman, 1996.
Sims Ezra, 'Tonality in my Harmonically based Microtonality', lecture presented at the 2nd Naturton-Symposion, Heidelberg, October 1992, (unpublished, 26 pages).
Sims Ezra, 'Reflections on This and That (Perhaps a Polemic)', *Perspectives of New Music*, Vol. 29, No. 1, Winter 1991, pp. 236-257.
Sims Ezra, 'Yet Another 72-Noter', *Computer Music Journal*, Vol. 12, No. 4, Winter 1988, pp. 28-45.
Sims Ezra, 'Harmonic Ordering in Quintet: A use of harmonics as horizontal and vertikal determinants', *Mikrotone III*, ed., Horst-Peter Hesse, Editions Helbling, Zunsbruck, 1990, pp. 123-137.
Sitsky Larry, *Music of the Repressed Russian Avant-garde 1900-1929*, Greenwood Press, Westport, 1994.
Skei Allen B., *Woodwind, Brass, and Percussion Instruments of the Orchestra: A bibliographic guide*, Garland, New York, 1985.
Sloboda John A., *The Musical Mind : The cognitive psychology of music*, Oxford, pp. 23-8.
Smith R.A. & **Mercer** D.M.A., 'Possible causes of woodwind tone-colour', *Journal of Sound Vibration*, Vol. 32, 1974, pp. 347-58.
Stowell Robin, ed., *The Cambridge Companion to the Violin*, Cambridge University Press, 1997.
Sundberg J.E.F. & **Lindqvist** J., 'Musical Octaves and Pitch', *Journal of the Acoustical Society of America*, Vol. 54, No. 4, 1973, pp. 922-9
Sundberg J., ed., *Harmony and Tonality*, Royal Swedish Academy of Music, No. 54, 1987.
Sundberg J., *The Science of Musical Sounds*, Academic Press, 1991.
Swift G.W & **Yunik** M., 'Modifying a trumpet to play 19 tone music', *Interval*, Spring-Summer 1980, p. 8.
Tenney J., *A History of 'Consonance' and 'Dissonance'*, Excelsior Music Publishing Co., New York, 1988.
Terhardt Ernst, 'Pitch shifts of harmonics, an explanation of the octave enlargement phenomenon', Proceedings of the 7th ICA, Budapest, 3, pp. 621-4.
Terhardt Ernst, 'Pitch, Consonance and Harmony', *Journal of the Acoustical Society of America*, 1974, Vol. 55, No 5, May 1974, pp. 1061-9.
Terhardt Ernst, 'The concept of musical consonance: a link between music and psychoacoustics', *Music Perception*, Spring 1984, Vol. 1, No. 3, pp. 276-295.
Thurston Frederick, Clarinet Technique, Oxford University Press, New York, 1977.
Schafer Tillman & **Piehl** W.A., 'Musical instruments in nineteen-tone equal temperament', *Journal of the Acoustical Society of America*, 19 (4), 1947, p.730.

Toff Nancy, *The Development of the Modern Flute*, Taplinger Publishing Company, New York, 1979.
Veale Peter, et al., *The Techniques of Oboe Playing: A Compendium with Additional Remarks on the Oboe d'Amore and the Cor Anglais*, Bärenreiter, 1995.
Vogel Martin, *On the Relations of Tone*, translated from the German by Vincent J. Kisselbach, edited by Carl A. Poldy, Verlag für systematische Musikwissenschaft, Bonn, 1993.
Vogel Martin, *Die Intonation der Blechbläser - neue wege im metallblas-instrumentenbau*, Düsseldorf, 1961, (not obtained).
Vorhees Jerry L., 'Notes on the Fingering Systems of 'Böhm' bassoons', *Galpin Society Journal*, No 29, 1976, pp. 51-63.
Wagge H. M., 'The intelligent keyboard', 1/1 (4):1, pp. 12-13;
Waters Paul, *A method of fretting the viol in 31 note equal temperament. Including performance of music composed by Nicola Vincentino.* Project completed as part of B.Sc., London Guildhall University, May 1997.
Ward W.D. & **Martin** D.W., 'Psychophysical comparison of just tuning and equal temperament in sequences of individual tones', Journal of the Acoustical Society of America, Volume 33, No. 5, May 1961, pp. 586-??
Ward W. Dixon, 'Musical Perception', in Jerry V. Tobias, ed., *Foundations of Auditory Theory*, New York, 1970, pp. 407-47.
Weinrich G., 'Coupled piano strings', *Journal of the Acoustical Society of America*, Vol. 62, 1977, pp. 1474-84.
Weyer R. D., 'Time-varying Amplitude-Frequency-Structures in the Attack Transients of Piano and Harpsichord Sounds - II', *Acoustica*, Vol. 36., No. 4, 1976-7, pp. 241-258.
Wightman Frederick L. and **Green** David M., 'The perception of Pitch', 1974, *American Scientist*, Vol. 62, pp. 208-215.
Wilkinson Scott R, *Tuning In*, Hal Leonard Books, 1988.
Wilson Erv, 'D'allessandro, like a hurricane', *Xenharmonikôn*, XII, Spring 1989, pp. 1-39.
Wilson Ervin, 'The Bosanquetian 7-rank keyboard after Poole and Brown', *Xenharmonikôn,* No. 1, Spring 1974, p. 3.
Wilson Ervin, 'On linear notations and the Bosanquet keyboard', *Xenharmonikôn* 3, Vol. 2, No. 1, 1975.
Winter R.S., 'Striking it rich: The significance of striking points in the evolution of the romantic piano', *Journal of Musicology,* 6, 1988, pp. 267-292.
Wishart Trevor, 'Connections', in 'Leaving the Twentieth Century - Ideas and Visions of New Musics', *Contemporary Music Review*, ed. Leigh Landy and Frank Denyer, Vol. 15, Parts 3-4, pp. 93-6.
Wood James, 'Microtonality: aesthetics and practicality', *Musical Times*, June 1986, p. 328-30.
Wood James, (project co-ordinator, various contributors) *Microtonality Manual*, co-produced by *The Centre for Microtonal Music* and the *Society for the Promotion of New Music*, London, July 1991.
Yasser Joseph, *A Theory of Evolving Tonality*, Da Capo Press, New York, 1975
Young Gayle, 'Pitch probe. Musical instruments for unusual tuning systems', *Contemporary Music Review*, Vol. 7, Part 1, p. 56, 1992, pp. 51-64.